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STATIONARY WAVES

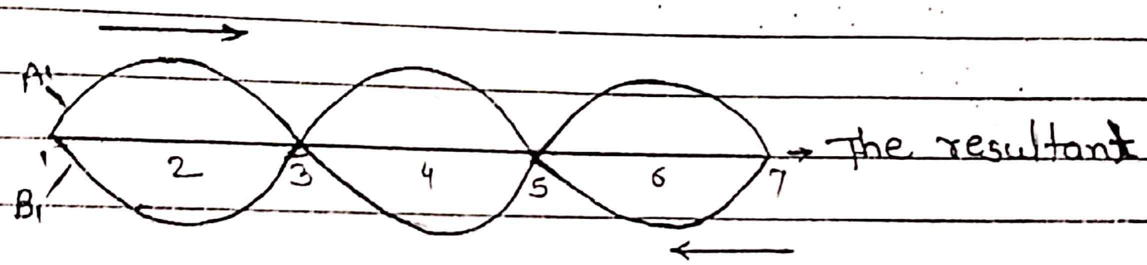
Introduction :-

When two simple harmonic waves of the same amplitude, frequancy and time peroid travel in a opposite directions in a straight line, the resultant wave is obtained is called a stationary or standing wave.

The formation of stationary wave is due to the superposition of the two waves on the particles of the medium.

The formation of stationary wave can be represented graphically as follows :-

1)



Consider two wave trains A and B of the same amplitude, frequency and wave length travelling in opposite directions.

At an instant time $t=0$ The resultant displacement curve is a straight line. All the particles of the medium are at their mean positions.

At $t = T/4$, the wave A will advance through a distance $\lambda/4$ towards right. and the wave B will advance through a distance $\lambda/4$ towards left.

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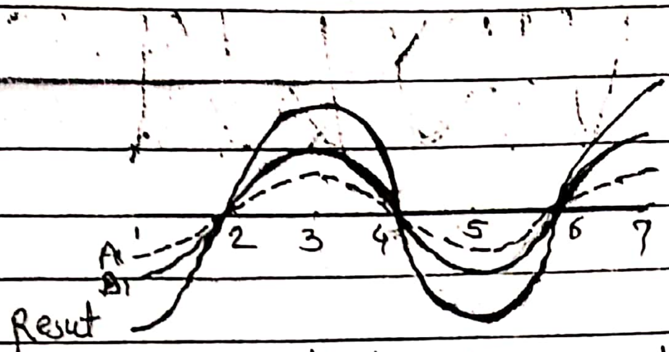
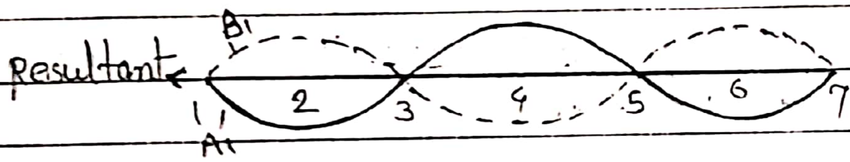
ii)



At particles 1,3,5 and 7 great their extre position and particles 2,4 and 6 are at their mean position.

At time $t = T/2$, the wave A will be advanc through a distance $\lambda/2$ towords ~~a~~right and the wave B will be advance through a distance $\lambda/2$ towords left. $(t=0)$

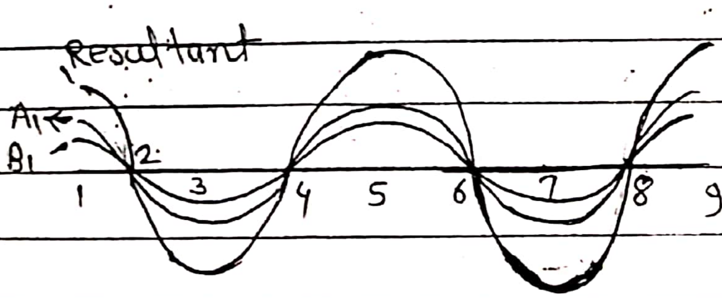
iii)



All the particle of the mean position in the medium.

At time $t = 3T/4$, the wave will advance thro ugh a distance $3\lambda/4$ towords right and the wave B will advance through a distance $3\lambda/4$ towords left.

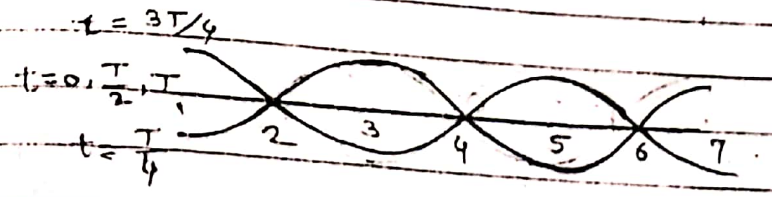
iv)



7 are the extreme position and 2,4,6 are at their mean position.

At the time $t = T$, the wave will be advance through a distance λ towords right and the wave B will advance through a distence λ towords left

The all particles are mean positions.



The position of the particles 2, 4, 6 etc. which always remain at their mean positions are called nodes. Node are position of zero displacement and a maximum strain.

The position of the particles 1, 3, 5, 7 etc. which vibrate simple harmonicaly with maximum amplitude are called a antinodes. At the antinodes strain is minimum.

properties of stationary wave :-

i) In stationary wave, different particles move with different amplitude.

ii) In a stationary wave, the particles at nodes are always remains in rest.

iii) In stationary wave, all the particle cross their mean position together.

iv) In stationary wave, all the particle between two successive node reach their extremposition.

v) In a stationary wave, energy of one region is always in that region.

vi) Nodes are the position where the particles are at their mean position having maximum strain.

vii) Antinodes are the position where the particles vibrate with maximum amplitude having minimum strain.

viii) Nodes and antinodes from alternately.

xi) velocity and accleration of particle which is seprated by λ is same at same instant.

x) D[illegible]jacent node & adjacent antinode is $\lambda/2$.

Analytical treatment of stationary Waves:
[closed end & open end pipe in other end]

Stationary waves are formed in an open end organ pipe or a closed end organ pipe. stationary waves are ~~called~~ with a stretched string fixed at one end and free at the other end or fixed at the other end.

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a) Open end organ pipe or string free at the other end:-

consider, the simple haromic wave is a given by the equation,

$$\boxed{y_1 = a \sin \frac{2\pi}{\lambda}(vt - x)} \quad ---- (1)$$

Incident wave.

The displacement of the ~~simple~~ particle at the same instant due to the reflected wave is given by

$$\boxed{y_2 = a \sin \frac{2\pi}{\lambda}(vt + x)} \quad ---- (11)$$

Reflected Wave.

The resultant displacement of the wave,

$$y = y_1 + y_2$$

$$\boxed{y = \left[a \sin \frac{2\pi}{\lambda}(vt - x) + a \sin \frac{2\pi}{\lambda}(vt + x) \right]} \quad - (3)$$

$$y = 2a \left[\sin \frac{2\pi}{\lambda}(vt - x) + \sin \frac{2\pi}{\lambda}(vt + x) \right]$$

$$y = 2a \sin \frac{2\pi}{\lambda}\left(\frac{vt - x + vt + x}{2}\right) \cdot \cos \frac{2\pi}{\lambda}\left(\frac{vt - x - vt - x}{2}\right)$$

$$\left[\because \sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C+D}{2}\right)\right]$$

$$y = 2a \sin \frac{2\pi}{\lambda}\left(\frac{2vt}{2}\right) \cos \frac{2\pi}{\lambda}\left(\frac{-2x}{2}\right)$$

$$y = 2a \sin \frac{2\pi}{\lambda}(vt) \cos \frac{2\pi}{\lambda}(-x)$$

$$[\because \cos(-\theta) = \cos\theta]$$

$$\boxed{y = 2a \sin \frac{2\pi}{\lambda}(vt) \cdot \cos \frac{2\pi}{\lambda}(x)} \quad \text{---- (IV)}$$

or

$$y = y_1 + y_2$$

$$y = \left[a \sin \frac{2\pi}{\lambda}(vt - x) + a \sin \frac{2\pi}{\lambda}(vt + x)\right] \quad \text{—— (3)}$$

$$y = a\left\{\sin \frac{2\pi}{\lambda}(vt - x) + \sin \frac{2\pi}{\lambda}(vt + x)\right]$$

$$y = a\left\{\left[\sin \frac{2\pi}{\lambda}(vt) \cos \frac{2\pi}{\lambda}(x) - \cos \frac{2\pi}{\lambda}(vt) \sin \frac{2\pi}{\lambda}(x)\right]\right.$$

$$+ \left[\sin \frac{2\pi}{\lambda}(vt) \cos \frac{2\pi}{\lambda}(x) + \cos \frac{2\pi}{\lambda}(vt) \sin \frac{2\pi}{\lambda}(x)\right]$$

$$y = a\left\{\sin \frac{2\pi}{\lambda}(vt) \cos \frac{2\pi}{\lambda}(x) - \cancel{\cos \frac{2\pi}{\lambda}(vt) \sin \frac{2\pi}{\lambda}(x)}\right.$$

$$\left. + \sin \frac{2\pi}{\lambda}(vt) \cos \frac{2\pi}{\lambda}(x) + \cancel{\cos \frac{2\pi}{\lambda}(vt) \sin \frac{2\pi}{\lambda}(x)}\right\}$$

$$y = a\left\{\sin\frac{2\pi}{\lambda}(vt)\cos\frac{2\pi}{\lambda}(x) + \sin\frac{2\pi}{\lambda}(vt)\cos\frac{2\pi}{\lambda}(x)\right\}$$

$$\boxed{y = 2a\sin\frac{2\pi}{\lambda}(vt)\cos\frac{2\pi}{\lambda}(x)} \quad \text{---} (v)$$

above eqn (iv)

The resultant vibration of a particles representing a simple harmonic wave of the same time peroid. The amplitude of the resultant wave is given by

$$\boxed{A = 2a\cos\frac{2\pi x}{\lambda}} \quad \text{------} (v)$$

The resultant amplitude is a function of x. The velocity of the particle at the any instant of time is dy/dt.

Diff' w.r.to time in equatin (iv)

$$\frac{dy}{dt} = \frac{d}{dt}\left[2a\sin\frac{2\pi}{\lambda}(vt)\cos\frac{2\pi}{\lambda}(x)\right]$$

$$= 2a\frac{d}{dt}\left[\sin\frac{2\pi}{\lambda}(vt)\cos\frac{2\pi}{\lambda}(x)\right]$$

Formula $[u.v = uv' + vu']$

$$= 2a\left\{\sin\frac{2\pi}{\lambda}(vt) - \sin\frac{2\pi}{\lambda}(x).(0)\right.$$

$$\left. + \cos\frac{2\pi}{\lambda}(x)\cos\frac{2\pi}{\lambda}(vt)\right.$$

$$\frac{2\pi v}{\lambda}$$

$$\frac{dy}{dt} = 2a.\cos\frac{2\pi}{\lambda}(x)\cos\frac{2\pi}{\lambda}(vt).\frac{2\pi v}{\lambda}$$

$$\boxed{\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi}{\lambda}(vt)} \quad \text{---- (V)}$$

The findout acceleration of the particle Diff' eqn (V) w.r.to 't', we get

$$\frac{d^2y}{dt^2} = \frac{d}{dt}\left[\frac{4\pi a v}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi}{\lambda}(vt)\right]$$

$[u.v = uv' + vu']$

$$= \frac{4\pi a v}{\lambda} \cos\frac{2\pi x}{\lambda} \; -\sin\frac{2\pi}{\lambda}(vt) . \frac{2\pi}{\lambda} v$$

$$+ \cos\frac{2\pi}{\lambda} vt . (0)$$

$$= \frac{-4\pi a v}{\lambda} \cos\frac{2\pi x}{\lambda} \sin\frac{2\pi vt}{\lambda} \frac{2\pi v}{\lambda}$$

$$\boxed{\frac{d^2y}{dt^2} = -\frac{8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi vt}{\lambda} \cos\frac{2\pi x}{\lambda}} \quad \text{---- (VI)}$$

The strain are compression at any point of resultant vibration is $\frac{dy}{dx}$.

Diff eqn (IV) w.r.t x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[2a \sin\frac{2\pi vt}{\lambda} \cos\frac{2\pi x}{\lambda}\right]$$

$$= 2a \sin\frac{2\pi vt}{\lambda} \; -\sin\frac{2\pi x}{\lambda} \frac{2\pi}{\lambda} + 0$$

$$\boxed{\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \sin\frac{2\pi vt}{\lambda} \sin\frac{2\pi x}{\lambda}} \quad \text{---- (VII)}$$

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Extra information

* **Antinodes :-** The point of medium, which vibrate with maximum amplitude, are called antinode.

Amplitude at antinodes is maximum,

$$A = \pm 2a.$$

$$A = 2a \cos \frac{2\pi}{\lambda}$$

$$\pm 2a = 2a \cos \frac{2\pi x}{\lambda}$$

$$\therefore \cos \frac{2\pi x}{\lambda} = \pm 1$$

$$\therefore \frac{2\pi x}{\lambda} = 0, \pi, 2\pi, \ldots$$

$$\therefore \frac{2\pi x}{\lambda} = p\pi$$

$$\therefore x = \frac{\lambda p}{2} = p\left(\frac{\lambda}{2}\right)$$

Where,

$$p = 0, 1, 2, \ldots$$

For. $x = 0, \lambda/2, \lambda, 3\lambda/2, \ldots$ antinodes are produced

The distance b/w two successive antinode is $\lambda/2$.

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* **Nodes :-** The point of medium, which vibrate with minmum amplitude are called Nodes.

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Amplitude at nodes is zero, $A=0$

But $A = 2a\cos\frac{2\pi x}{\lambda}$

$$2a\cos\left(\frac{2\pi x}{\lambda}\right)=0$$

$$\therefore \cos\left(\frac{2\pi x}{\lambda}\right)=0$$

$$\therefore \frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots$$

$$\therefore x=(2p-1)\lambda/4$$

Where, $p = 1, 2, 3, \ldots\ldots$

For, $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots\ldots$ nodes are produced.

The distance between any two successive nodes is $\lambda/2$

The distance between nodes and adjacent antinode is $\lambda/4$

Changes with respect to Position :-

1) Consider the Position,

Where, $\sin\frac{2\pi x}{\lambda} = 0$ and $\cos\frac{2\pi x}{\lambda} = \pm 1$

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From equations (IV) and (VIII)

Displacement:-

$$y = 2a\cos\frac{2\pi x}{\lambda}\ \sin\frac{2\pi vt}{\lambda}$$

$$y = 2a\ (1)\ \sin\frac{2\pi vt}{\lambda}$$

$$\boxed{y = 2a\sin\frac{2\pi vt}{\lambda}} \quad \text{----- (IX)}$$

Amplitude:-

$$A = 2a\cos\frac{2\pi x}{\lambda}$$

$$A = 2a\ (\pm 1)$$

$$\boxed{A = \pm 2a} \quad \text{----- (X)}$$

Velocity:-

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda}\cos\frac{2\pi x}{\lambda}\ \cos\frac{2\pi vt}{\lambda}$$

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda}\ \pm(1)\ \cos\frac{2\pi vt}{\lambda}$$

$$\boxed{\frac{dy}{dt} = \pm\frac{4\pi a v}{\lambda}\ \cos\frac{2\pi vt}{\lambda}} \quad \text{----- (XI)}$$

Acceleration :-

$$\frac{d^2y}{dt^2} = \frac{-8\pi^2 a v^2}{\lambda^2} \cos\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$\frac{d^2y}{dt^2} = \frac{-8\pi^2 a v^2}{\lambda^2} (1) \sin\frac{2\pi v t}{\lambda}$$

$$\boxed{\frac{d^2y}{dt^2} = \frac{-8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi v t}{\lambda}} \quad \text{---} \; (XII)$$

Strain :-

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda} (0) \sin\frac{2\pi v t}{\lambda}$$

$$\boxed{\frac{dy}{dx} = 0} \quad \text{-----} \; (XIII)$$

As the strain is zero, these positions corresponds to antinodes.

$$\sin\frac{2\pi x}{\lambda} = 0$$

but $\sin m\pi = 0$

Where, $m = 0, 1, 2, \ldots$ etc

$$\therefore \frac{2\pi x}{\lambda} = m\pi$$

$$x = \frac{m\lambda}{2}$$

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \text{etc}$$

Thus the antinodes are equidistant and separated by $\lambda/2$. at $x=0$, the position of interface is antinodes.

Therefore, the position of the open end of the open end organ pipe or free end of a stretched string corresponds to an antinodes.

II) Consider the possition,

where

$$\sin\frac{2\pi x}{\lambda} = \pm 1, \quad \text{and} \quad \cos\frac{2\pi x}{\lambda} = 0$$

From equation (IV) and (VIII)

Displacement :-

$$y = 2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$

$$y = 2a(0)\sin\frac{2\pi vt}{\lambda}$$

$$\boxed{y=0} \quad \text{-------} (XIV)$$

Amplitude :-

$$A = 2a\cos\frac{2\pi x}{\lambda}$$

$$A = 2a(0)$$

$$\boxed{A=0} \quad \text{-------} (XV)$$

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velocity :-

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} (0) \cos\frac{2\pi v t}{\lambda}$$

$$\boxed{\frac{dy}{dt} = 0} \quad \cdots\cdots \text{(XVI)}$$

Acceleration :-

$$\frac{d^2y}{dt^2} = -\frac{8\pi^2 a v^2}{\lambda^2} \cos\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$\frac{d^2y}{dt^2} = -\frac{8\pi^2 a v^2}{\lambda^2} (0) \sin\frac{2\pi v t}{\lambda}$$

$$\boxed{\frac{d^2y}{dt^2} = 0} \quad \cdots\cdots \text{(XVII)}$$

strain :-

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} (\pm 1) \sin\frac{2\pi v t}{\lambda}$$

$$\boxed{\frac{dy}{dx} = \pm \frac{4\pi a}{\lambda} \sin\frac{2\pi v t}{\lambda}} \quad \cdots\cdots \text{(XVIII)}$$

These positions correspond to nodes.

$$\therefore \cos \frac{2\pi x}{\lambda} = 0$$

But $\cos (2m+1)\frac{\pi}{2} = 0$

Where, $n = 0, 1, 2, 3, \ldots$ etc.

$$\therefore \frac{2\pi x}{\lambda} = (2m+1)\frac{\pi}{2}$$

$$x = \frac{(2m+1)\lambda}{4}$$

$$\therefore x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots \text{etc}$$

Thus the antinodes are also equidistant and separated by a distance of $\lambda/2$.

It is clear that between two consecutive antinodes there is a node. Distance b/w a node and an antinodes is $\frac{\lambda}{4}$.

Change with respect to time :-

1) ~~Change~~ Consider the instant of time,
When $\sin \frac{2\pi v t}{\lambda} = 0$ and $\cos \frac{2\pi t v}{\lambda} = \pm 1$

from equation ⓥ and (VIII)

Displacement :-

$$y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi v t}{\lambda}$$

$$y = 2a\,(0)\,\sin\frac{2\pi vt}{\lambda}$$

$$\boxed{y = 0} \quad \text{------ (i)}$$

Amplitude :-

$$\boxed{A = 2a\cos\frac{2\pi x}{\lambda}} \quad \text{------ (ii)}$$

velocity :-

$$\frac{dy}{dt} = \pm\frac{4\pi a v}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$$

$$\frac{dy}{dt} = \pm\frac{4\pi a v}{\lambda}\cos\frac{2\pi x}{\lambda}\,(1)$$

$$\boxed{\frac{dy}{dt} = \pm\frac{4\pi a v}{\lambda}\cos\frac{2\pi x}{\lambda}} \quad \text{---- (iii)}$$

Acceleration :-

$$\frac{d^2y}{dt^2} = \frac{-8\pi^2 a v^2}{\lambda^2}\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$

$$\frac{d^2y}{dt^2} = \frac{-8\pi^2 a v^2}{\lambda^2}\cos\frac{2\pi x}{\lambda}\,(0)$$

$$\text{----- (IV)}$$

$$\boxed{\frac{d^2y}{dt^2} = 0}$$

strain :-

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} (0)$$

$$\boxed{\frac{dy}{dx} = 0} \quad \text{------ (v)}$$

$$\therefore \sin\frac{2\pi v t}{\lambda} = 0$$

$$\sin m\pi = 0$$

Where, $m = 0, 1, 2, 3, \ldots$ etc

$$\therefore \frac{2\pi v t}{\lambda} = m\pi$$

or $\quad t = \frac{m\lambda}{2v}$

but $\quad \frac{v}{\lambda} = n \text{ (frequancy)} = \frac{1}{T}$

$$\therefore t = \frac{mT}{2}$$

or $\quad t = 0, T/2, T, 3T/2, \ldots$ etc

It follows that twice in each time period the particles pass through their mean positions.

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ii) Consider the instant of time :-

When, $\sin \frac{2\pi vt}{\lambda} = \pm 1$ and $\cos \frac{2\pi vt}{\lambda} = 0.$

From equation (IV) to (VIII)

Displacement :-

$$y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda}.$$

$$y = 2a \cos \frac{2\pi x}{\lambda} \quad (1)$$

$$\boxed{y = \pm 2a \cos \frac{2\pi x}{\lambda}} \quad \text{------ (I)}$$

Amplitude :-

$$\boxed{A = 2a \cos \frac{2\pi x}{\lambda}} \quad \text{------ (II)}$$

Velocity :-

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \cos \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda}.$$

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \cos \frac{2\pi x}{\lambda} (0)$$

$$\boxed{\frac{dy}{dt} = 0.} \quad \text{------ (III)}$$

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Acceleration :-

$$\frac{d^2y}{dt^2} = \frac{-8\pi^2 a v^2}{\lambda^2} \cos\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$\frac{d^2y}{dt^2} = \frac{-8\pi^2 a v^2}{\lambda^2} \cos\frac{2\pi x}{\lambda} (\pm 1)$$

$$\boxed{\frac{d^2y}{dt^2} = \mp \frac{8\pi^2 a v^2}{\lambda^2} \cos\frac{2\pi x}{\lambda}} \quad \text{----- (iv)}$$

The strain or compression,

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} (\pm 1)$$

$$\boxed{\frac{dy}{dx} = \pm \frac{4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda}} \quad \text{----- (v)}$$

$$\sin\frac{2\pi v t}{\lambda} = \pm 1 \quad \text{and} \quad \cos\frac{2\pi v t}{\lambda} = 0$$

But, $\sin(2m+1)\frac{\pi}{2} = \pm 1$

Where, $m = 0, 1, 2, \text{-----} \text{etc}$

$$\therefore \frac{2\pi v t}{\lambda} = (2m+1)\frac{\pi}{2}$$

or $t = \dfrac{(2m+1)\lambda}{4v}$

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But $\frac{v}{\lambda} = n = \frac{1}{T}$

$\therefore t = \frac{(2m+1)T}{4} = \frac{mT}{2} + \frac{T}{4}$

or $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots$ etc

At these instants, the displacement, acceleration and strain are maximum at all positions and the velocity of the particles is zero. At these instants, each particles is at its extreme position and the pattern is stationary at that instant.

This instant is called the stationary instant.

Imp

Closed end organ pipe or string fixed at the other end :-

Consider a simple harmonic wave given by the equation

$$\boxed{y_1 = a \sin \frac{2\pi}{\lambda}(vt-x)} \text{ Incident wave} \quad \cdots (i)$$

The displacement of the same particles at the same instant due to the reflected wave is given by

$$\boxed{y_2 = -a \sin \frac{2\pi}{\lambda}(vt+x)} \text{ Reflected wave} \quad \cdots (ii)$$

$$y = y_1 + y_2$$

$$y = \left[a \sin\frac{2\pi}{\lambda}(vt-x)\right] + \left[-a\sin\frac{2\pi}{\lambda}(vt+x)\right]$$

$$y = \left[a \sin\frac{2\pi}{\lambda}(vt-x) - a\sin\frac{2\pi}{\lambda}(vt+x)\right]$$

$$y = a\left[\sin\frac{2\pi}{\lambda}(vt-x) - \sin\frac{2\pi}{\lambda}(vt+x)\right]$$

$$y = a\left[\sin\frac{2\pi}{\lambda}vt \cdot \cos\frac{2\pi}{\lambda}x - \cos\frac{2\pi}{\lambda}vt \sin\frac{2\pi}{\lambda}x\right.$$

$$\left. - \left[\sin\frac{2\pi}{\lambda}vt \cos\frac{2\pi}{\lambda}x + \cos\frac{2\pi}{\lambda}vt \sin\frac{2\pi}{\lambda}x\right]\right]$$

$[\sin(A-B) \;\&\; \sin(A+B)]$

$$y = a\left[\cancel{\sin\frac{2\pi}{\lambda}vt \cdot \cos\frac{2\pi}{\lambda}x} - \cos\frac{2\pi}{\lambda}vt \cdot \sin\frac{2\pi}{\lambda}x\right.$$

$$\left. - \cancel{\sin\frac{2\pi}{\lambda}vt \cos\frac{2\pi}{\lambda}x} - \cos\frac{2\pi}{\lambda}vt \sin\frac{2\pi}{\lambda}x\right]$$

$$y = a\left[-\cos\frac{2\pi}{\lambda}vt \sin\frac{2\pi}{\lambda}x - \cos\frac{2\pi}{\lambda}vt \sin\frac{2\pi}{\lambda}x\right]$$

$$\boxed{y = -2a\sin\frac{2\pi}{\lambda}x \cdot \cos\frac{2\pi}{\lambda}vt} \quad \text{---- (iii)}$$

or

$$y = y_1 + y_2$$

$$y = \left[a\sin\frac{2\pi}{\lambda}(vt-x)\right] + \left[-a\sin\frac{2\pi}{\lambda}(vt+x)\right]$$

$$y = \left[a \sin \frac{2\pi}{\lambda}(vt-x) - a \sin \frac{2\pi}{\lambda}(vt+x)\right]$$

$$y = a\left[2\cos\frac{2\pi}{\lambda}\left(\frac{vt-x+vt+x}{2}\right)\sin\left(\frac{vt-x-vt-x}{2}\right)\right]$$

$$y = 2a\cos\frac{2\pi}{\lambda}\left(\frac{\cancel{2}vt}{\cancel{2}}\right)\sin\left(\frac{-\cancel{2}x}{\cancel{2}}\right)$$

$$y = 2a\cos\frac{2\pi}{\lambda}(vt)\cdot\sin\frac{2\pi}{\lambda}(-x)$$

$$\boxed{y = -2a\sin\frac{2\pi}{\lambda}x \cdot \cos\frac{2\pi}{\lambda}vt} \quad \text{------ (iii)}$$

The resultant vibration of the particle represents a simple harmonic wave of the same time period. The amplitude of the resultant wave is given by

$$\boxed{A = 2a\sin\frac{2\pi x}{\lambda}} \quad \text{------ (iv)}$$

The resultant amplitude is a function of x. The velocity of the portical at any instant of time is dy/dt

Diff equation (iv) and w.r.to time,

$$\frac{dy}{dt} = \frac{d}{dt}\left[-2a\sin\frac{2\pi x}{\lambda}\cdot\cos\frac{2\pi}{\lambda}vt\right]$$

$$= -2a\left[\sin\frac{2\pi x}{\lambda} - \sin\frac{2\pi}{\lambda}vt\ \frac{2\pi v}{\lambda}\right.$$

$$+ \cos\frac{2\pi}{\lambda} vt \; (0) \Big]$$

$$= +2a \sin\frac{2\pi}{\lambda}x \cdot \sin\frac{2\pi}{\lambda}vt \cdot \frac{2\pi}{\lambda}v$$

$$\boxed{\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \sin\frac{2\pi}{\lambda}x \sin\frac{2\pi}{\lambda}vt} \quad \text{----} \; (v)$$

Now, acceleration of the particale at any instant of time is $\frac{d^2y}{dt^2}$

Diff' eqn (v) wrt t; we get

$$\frac{d^2y}{dt^2} = \frac{4\pi a v}{\lambda}\left[\frac{d}{dt} \sin\frac{2\pi}{\lambda}x \cdot \sin\frac{2\pi}{\lambda}vt\right]$$

$$= \frac{4\pi a v}{\lambda}\left[\sin\frac{2\pi}{\lambda}x \cdot \cos\frac{2\pi}{\lambda}vt \cdot \frac{2\pi}{\lambda}v + 0\right]$$

$$\boxed{\frac{d^2y}{dt^2} = \frac{8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi}{\lambda}x \cos\frac{2\pi}{\lambda}vt} \quad \text{----} \; (vi)$$

The strain or compression at any point of resultant vibration is dy/dx

Diff' wrt x in eqn (iii)

$$\frac{dy}{dx} = \frac{d}{dx}\left[-2a \sin\frac{2\pi}{\lambda}x \cos\frac{2\pi}{\lambda}vt\right]$$

$$\frac{dy}{dx} = -2a\Big[\sin\frac{2\pi}{\lambda}x \; -\sin\frac{2\pi}{\lambda}vt \; (0)$$

$$+ \cos\frac{2\pi}{\lambda}vt \cos\frac{2\pi}{\lambda}x \cdot \frac{2\pi}{\lambda}$$

$$\boxed{\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos\frac{2\pi}{\lambda}vt \cos\frac{2\pi}{\lambda}x}$$

Changes with respect to position :-

1) Consider the positions,

Where $\sin\frac{2\pi x}{\lambda} = 0$ and $\cos\frac{2\pi x}{\lambda} = \pm 1$

from equations (IV) and (VIII)

Displacement :-

$$y = -2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$$

$$y = -2a(0)\cos\frac{2\pi vt}{\lambda}$$

$$\boxed{y = 0} \quad \text{------} \quad (IX)$$

Amplitude :-

$$A = 2a\sin\frac{2\pi x}{\lambda}$$

$$A = 2a(0)$$

$$\boxed{A = 0} \quad \text{------} \quad (X)$$

Velocity :-

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} (0)$$

$$\boxed{\frac{dy}{dt} = 0} \quad ---- \text{(x1)}$$

Acceleration :-

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi x}{\lambda} \cos\frac{2\pi v t}{\lambda}$$

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a v^2}{\lambda^2} (0) \cos\frac{2\pi v t}{\lambda}$$

$$\boxed{\frac{d^2y}{dt^2} = 0} \quad ----- \text{(x11)}$$

strain :-

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda} (\pm 1) \cos\frac{2\pi v t}{\lambda}$$

$$\boxed{\frac{dy}{dx} = \frac{\pm 4\pi a}{\lambda} \cos\frac{2\pi v t}{\lambda}} \quad ----- \text{(x111)}$$

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from equation (IX) to (XIII), it is clear that at these positions, displacement, amplitude, velocity and acceleration are zero but the strain is maximum. These positions correspond to nodes.

$$\because \sin\frac{2\pi x}{\lambda} = 0$$

But $\sin m\pi = 0$

Where. $m = 0, 1, 2, \ldots$ etc.

$$\therefore \frac{\pm 2\pi x}{\lambda} = m\pi$$

$$x = \frac{m\lambda}{2}$$

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \text{etc.}$$

Thus,

the nodes are equidistant and separated by $\frac{\lambda}{2}$. At $x=0$, the position of interface is a node. Therefore, the position of the closed end organ pipe or fixed end of a stretched string correspond to a node.

ii) Consider the positions, where

$$\sin\frac{2\pi x}{\lambda} = \pm 1 \quad \text{and} \quad \cos\frac{2\pi x}{\lambda} = 0$$

From equation (IV) and (VIII)

Displacement :-

$$y = -2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi v t}{\lambda}$$

$$y = -2a(\pm 1) \cos \frac{2\pi v t}{\lambda}$$

$$\boxed{y = \mp 2a \cos \frac{2\pi v t}{\lambda}} \quad \text{------ (xiv)}$$

Amplitude :-

$$A = 2a \sin \frac{2\pi x}{\lambda}$$

$$A = 2a(\pm 1)$$

$$\boxed{A = \pm 2a} \quad \text{------ (xv)}$$

velocity :-

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} (\pm 1) \sin \frac{2\pi v t}{\lambda}$$

$$\boxed{\frac{dy}{dt} = \pm \frac{4\pi a v}{\lambda} \sin \frac{2\pi v t}{\lambda}} \quad \text{------ (xvi)}$$

Acceleration :-

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi x}{\lambda} \cos\frac{2\pi vt}{\lambda}$$

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a v^2}{\lambda^2} (\pm 1) \cos\frac{2\pi vt}{\lambda}$$

$$\boxed{\frac{d^2y}{dt^2} = \pm\frac{8\pi^2 a v^2}{\lambda^2} \cos\frac{2\pi vt}{\lambda}} \quad \text{------ (XVII)}$$

Strain :-

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi vt}{\lambda}$$

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} (0) \cos\frac{2\pi vt}{\lambda}$$

$$\boxed{\frac{dy}{dx} = 0} \quad \text{------ (XVIII)}$$

As the strain in zero, these positions correspond to antinodes.

$$\because \sin\frac{2\pi x}{\lambda} = \pm 1$$

Also, $\sin\frac{(2m+1)\pi}{2} = \pm 1$

Where $m = 0, 1, 2, \ldots$ etc.

$$\therefore \frac{2\pi x}{\lambda} = \frac{(2m+1)\pi}{2}$$

or $x = \frac{(2m+1)\lambda}{4}$

or $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$ etc

Thus, The antinodes are equidistant and separated by $\lambda/2$. The first antinode is at a distance of $\lambda/4$ from the closed end to the closed end pipe or fixed end of a stretched string.

Change with respect to time :-

i) At the instant of time,

When $\sin \frac{2\pi vt}{\lambda} = 0$ and $\cos \frac{2\pi vt}{\lambda} = \pm 1$

From equation (vi) and (viii)

Displacement :-

$$y = -2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda}$$

$$y = -2a \sin \frac{2\pi x}{\lambda} (\pm 1)$$

$$\boxed{y = \mp 2a \sin \frac{2\pi x}{\lambda}} \quad \text{------ (1)}$$

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Amplitude :-

$$\boxed{A = 2a\sin\frac{2\pi x}{\lambda}} \quad \text{.....(II)}$$

velocity :-

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \sin\frac{2\pi x}{\lambda} (0)$$

$$\boxed{\frac{dy}{dt} = 0} \quad \text{.....(III)}$$

Acceleration :-

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi x}{\lambda} \cos\frac{2\pi v t}{\lambda}$$

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi x}{\lambda} (\pm 1)$$

$$\boxed{\frac{d^2y}{dt^2} = \pm \frac{8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi x}{\lambda}} \quad \text{....(IV)}$$

strain :-

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \cos\frac{2\pi x}{\lambda} \; (\pm 1)$$

$$\boxed{\frac{dy}{dx} = \mp\frac{4\pi a}{\lambda} \cos\frac{2\pi x}{\lambda}} \quad \text{----} \; (v)$$

These equations show that at these instant the displacement, acceleration and strain are maximum at all positions and the velocity of the particles is zero.

Here, $\cos\frac{2\pi v t}{\lambda} = \pm 1$

Also $\cos m\pi = \pm 1$

$$\therefore \frac{2\pi v t}{\lambda} = m\pi$$

But $\frac{v}{\lambda} = \frac{1}{T}$

$$t = \frac{mT}{2}$$

$$t = 0, \frac{T}{2}, T, \frac{3T}{2}, \text{----} \; etc.$$

At these instant, each particle is at its extrem position and the pattern is stationary at that instant. The instant is called the stationary instant.

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ii) At the instant of time,

When, $\sin\frac{2\pi vt}{\lambda} = \pm 1$

and $\cos\frac{2\pi vt}{\lambda} = 0$

From equation (iv) and (viii)

Displacement :-

$$y = -2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda}$$

$$y = -2a\sin\frac{2\pi x}{\lambda}(0)$$

$$\boxed{y = 0} \quad \text{------ (vi)}$$

Amplitude :-

$$\boxed{A = 2a\sin\frac{2\pi x}{\lambda}} \quad \text{------ (vii)}$$

velocity :-

$$\frac{dy}{dt} = \frac{4\pi av}{\lambda}\sin\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$

$$\frac{dy}{dt} = \frac{4\pi av}{\lambda}\sin\frac{2\pi x}{\lambda}(\pm 1)$$

$$\boxed{\frac{dy}{dt} = \pm\frac{4\pi av}{\lambda}\sin\frac{2\pi x}{\lambda}} \quad \text{------ (viii)}$$

Acceleration :-

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi x}{\lambda} \cos\frac{2\pi v t}{\lambda}$$

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi x}{\lambda} (0)$$

$$\boxed{\frac{d^2y}{dt^2} = 0} \quad \text{------} \ (\text{ix})$$

strain :-

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi v t}{\lambda}$$

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda} \cos\frac{2\pi x}{\lambda} (0)$$

$$\boxed{\frac{dy}{dx} = 0} \quad \text{--------} \ (\text{x})$$

This equations show that displacement, ~~velocity~~, acceleration, and strain are zero.

$\because$ Here $\sin\frac{2\pi v t}{\lambda} = \pm 1$

Also $\sin\frac{(2m+1)\pi}{2} = \pm 1$

$$\therefore \frac{2\pi v t}{\lambda} = \frac{(2m+1)\pi}{2}$$

But,

$$\frac{v}{\lambda} = \frac{1}{T}$$

$$t = \frac{(2m+1)T}{4}$$

$$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \text{.....etc.}$$

All the instant of time, all the particles are at their mean positions. If it follows that twice each in time peroid the particles pass through their mean position.

Imp

Investigation of pressure and density change at displacement nodes and antinodes :-

When a wave is propagated through a fluid,

$$E = \frac{-P}{dy/dx}$$

Where, E is the bulk modulus of elasticity, P is the excess of pressure and $\frac{dy}{dx}$ is the volumetic strain.

$$\therefore \boxed{P = -E\frac{dy}{dx}} \quad \text{-----} (1)$$

In the case of an open end organ pipe, strain is given by

$$\frac{dy}{dx} = \frac{-4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi vt}{\lambda}$$

Also

$$v = \sqrt{\frac{E}{\rho}}$$

$$v^2 = \frac{E}{\rho}$$

$$\therefore E = v^2 \rho$$

v = is volume elasticity

$\frac{dy}{dx}$ = denotes volumetrics strain.

ρ = density of material.

substituting the values of $\frac{dy}{dx}$ and E in equation (1)

$$p = -v^2\rho - \frac{4\pi a}{\lambda} \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi v t}{\lambda}$$

$$\boxed{p = v^2\rho \frac{4\pi a}{\lambda} \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi v t}{\lambda}} \quad \text{---- (II)}$$

When, $\sin \frac{2\pi x}{\lambda} = 1$ and $\sin \frac{2\pi v t}{\lambda} = 1$,

The excess of pressure is maximum

$$\boxed{p = P_0 = v^2\rho \cdot \frac{4\pi a}{\lambda}} \quad \text{---- (III)}$$

From equation (II) and (III)

$$\therefore \boxed{p = P_0 \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi v t}{\lambda}} \quad \text{--- (IV)}$$

Taking

$$P_0 \sin \frac{2\pi x}{\lambda} = P_x$$

$$\boxed{P = P_x \sin \frac{2\pi v t}{\lambda}} \quad \text{----} \ (v)$$

This is represents pressure at a point distance of 'x' from interface.

This expression for the excess pressure (ii) & (v) indicates that pressure varies harmonically with time whose time period is

$$T = \frac{2\pi}{2\pi v/\lambda} = \frac{\lambda}{v} \qquad \left[\because n = \frac{1}{T} = \frac{v}{\lambda}\right]$$

Which is a same as that of component wave.

Since at a antinode $\frac{dy}{dx} = 0$, $\sin \frac{2\pi x}{\lambda} = 0$

$$P = P_{max} \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi v t}{\lambda} = 0$$

$$P_x = 0$$

Hence, it indicates that there are no change of pressure and hence density at these points. ie at antinodes pressure and denosity remains normal at nodes.

$\frac{dy}{dx} =$ maximum and $\sin \frac{2\pi x}{\lambda} = \pm 1$

$$P_x = \pm P_{max}$$

$$P = \pm P_{max} \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi v t}{\lambda}$$

Density may be greater than or less than normal value. at all other points pressure variation is,

$$P = P_{max} \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi v t}{\lambda}$$

Thus we see that antinodes displacement not undergoes pressure and nodes are displacement undergoes a maximum variations of pressure.

Imp

Energy is not transferred in a stationary Wave:-

In stationary wave is no transfer of energy across plane thus resultant can be obtain mathematically as follows.

Excess pressure in stationary wave set up in a fluid is given by

$$\boxed{P = P_x \sin \frac{2\pi v t}{\lambda}} \quad \text{----} (1)$$

velocity of a partical in case of open end pipe is given by,

$$u = \frac{dy}{dt} = \frac{4\pi a v}{\lambda} \cos \frac{2\pi x}{\lambda} \cos \frac{2\pi v t}{\lambda}$$

$$u = U_x \cos \frac{2\pi v t}{\lambda}$$

Where, $U_x = \frac{4\pi a v}{\lambda} \cos \frac{2\pi x}{\lambda}$

Workdone or energy transfered per unit area in small time dt is given by.

$= P.u.dt$

The energy is transfered in periodic time t is

$$\int_0^T P.u.dt$$

The rate of transfer of energy

$$C = \frac{1}{T}\int_0^T P.dt.u.$$

$$C = \frac{1}{T}\int_0^T P_x \sin\frac{2\pi vt}{\lambda} + U_x \cos\frac{2\pi vt}{\lambda}\, dt$$

Since,

Px and Ux are not the functions of time They can be taken out of the limits integration such as,

$$\frac{P_x U_x}{2T}\int_0^T 2\sin\frac{2\pi vt}{\lambda}\cos\frac{2\pi vt}{\lambda}\, dt$$

Multiplying and dividing by 2

$$C = \frac{P_x U_x}{2T}\int_0^T \sin\frac{4\pi vt}{\lambda}\, dt$$

Since, $\int_0^T \sin\frac{4\pi vt}{\lambda}\, dt = 0$ [any angle in multiple of 2π of sin is zero].

$$C = \frac{P_x U_x}{2T} \times 0$$

$$\boxed{C = 0}$$

is equal to zero.

ie Rate ~~of~~ transfer energy is equal to zero.

∴ In case of stationary vibration no energy is transfered across any section.

Distribution of energy in a stationary wave:-

The distribution of the energy in stationary waves is not same but it is same in progressive waves. The total energy per unit length is double i stationary waves because it is compound of two wave. But morever total energy. ie algebric sum of energy current in the direction is zero. In progressive wave, $\frac{1}{2}$ is K.E. and $\frac{1}{2}$ is P.E

But is a stationary wave ratio of K.E to P.E is same at every point of instant. At the equilibrium position of wave the entire energy is kinetic and when the particle at the extreme end string ie nodes then the energy is potential because all the particle of the medium are at rest.

At the intermediate of the wave the energy K.E and P.E

At open end pipe the displacement of the layer at any instant is given by

$$\boxed{y = 2a \sin \frac{2\pi v t}{\lambda} \cos \frac{2\pi x}{\lambda}} \quad \text{-----} (1)$$

Diff' w.r.to velocity of a layer

$$\frac{dy}{dt} = 2a \frac{d}{dt}\left[\sin \frac{2\pi v t}{\lambda} \cos \frac{2\pi x}{\lambda}\right]$$

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$$\boxed{\frac{dy}{dx} = \frac{4\pi a v}{\lambda} \cos\frac{2\pi v t}{\lambda} \cos\frac{2\pi x}{\lambda}} \quad \text{-----} (ii)$$

Now, Consider unit area of the layer whose mass of having thickness δx and ρ is density

$$\boxed{\text{Mass} = \rho\,\delta x = (\text{density} \times \text{volume})} \quad \text{-----} (iii)$$

K.E of the unit area of layer at any instant is,

$$\delta K = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 \delta x$$

$$= \frac{1}{2} \rho \left(\frac{4\pi a v}{\lambda}\right)^2 \left[\cos^2\frac{2\pi v t}{\lambda} \cos^2\frac{2\pi x}{\lambda}\right] \delta x$$

$$= \frac{1}{\cancel{2}} \rho . \cancel{4} \left(\frac{2\pi a v}{\lambda}\right)^2 \left[\cos^2\frac{2\pi v t}{\lambda} \cos^2\frac{2\pi x}{\lambda}\right] \delta x$$

$$= 2\rho \left(\frac{2\pi a v}{\lambda}\right)^2 \left[\cos^2\frac{2\pi v t}{\lambda} \int \cos^2\frac{2\pi x}{\lambda}\,\delta x - \right.$$

$$\left. \int \left[\frac{d}{dx}\left(\cos^2\frac{2\pi v t}{\lambda}\right) \int \cos^2\frac{2\pi x}{\lambda}\,\delta x\right] \delta x \right]$$

$$\because \left[\int u v\,\delta x = u \int v\,\delta x - \int \left[\frac{d}{dx}(u) \int v\,\delta x\right] \delta x\right]$$

$$\delta K = 2\rho \left(\frac{2\pi a v}{\lambda}\right)^2 \left[\cos^2\frac{2\pi v t}{\lambda} \frac{1}{2} \int 2\cos^2\frac{2\pi x}{\lambda}\,\delta x - \right.$$

$$\left. \int -\sin^2\frac{2\pi v t}{\lambda} . (0) \frac{1}{2} \int 2\cos^2\frac{2\pi x}{\lambda}\,\delta x \right]$$

$$\delta K = \cancel{2}\rho \left(\frac{2\pi a v}{\lambda}\right)^2 \left[\cos^2\frac{2\pi v t}{\lambda} \frac{1}{\cancel{2}} \int \left(1 + \cos\frac{4\pi x}{\lambda}\right) \delta x \right.$$

$$\left. - 0 \right] \qquad \because \left[2\cos^2\frac{2\pi x}{\lambda} = 1 + \cos\frac{4\pi x}{\lambda}\right]$$

$$\delta K = \rho\left(\frac{2\pi a v}{\lambda}\right)^2\left[\cos^2\frac{2\pi v t}{\lambda}\int\left(1+\cos\frac{4\pi x}{\lambda}\right)\delta\right.$$

$$\delta K = \rho\left(\frac{2\pi a v}{\lambda}\right)^2\left\{\cos^2\frac{2\pi v t}{\lambda}\left[\int 1\,\delta x + \int \cos\frac{4\pi}{\lambda}\right.\right.$$

$$\delta K = \rho\left(\frac{2\pi a v}{\lambda}\right)^2\left\{\cos^2\frac{2\pi v t}{\lambda}\left[x + \frac{\sin\frac{4\pi x}{\lambda}}{\frac{4\pi}{\lambda}}\right]\right.$$

$$\delta K = \rho\left(\frac{2\pi a v}{\lambda}\right)^2\cos^2\frac{2\pi v t}{\lambda}\left[x\right]_0^l + \left[\frac{\sin\frac{4\pi x}{\lambda}}{4\pi/\lambda}\right]_0^l$$

above eq^n put in limit [0 to L]

$$\delta K = \rho\left(\frac{2\pi a v}{\lambda}\right)^2\cos^2\frac{2\pi v t}{\lambda}\left[l-0\right] + (0)$$

$$[\sin 4\pi = 0]$$

$$\delta K = \rho\left(\frac{2\pi a v}{\lambda}\right)^2\cos^2\frac{2\pi v t}{\lambda}$$

$$\boxed{\text{K.E per unit length, } KE = \rho\left(\frac{2\pi a v}{\lambda}\right)^2\cos^2\frac{2\pi v t}{\lambda}} \quad \text{---- (IV)}$$

$$\boxed{\text{Averge T.E per unit length per area } TE = \rho\left(\frac{2\pi a v}{\lambda}\right)^2} \quad \text{--- (V)}$$

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By comparing with progressing wave, stationary wave is double

$$P.E = T.E - K.E$$

$$= \rho\left(\frac{2\pi a v}{\lambda}\right)^2 - \rho\left(\frac{2\pi a v}{\lambda}\right)^2 \cos^2\frac{2\pi v t}{\lambda}$$

[from (4) and (5)]

$$\boxed{P.E = \rho\left(\frac{2\pi a v}{\lambda}\right)^2\left[1-\cos^2\frac{2\pi v t}{\lambda}\right]} \quad \text{----- (VI)}$$

Total energy is totally kinetic energy when $\cos\frac{2\pi v t}{\lambda}$ is equal to zero or

$$\boxed{\sin\frac{2\pi v t}{\lambda} = \pm 1}$$

The total energy is totally potential ie ratio betn K.E and P.E varies from instant to instant.