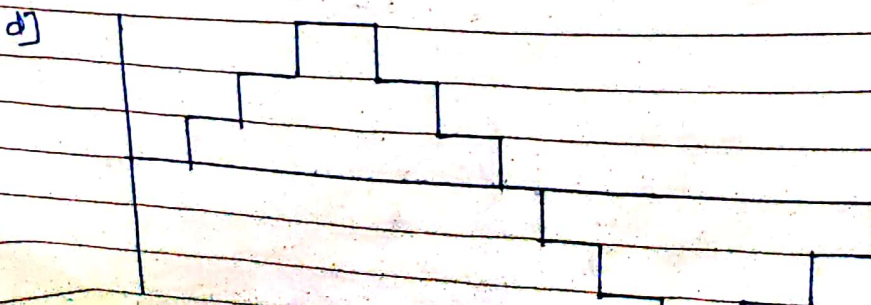
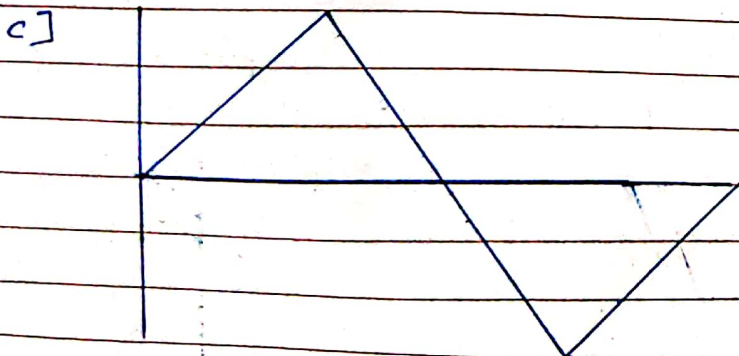
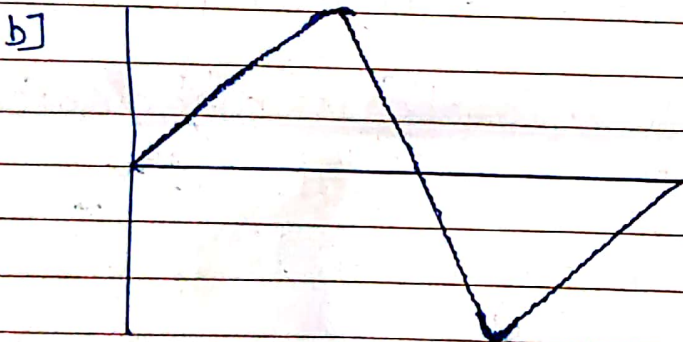
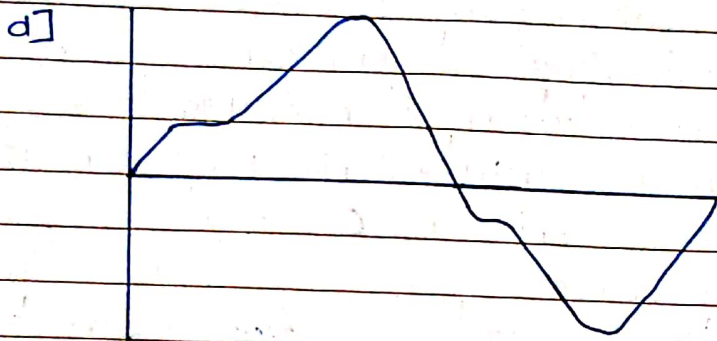


AC-Fundamentals

- 1] Types of ac waveforms
- 2] cycle, Time period, frequency, Amplitude of AC-voltage/current.
- 3] characteristics of sine wave
- 4] Different values of sinusoidal voltage/current
- 5] Phase of AC, Phase difference
- 6] Vector representation of an AC quantity
- 7] R-L circuit, R-C circuit, series R-L-C circuit, resonance in series R-L-C circuit.
- 8] Resonance curve
- 9] Bandwidth & Q-factor of series resonant circuit.
- 10] Parallel resonance, Resonance Curve
- 11] Q-factor
- 12] Band width of parallel resonant circuit
- 13] Transformer & its working.

1. Simple waveforms:

The shape of the curve obtained by plotting the instantaneous values of voltage or current as the ordinate against time as abscissa is called its waveform or wave-shape



An alternating voltage or current may not always take form of a systematical or a smooth wave as shown in figure. Figure also represents alternating waves. But while it is scarcely possible for manufacturers to produce sine wave generators or alternators, yet sine wave is the ideal form sought by designers and is the accepted standard. The waves deviating from the standard sine wave are termed as distorted waves.

In general, however, 'An alternating current or voltage is one the circuit direction of which reverses at regularly recurring intervals'

## 2. Complex Waveforms:

Complex waves are those which depart from the ideal sinusoidal form of figure. All alternating complex waves, which are periodic and have equal positive & negative half cycles can be shown to be made up of number of pure sine waves, having different frequencies but all these frequencies are integral multiples of that of the lowest alternating wave, called the 'fundamental' [or first harmonic].

These waves of higher frequencies are called 'Harmonics'. If the fundamental frequency is 50 Hz, then the frequency of 2<sup>nd</sup> harmonic is 100 Hz. & of third is 150 Hz & so on.

\* Cycle:

one complete set of positive & negative values of alternating quantity is known as cycle. Hence each diagram represents one complete cycle.

A cycle may also be sometimes specified in terms of angular measure. In that case one complete cycle is said to spread over  $360^\circ$  or  $2\pi$  radians

\* Time period:

The time taken by an alternating quantity to complete one cycle is called its time period  $T$ .

\* Frequency:

The number of cycles/second is called frequency of alternating quantity

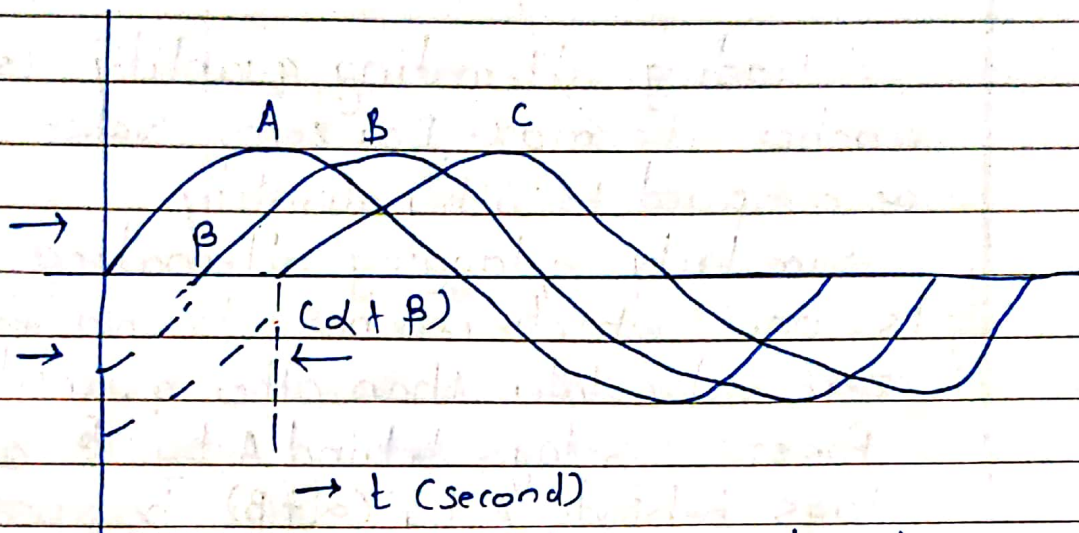
\* Amplitude:

The maximum value, positive or negative, of an alternating quantity is known as its amplitude.

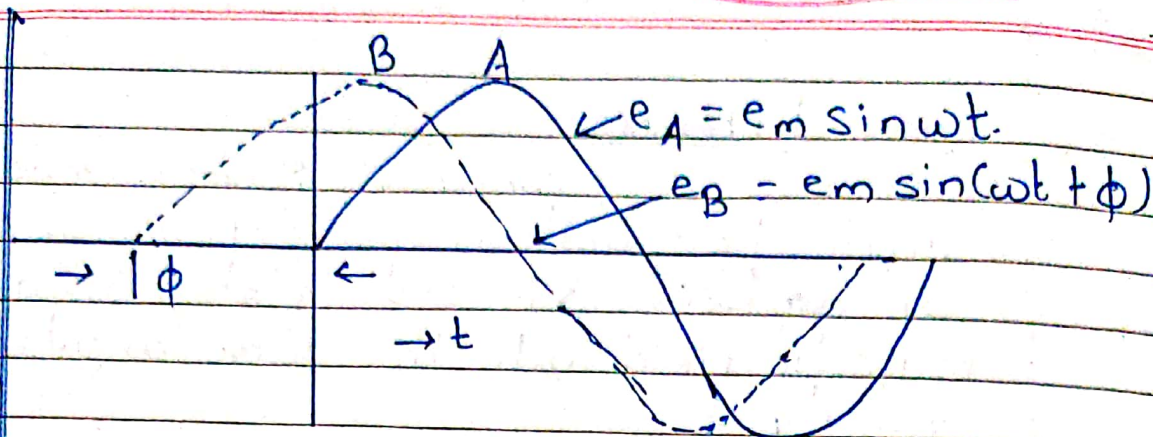
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## Phase difference:

consider three similar single-turn coils displaced from each other by angles  $\alpha$  &  $\beta$  and rotating in a uniform magnetic field with the same angular velocity



In this case the value of induced e.m.f. in 3 coils are same but there is one important difference. The e.m.f.s in these coils don't reach their maximum or zero values simultaneously but one after another. The 3 sinusoidal waves are shown in figure. It is seen that curves B and C are displaced from curve A and angles  $\beta$  and  $(\alpha + \beta)$  respectively. Hence, it means that phase difference b/w A & B is  $\beta$  and between B and C is  $\alpha$  but between A & C is  $(\alpha + \beta)$ . The statement however doesn't give indication to which e.m.f. reaches max. value first. This deficiency is supplied by using the terms 'lag' or 'lead'



A leading alternating quantity is one which reaches its max. [or zero] value earlier as compared to other quantity

Similarly a lagging alternating quantity is one which reaches its maximum or zero value later than other quantity.

For ex: B lags behind A by  $\beta$  and C lags behind A by  $(\alpha + \beta)$  because they reach their maximum values later.

The 3 equations for the instantaneous induced e.m.f's are:

$$e_A = E_m \sin \omega t \dots\dots \text{reference quantity}$$

$$e_B = E_m \sin(\omega t - \beta)$$

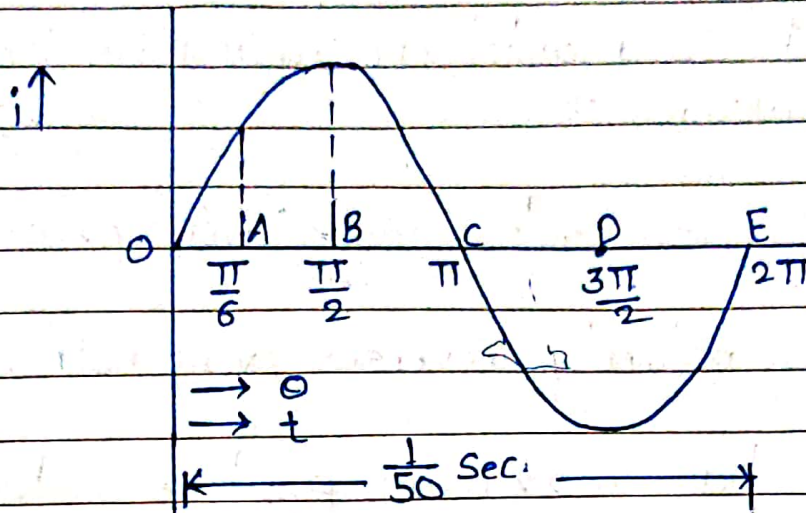
$$e_C = E_m \sin[\omega t - (\alpha + \beta)]$$

In above figure quantity B leads A by an angle  $\phi$ . Hence their equations are

$$e_A = E_m \sin \omega t \dots\dots \text{reference quantity.}$$

$$e_B = E_m \sin(\omega t - \phi)$$

## \*] Phase of An AC:



The fraction of a cycle or time period that has elapsed since an ac current [or voltage] last passed a given reference point [usually starting point] is called its phase. It may be expressed in

- 1) Time measured in seconds for
- 2) Fraction of a time period or
- 3) Angle expressed either in radians or degrees.

Eg:

Phase of AC at point 'A'

- 1)  $\pi/6$  radians or  $30^\circ$  or
- 2)  $\frac{1}{12}$  th of a cycle or  $\frac{1}{12}$  sec or
- 3)  $\frac{1}{12} \times 50 = \frac{1}{600}$  sec.

Phase at point 'E' is  $2\pi$  radians or zero because AC current is exactly in same condition as it was at starting point O

## \*] characteristics of sine wave:

A sinusoidal a.c. waveform has following characteristics:

- 1) Its one cycle spreads over  $360^\circ$  or  $2\pi$  radians
- 2) Its polarity reverses every half cycle.
- 3) It has maximum positive value at  $90^\circ$  and maximum negative value at  $270^\circ$
- 4) It has zero value at  $0^\circ$  and  $180^\circ$
- 5) It changes its values fastest when it crosses the zero axis.
- 6) It changes its value the slowest when near its maximum value whether positive or negative.



\*] Different values of sinusoidal voltage and current:

1] Instantaneous value:

It is value of the current that exists at any instant of time measured from some reference point. It can have any value between plus maximum value  $+I_m$  and negative maximum value  $-I_m$  and is denoted "i". In fact, the entire sine wave is made up of instantaneous values. Mathematically its given by:

$$i = I_m \sin \theta = I_m \sin \omega t = I_m \sin 2\pi f t \\ = I_m \sin 2\pi T t$$

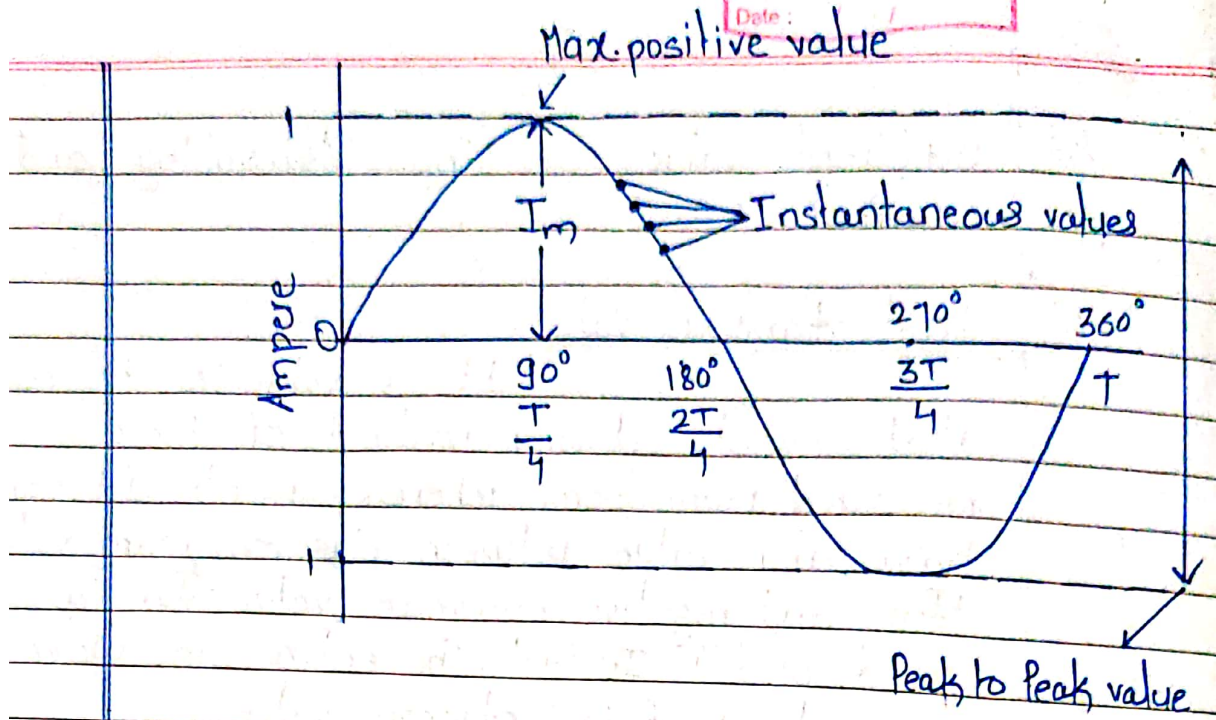
2] Peak value or Maximum value:

This the highest value reached by current in one cycle.

3] Peak-to-Peak value:

It is sum of positive peak and negative peak values usually written as P-P value.

It is obvious that positive and negative peak values can't occur at same time and may not have same value in all waveforms.



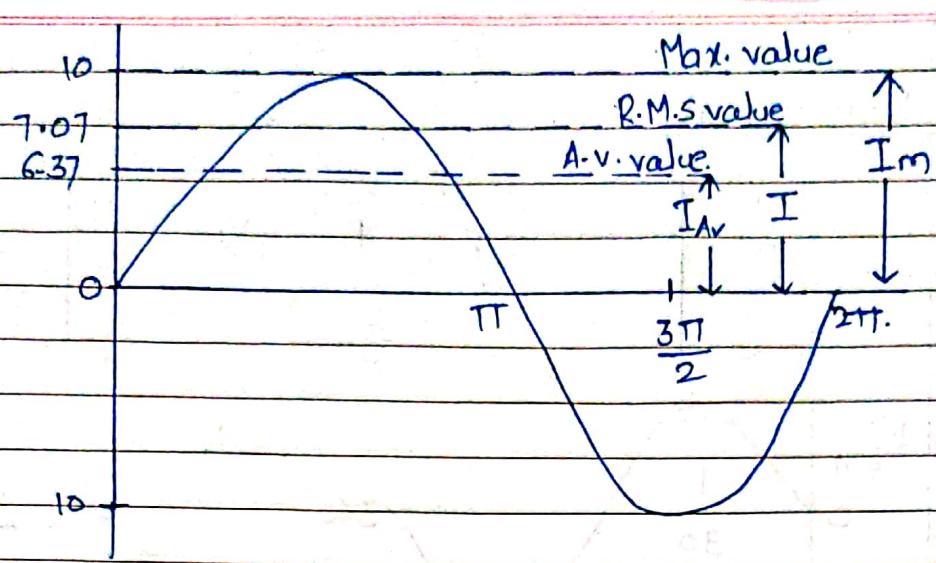
4] Root Mean Square [RMS] value:

It is also called the effective value. It is the value of current at  $\theta = 45^\circ$  which equals  $0.707 I_m$  or 70.7 percent of max. value.

In general;

$$\text{RMS value} = \frac{\text{Max. Value}}{\sqrt{2}}$$

$$I = \frac{I_m}{\sqrt{2}}$$



5] Average value:

It is mathematical avg. of all instantaneous values in one half cycle of the wave

$$\therefore I_{av} = 0.637 \times \text{Max. value} = \frac{2 \times I_m}{\pi}$$

⇒ for sine waves only

6] Form factor:

It is defined as the ratio

$$K_f = \frac{\text{rms value}}{\text{Avg. value}} = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

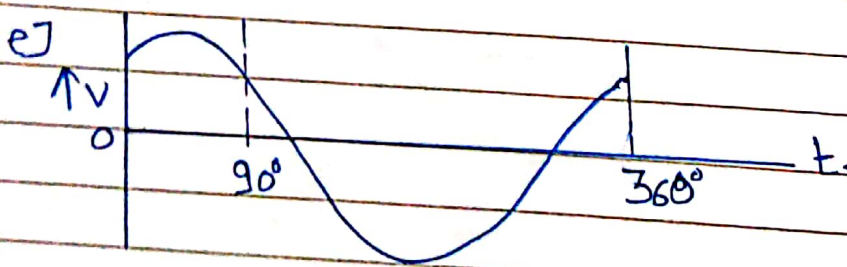
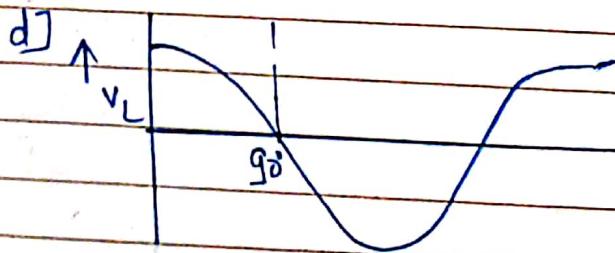
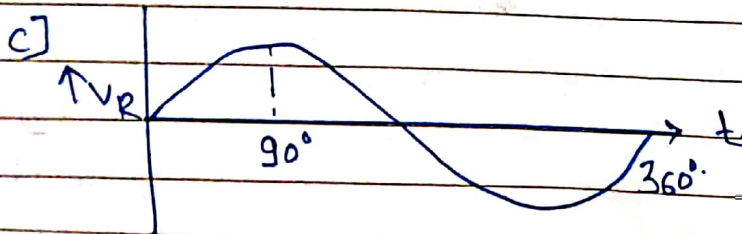
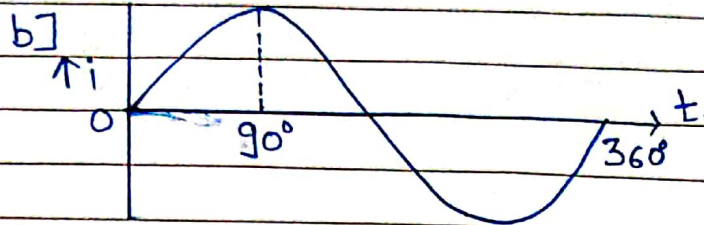
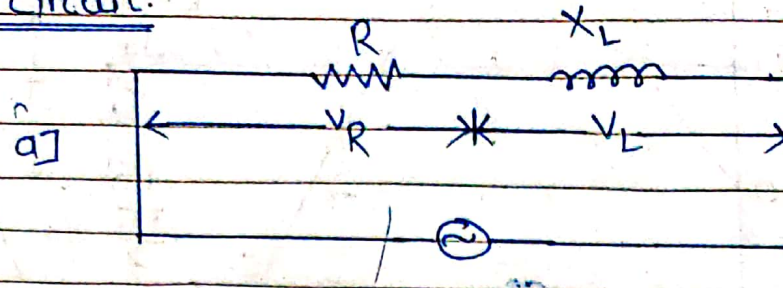
7] crest or Peak factor:

It is defined as the ratio

$$K_c = \frac{\text{Max. value}}{\text{RMS value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$$

⇒ for sine wave only

\*] R-L circuit:



Suppose a pure coil of inductance 'L' is connected in series with a pure resistance 'R' and is energised by a sinusoidal voltage of r.m.s value 'V'. If we take current as the reference quantity then it will give rise to sinusoidal voltage drop  $V_R$  across 'R' which will be in phase with it. as shown in fig [C]

The voltage drop  $V_L$  across the coil will lead the current by  $90^\circ$  as shown in fig [D]

In other words current through a coil lags behind voltage across it. The resultant voltage is the vector [or phasor] sum of  $V_R$  and  $V_L$  and as shown in fig [E] leads current by some angle  $\phi$ . In other words circuit current lags behind the applied voltage by an angle  $\phi$ .

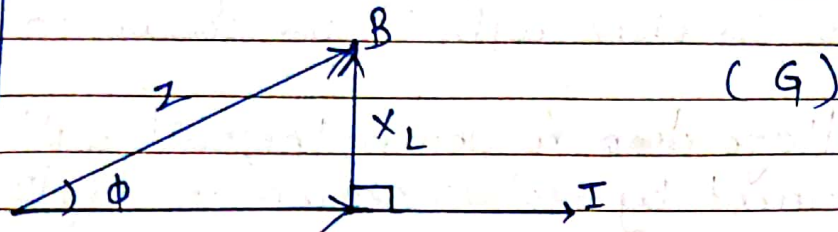
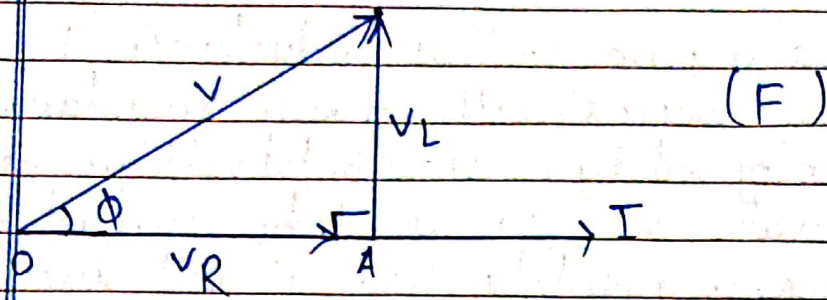
Since in a series combination, same current flows through the two components, it has been taken as the reference quantity. That is why current vector has been taken along the horizontal or reference axis.

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I \sqrt{R^2 + X_L^2} = IZ \text{ or } I = V/Z$$

The quantity  $(R^2 + X_L^2)^{1/2}$  is called impedance (Z) of the circuit.

'Z' is vector sum of R &  $X_L$  as shown in impedance triangle of fig (F)



$V =$  r.m.s value of applied voltage

$I =$  r.m.s value of circuit current

$$V = IR$$

$=$  r.m.s voltage drop across 'R'

(In phase with I)

$$V = IX_L$$

$=$  r.m.s voltage drop across L.

(at right angles to I)

It is also seen that current I lags behind applied voltage 'V' by angle  $\phi$  such that,

$$\tan \phi = \frac{X_L}{R}$$

Since pure inductive coil consumes no power, the power drawn by the circuit is the same as the dissipated by 'R'.

$$\therefore P = I^2 R$$

$$= I \cdot I \cdot R = I \cdot \frac{V}{R} \cdot R$$

$$= VI \cos \phi$$

The term  $\cos \phi$  is called the power factor of R-L circuit.

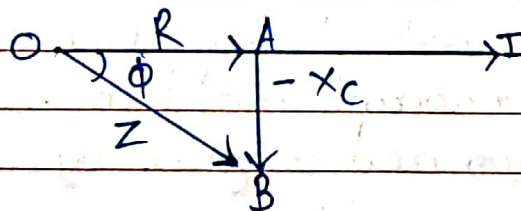
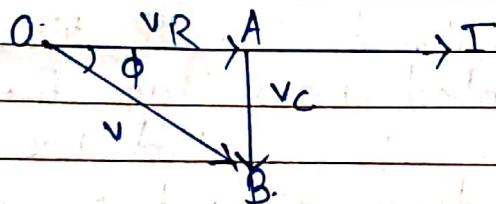
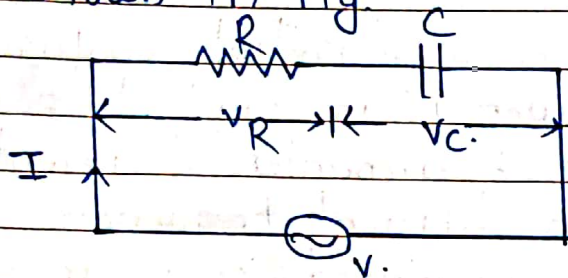
★ R-C circuit:

Such a series combination connected across an a.c voltage of r.m.s value  $v$  is shown in fig.

$$V_R = IR = \text{drop across } R \text{ (In phase with } I)$$

$$V_C = IX_C = \text{drop across } C \text{ (lagging } I \text{ by } 90^\circ)$$

Since capacitive reactance  $X_C$  is taken negative,  $V_C$  is drawn along negative  $Y$ -axis as shown in fig.



$$\therefore V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$= I \sqrt{R^2 + X_C^2}$$

$$V = IZ$$

$$\therefore I = \frac{V}{Z} \quad \therefore I = \frac{V}{Z}$$

It is seen that current leads the applied voltage by an angle  $\phi$  such that

$$\tan \phi = \frac{-X_C}{R}$$

capacitor  
 Since a pure circuit consumes no power, the entire circuit power consumption is due to resistor only.

$$\therefore P = I^2 R = VI \cos \phi$$

\*] R-L-C circuit

\* Resonance in R-L-C circuit:

If a sinusoidal voltage of variable frequency is applied across an R-L-C circuit it encounters different impedance at different frequencies. As frequency is increased  $X_L$  is increased but  $X_C$  is decreased. There is a certain frequency of the applied a.c. voltage for which  $X_L = X_C$ . It is called the resonance

The only impedance offered by circuit is R and is lowest it can offer

Hence under resonant condition, circuit current is maximum and is given by,

$$I_m = V/R$$

Moreover this current is in phase with the applied voltage. Hence the circuit behaves like a purely resistive circuit with a power factor of unity. The resonant frequency can be found from the condition

$$X_L = X_C \quad \text{or} \quad 2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

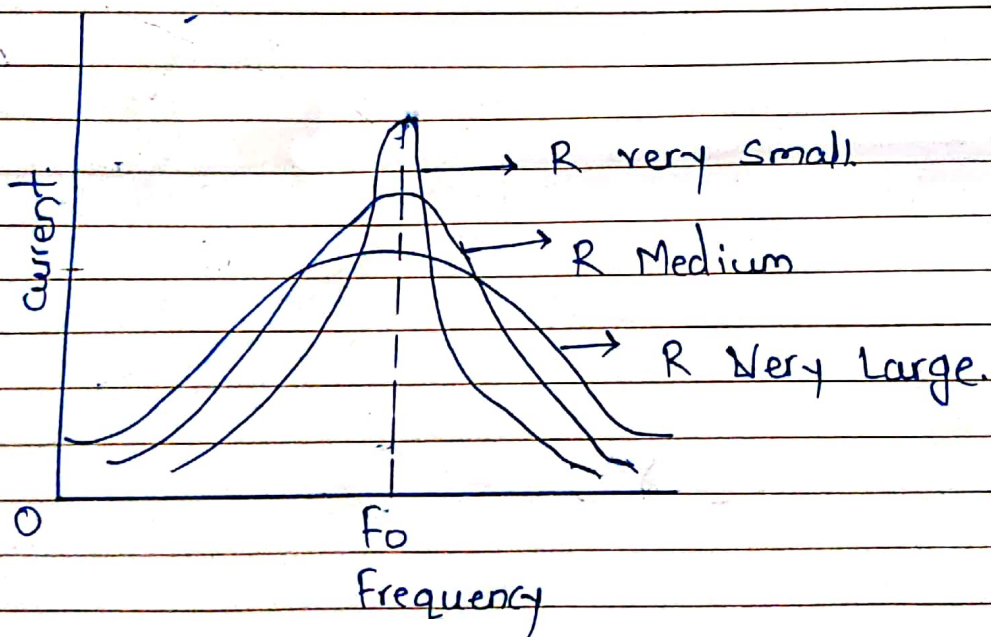
$$\therefore f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{0.159}{\sqrt{LC}} \approx \frac{0.16}{\sqrt{LC}}$$



If  $L$  is in henrys and  $C$  in farads, then  $f_0$  is in hertz.

' $f_0$ ' can be not be changed by changing either ' $L$ ' or ' $C$ '. It should be noted that resistance ' $R$ ' plays no part in determining the resonant frequency. Although it does limit the current at resonance and also affects the off resonance behaviour of circuit.

\*] Resonance Curve:



It is the curve which shows variation in circuit current with change in the frequency of the applied voltage. The shape of such a curve for various values of ' $R$ ' is shown in fig.

For smaller values of  $R$ , the resonance curve is not only sharply peaked but also has very steep sides. For large values of  $R$  the curve is flat and broad sided.

It should be particularly noted that the value of  $R$  not only affects the value of circuit current  $I$  it also affects the shape of resonance curve itself.

For low values of ' $R$ ' sides of the curve are very steep which means that the current falls off very rapidly as the frequency changes from resonance to off-resonance value. For large values of  $R$ , curve is broad sided which means limited change in current for resonance and off resonance condition.

As seen from above smaller the ' $R$ ', steeper the sides of the resonance curve and consequently sharper the tuning of the circuit.

## \*] Q-factor of a coil:

The quality factor 'Q' of a coil is given by,

$$Q = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi f L}{R}$$

The Q-value of a coil may vary from less than 10 to about 1000.

The radio frequency [RF] coils usually have Q of about 30 to 500. at low frequencies R is just equal to the d.c resistance of a coil but at radio frequencies, it represents the a.c effective resistance  $R_e$  of the coil which is much greater than 'R'. The factors which make  $R_e$  greater than 'R' are

- i) skin effect
- ii) eddy currents
- iii) hysteresis loss.

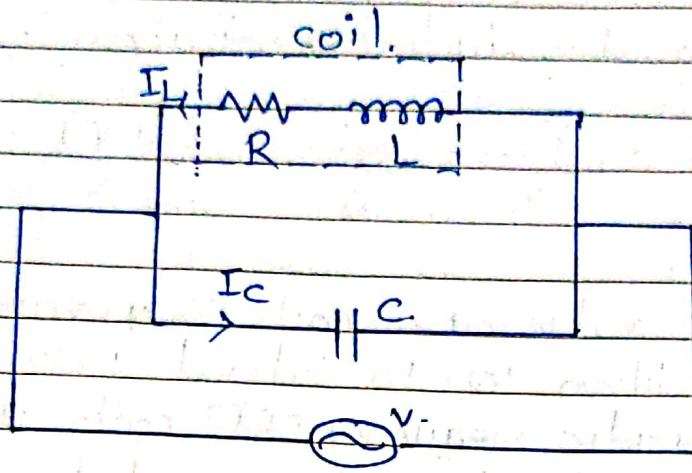
Hence Q of the coil is decreased at high frequencies because it is given by;

$$Q = \frac{X_L}{R_e}$$

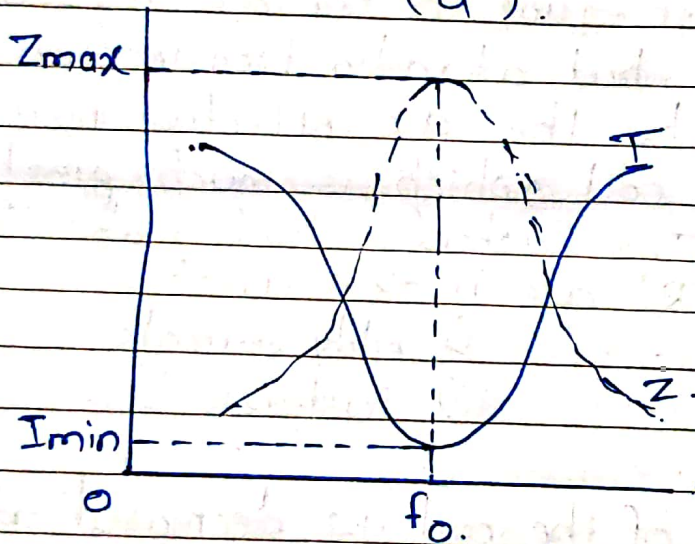
The Q of a coil can be measured with the help of Q-meter

\*7

Parallel Resonance:



(a)



(b)

In figure (a) shown that a circuit consisting of a capacitor in parallel with a coil of negligible small resistance. When fed from an AC voltage source, the capacitor draws leading current whereas coil draws a lagging current.

This circuit resonates to a frequency which makes  $X_L = X_C$  so that the two branch currents are equal but opposite.

Hence they cancel out with the result that current drawn from the supply is zero.

In practice, however, line current drawn is not zero but has minimum value due to small resistance 'R' of the coil.

Since current drawn by the circuit is minimum, it means it offers maximum impedance to the applied voltage under resonance condition.

If 'R' is neglected, then,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$I_{\min} = \frac{V}{L/CR} \quad \text{and} \quad Z_{\max} = \frac{L}{CR}$$

As seen fig.(b); current is minimum at  $f_0$  but increases for off-resonance frequencies due to decrease in the circuit impedance.

Main application of parallel resonant circuit is to act as a load in output circuit of an RF amplifier. Since impedance is max. at  $f_0$ ; amplifier gain is also max. at  $f_0$ .

When R is coil resistance;

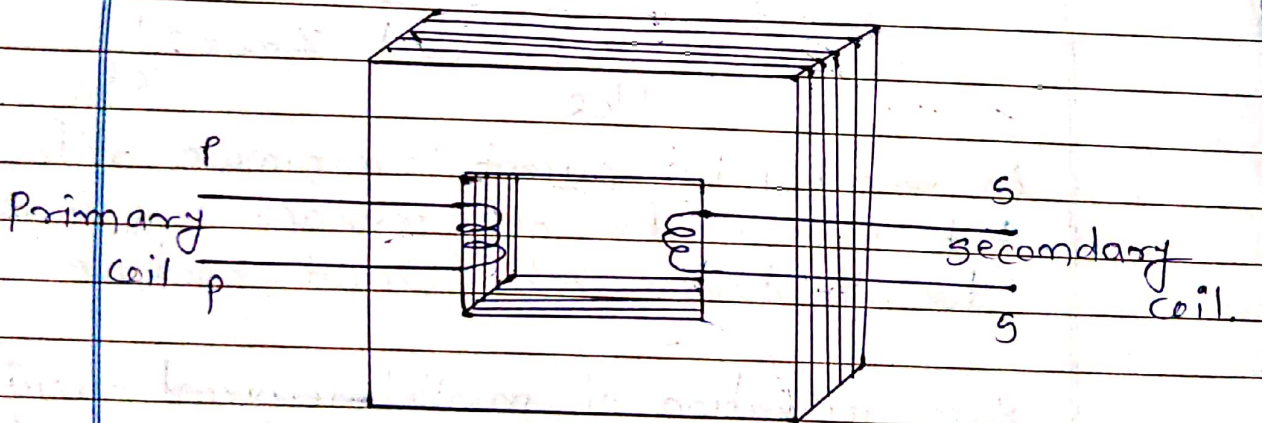
In the case of resonant parallel circuit, the band width is defined by the two points on either side of resonant frequency where value of impedance drops to 0.707 or  $1/\sqrt{2}$  of its max. value at resonance.

## \* Transformer :-

A transformer is an electrical appliance based on principle of mutual induction for converting a low a.c voltage into a high a.c voltage and vice-versa with very small loss of energy.

### Construction :-

It essentially consist of two coils known as primary and secondary wound on a rectangular or (circular) core of soft iron in the form of laminas insulated from one another. The coil to which energy is



Supplied is called the primary and that from which energy is delivered to the outer circuit is called the secondary. An alternating current in the primary coil sets up an alternating magnetic flux in the core.

This change in flux by Faraday's law induces an alternating e.m.f in the secondary coil.

The coils are so wound as to avoid leakage of magnetic flux. The soft iron used as a core has high permeability and low hysteresis loss. The core is laminated to minimise the loss of energy due to eddy currents.

Principle:

Let,  $N_p$  and  $N_s$  be the number of turns of primary and secondary coils respectively. Then the magnetic flux which is linked with the primary coil is  $N_p \phi_B A$ . Where  $A$  is area, and  $\phi_B$  is the magnetic flux linked with each turn of both the coils. The induced e.m.f.  $E_p$  in primary and  $E_s$  in the secondary is given by Faraday's law.

$$E_p = - \frac{d(N_p \phi_B A)}{dt} = - N_p A \frac{d\phi_B}{dt} \quad \text{--- (1)}$$

and

$$E_s = - \frac{d(N_s \phi_B A)}{dt} = - N_s A \frac{d\phi_B}{dt} \quad \text{--- (2)}$$

Dividing eqn (2) by eqn (1), we get

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} \quad \text{--- (3)}$$

Now, since, the resistance of primary coil is very low so that there is no energy loss, then induced e.m.f.  $E_p$  in primary will be equal to applied voltage  $V_p$  across the primary. Since the secondary may be considered to be open, this voltage  $V_s$ .

Across the secondary will be equal to the induced e. m. f.  $E_s$ . Thus, under ideal conditions,

$$\frac{V_s}{V_p} = \frac{E_s}{E_p} = \frac{N_s}{N_p} = k \quad \text{--- (4)}$$

Where  $k$  is called 'transformation ratio'

If  $I_p$  and  $I_s$  are the currents in the primary and secondary then power in secondary = power in primary.

$$V_s \cdot I_s = V_p \cdot I_p$$

$$\therefore \frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = k$$

If  $k > 1$  then the transformer is called step-up transformer, because  $V_s > V_p$

If  $k < 1$  then the transformer is called step-down transformer, because  $V_s < V_p$