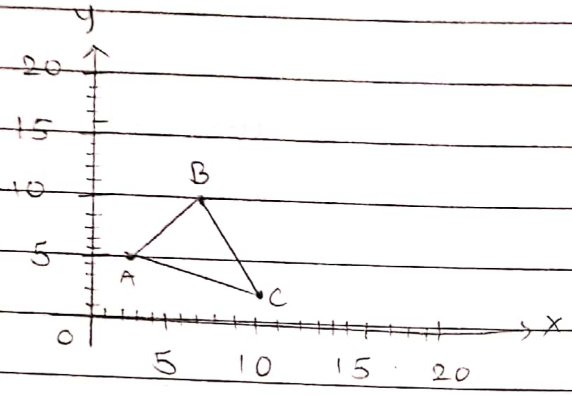
Unit III

06-01-20

Transformation



3 units x
3 unit y.

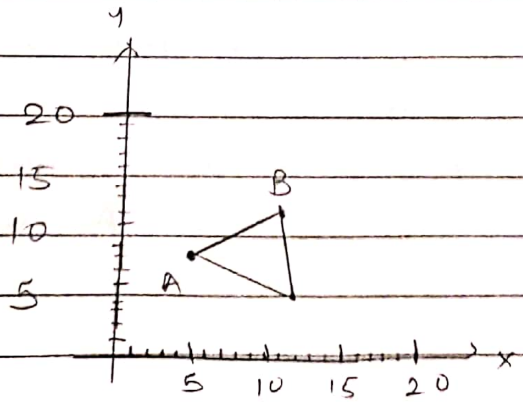
A (2, 5) ∴ A' (5, 8)
B (7, 10) B' (10, 13)
C (10, 2) C' (13, 5).

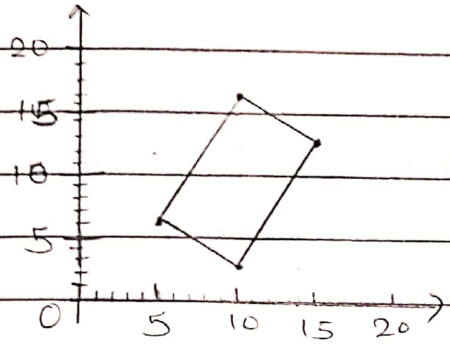
$P = \begin{bmatrix} x \\ y \end{bmatrix}$ $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ $T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$

$A' = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} \cancel{8} \\ \cancel{8} \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

$B' = \begin{bmatrix} 7 \\ 10 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 13 \end{bmatrix}$

$C' = \begin{bmatrix} 10 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$





4units x
4units y

A (5, 6) A' = (9, 10)
B (10, 17) B' = (14, 21)
C (15, 13) C' = (19, 17)
D (10, 3) D' = (14, 7)

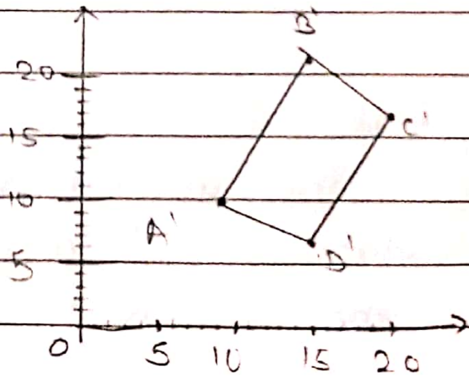
$P = \begin{bmatrix} x \\ y \end{bmatrix}$ $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ $T = \begin{bmatrix} tx \\ ty \end{bmatrix}$

$A' = \begin{bmatrix} 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \end{bmatrix}$

$B' = \begin{bmatrix} 10 \\ 17 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 14 \\ 21 \end{bmatrix}$

$C' = \begin{bmatrix} 15 \\ 13 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 19 \\ 17 \end{bmatrix}$

$D' = \begin{bmatrix} 10 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 14 \\ 7 \end{bmatrix}$



☆ Introduction :

Almost all graphics system allow programmer to define a picture that include a variety of transformation.

A programmer is able to magnify a picture so that details appear more clearly or reduce it so that more of picture is visible.
Programmer is also able to rotate the picture in different angles.

☆ Transformations :

Transformation means changing objects physically.
There are 2 types of transformation

I) Basic :
- i) Translate
- ii) Rotate
- iii) Scaling

II] Others
- i) Reflection
- ii) Shear.

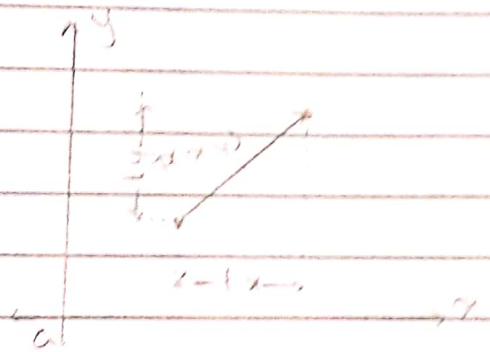
I] Basic :

The general procedure for applying translate, rotate & Scaling parameters to reposition & resize the 2D objects.

i] Translation:

It is a process of changing the position of an object in a straight line path from one co-ordinate location to another.

We can translate 2D point by adding translation distance (t_x, t_y) to the original co-ordinate position (x,y) to move the point to a new position (x', y') as shown in fig.



$$x' = x + t_x \quad \text{--- (1)}$$
$$y' = y + t_y \quad \text{--- (2)}$$

The translate distance pair (t_x, t_y) is called translation vector or shift vector.

It is possible to express the translation equation (1) & (2) as a single matrix equation by using column vector to represent co-ordinates position and translation vector.

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

These allows us to write the 2-D translation equation in the matrix form.

$$P' = P + T.$$

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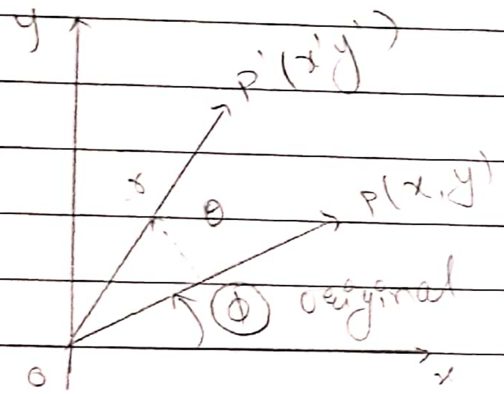
ii) Rotation

A 2-D rotation is applied to an object by repositioning it along a circular path in the (x,y) plane. To generate a rotation we specify rotation angle (θ) & the position of the angle rotation point about which the object is rotated.



Let us consider the rotation of object about the origin

Here 'r' is the constant distance of the point from the origin angle ϕ is the original angular position of the point from the horizontal & the θ is the rotation angle.

Using standard trigonometric equation we can express the transform co-ordinates in terms of angles θ & ϕ

$$\left.\begin{aligned} x' &= r\cos(\phi+\theta) = r\cos\phi\cos\theta) - r\sin\phi\sin\theta \\ y' &= r\sin(\phi+\theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta \end{aligned}\right\} \text{—}(1)$$

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The original co-ordinates of the point in polar co-ordinates are given as

$$\left.\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \end{aligned}\right\} \quad (2)$$

Substituting eqⁿ (1) into (2), we get the transformation equation for rotating point x, y through an angle θ about the origin

$$\left.\begin{aligned} x' &= x\cos\theta - y\sin\theta \\ y' &= x\sin\theta + y\cos\theta \end{aligned}\right\} \quad (3)$$

~~Fara~~ The above equation can be represented in the matrix form as given below:

$$[x'\ y'] = [x\ y]\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \text{--- } (4)$$

$$[P'] = P \cdot R \quad \text{--- } (5)$$

Where, 'R' is the rotation matrix & it is given as

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \text{--- } (6)$$

It is important to note that positive values for the rotation angle define counter clockwise rotation about the rotation point & negative values rotate objects in the clockwise sense.

For negative values, θ i.e, for clockwise rotation, rotation matrix become

$$R = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$\because = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\because \left.\begin{aligned} \cos(-\theta) &= \cos\theta \\ &\& \\ \sin(-\theta) &= -\sin\theta \end{aligned}\right\} \quad (7)$$

For eg:

A point (4,3) is rotated counter clockwise by angle 45° find the rotation matrix & resultant points.

$$[x' \; y'] = [x \; y] \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{7}{\sqrt{2}} \end{bmatrix}$$

$$= [4 \; 3] \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 2\sqrt{2} & 2\sqrt{2} \\ -2\sqrt{2} & 2\sqrt{2} \end{bmatrix} \begin{bmatrix} 3/\sqrt{2} & 3/\sqrt{2} \\ -3/\sqrt{2} & 3/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 4\sqrt{2} & 4\sqrt{2} \\ -3/\sqrt{2} & +3/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 4\sqrt{2} & -3/\sqrt{2} \\ 4\sqrt{2} & +3/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{7}{\sqrt{2}} \end{bmatrix}$$

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eg. (4, 3) angle $60°$

$$[x' \ y'] = [x \ y] \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= [4 \ 3) \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$$

$$= \cancel{4/2} \quad \cancel{\sqrt{3}/2}$$

$$= \begin{bmatrix} 4/2 & 2\sqrt{3} \\ -3\sqrt{3}/2 & 3/2 \end{bmatrix} = \begin{bmatrix} 4/2 & -3\sqrt{3}/2 \\ 2/\sqrt{3} & 3/2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4-3\sqrt{3}}{2} & \frac{4\sqrt{3}+3}{2} \end{bmatrix}$$

eg) (4, 3) angle $90°$

$$= [4, 3] \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= [0-3 \quad 4-0]$$

$$= [-3 \quad 4].$$

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* Scaling :

A scaling transformation changes the size of an object. This operations can be carried out for polygons by multiplying the co-ordinate values (x, y) of each vertex by scaling factor. (S_x, S_y) to produce the transform co-ordinates (x', y').

$$x' = \cancel{x} \cdot S_x \quad \text{——— (1)}$$

$$y' = y \cdot S_y$$

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The scaling factor S_x scales object in the x direction & scaling factor S_y scales object in the y direction.

The above eqn can be written in the matrix form as given below

$$[x' \quad y'] = [x \quad y] \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$= [x \cdot S_x \quad y \cdot S_y]$$

$$= P \cdot S.$$

Any positive numeric values are valid for scaling factor S_x & S_y. Values less than 1 reduce the size of the object & values greater than 1 produce an enlarge object.

For both S_x & S_y values equal to 1, the size of object does not change.

To get uniform scaling it is necessary to assign same values for S_x & S_y. Unequal values for S_x & S_y result in a differential scaling

Eg: Scale the polygon with co-ordinates

A (2, 5)

B (7, 10)

C (10, 2) by two units in x direction & two units in y direction.

$S_x = 2$

$S_y = 2.$

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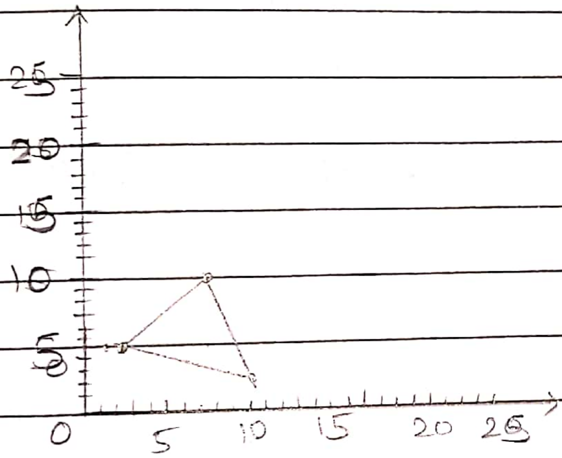
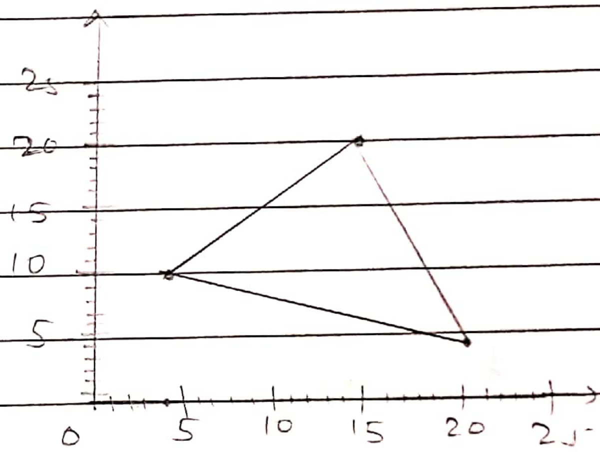
$$\begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 2 & 5 \\ 7 & 10 \\ 10 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

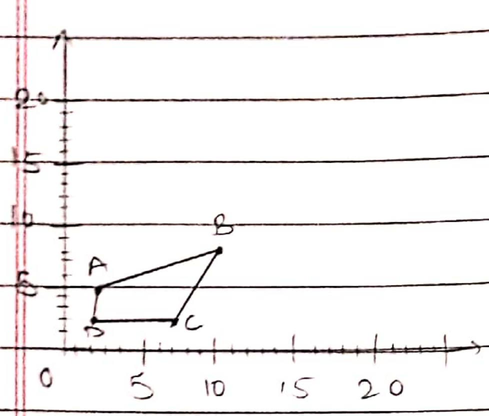
$$P = P' = \begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 4 & 10 \\ 14 & 20 \\ 20 & 4 \end{bmatrix}$$



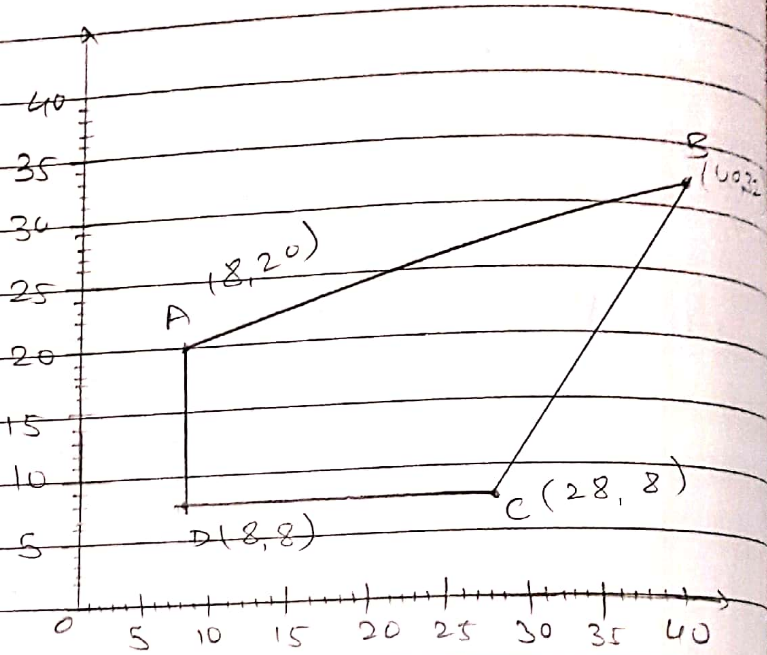


Eg: A(2, 5) units
B(10, 8) $S_x = 4$
C(7, 2) $S_y = 4$
d(2, 2)

$$P = P' = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 2 & 5 \\ 10 & 8 \\ 7 & 2 \\ 2 & 2 \end{bmatrix} = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 8 & 20 \\ 40 & 32 \\ 28 & 8 \\ 8 & 8 \end{bmatrix}$$



Before Scaling



After Scaling.

★ Other Transformation :

The 3 basic transformation translate, rotate & scaling are the most useful & most common. There are some other transformation which are useful in certain activations.

Such transformation are

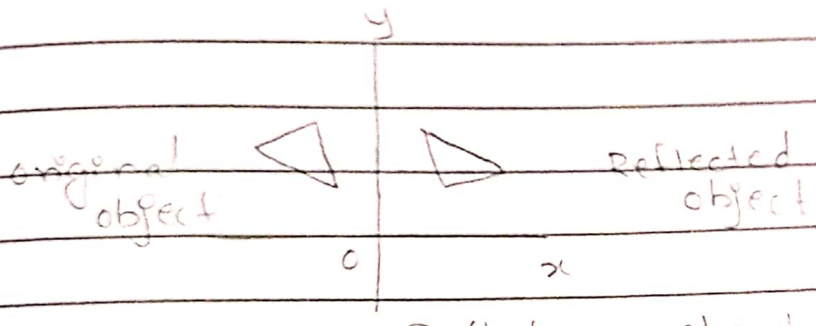
i) Reflection

ii) Shear

i] Reflection.

A reflection is a transformation that to produce a mirror image of an object relative to an axis of reflection

We can choose an axis of reflection in the (x, y) plane or perpendicular to the x, y plane.
Following table gives example of some common reflection.



Reflection about y axis

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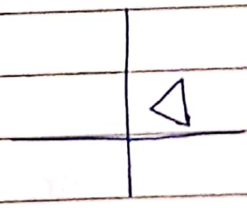
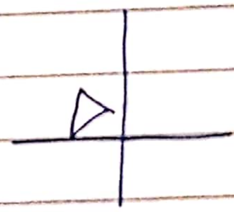
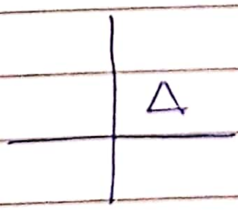
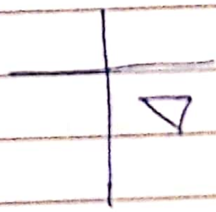
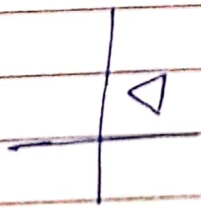
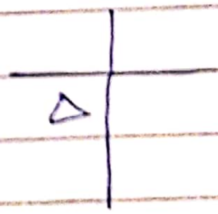
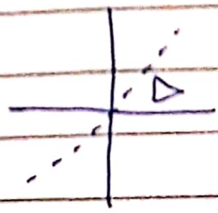
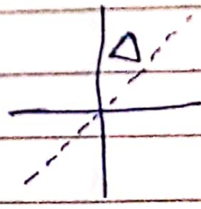
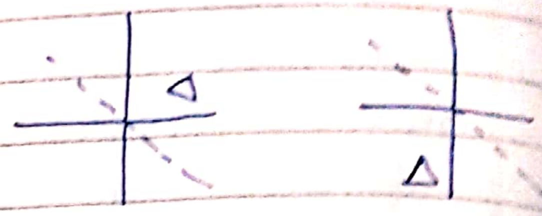
Reflection	T. Matrix	OI	RI
Y axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		
X axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		
Reflection about origin	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		
Reflection about $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		

Reflection about $y = -x$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



ii] Shear :

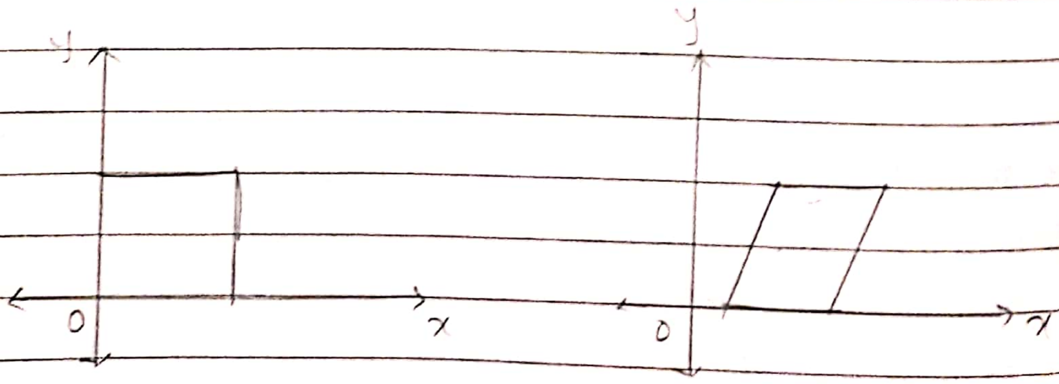
A transformation that slant (slope) the shape of an object is called the shear transformation.

There are 2 common shearing transformation are used.

One shift x- co-ordinate values & other shift y co-ordinate values, but in both case only one co-ordinates (x or y) changes its co-ordinates & other preserve its values.

i] x- Shear :

The x-shear preserve the y co-ordinates but changes the x- values which causes vertical lines to tilt right or left as shown in following fig.



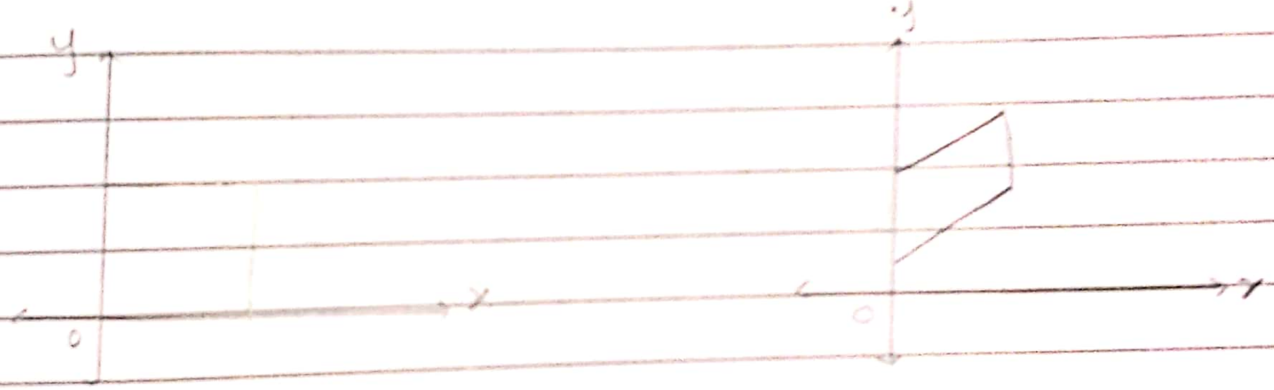
original object

after x shear

x_sh $\begin{bmatrix} 1 & 0 & 0 \\ sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$x' = x + sh_x \cdot y$
$y' = y$.

ii] Y- shear :



Original object   after y shear.

The y- shear preserve the x- co-ordinates but changes the y- values which causes horizontal lines to transform into lines which slope up or down as shown in above fig.

y_shear $\begin{bmatrix} 1 & sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$x' = x$
$y' = y + sh \cdot x$

We can apply x & y shear transformation relative to other reference line. In x-shear transformation we can use y reference line & in y-shear transformation we can use x-reference line.
The transformation matrices for both are given below.

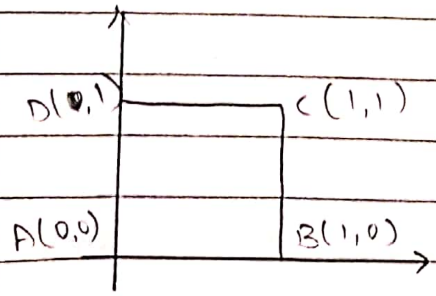
X shear with y reference line
$$\begin{bmatrix} 1 & 0 & 0 \\ -sh_x & 1 & 0 \\ -sh_x \cdot y_{ref} & 0 & 1 \end{bmatrix}$$

y shear with x reference line
$$\begin{bmatrix} 1 & sh_y & 0 \\ 0 & 1 & 0 \\ 0 & -sh_y \cdot x_{ref} & 1 \end{bmatrix}$$

Q: Apply the shearing transformation to square with A(0, 0) B(1, 0) C(1, 1) D(0, 1) as given below.

a) shear parameter values ~~0.5~~ → 0.5 relative to the line y-reference = -1.

b) shear parameter values 0.5 relative to line x-reference = -1



A(0, 0)
B(1, 0)
C(1, 1)
D(0, 1)

$Sh_x = 0.5$ & $y_{ref} = -1$

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ -Sh_x \cdot y_{ref} & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & 1 \\ 1+0.5 & 0 & 1.5 \\ 1+0.5+0.5 & 1 & 1 \\ 0.5+0.5 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & 1 \\ 1.5 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(A (0.5, 0) B (1.5, 0) C(2, 1) D(1, 1)