# Formulating Hypotheses Parametric Tests

Business Research Methodology

MBA : $2^{nd}$ Semester

# Formulating Hypotheses

- Begins with an assumption called Hypothesis.

- Hypothesis
- Claim (assumption) about a population parameter.
- An unproven proposition or supposition that tentatively explains certain facts or phenomena.
- A proposition that is empirically testable.

# What is Hypothesis testing

- A set of logical and statistical guidelines used to make decisions from sample statistics to population characteristics.

# Hypotheses Testing

- The intent of hypothesis testing is to formally examine two opposing conjectures (hypotheses), $H_o$ and $H_A$.
- These two hypotheses are mutually exclusive and exhaustive.
- Sample information is collected and analysed.

# Basic Concepts in Hypotheses Testing

- Null Hypotheses &Alternate Hypotheses
- Level of Significance
- Critical Region
- Decision Rule(Test of Hypothesis)
- Type I & Type II Errors
- Power of Test
- Two Tailed & One Tailed Tests
- One Sample & Two Sample Tests
- Parametric & Non Parametric Tests

# Null Hypothesis

- Specific statement about a population parameter made for the purposes of argument.
- States the assumption to be tested, is a status quo.
- Is always about a population parameter, not about a sample statistic.

# Alternate Hypothesis

- Represents all other possible parameter values except that stated in the null hypothesis.

- Challenges the status quo.

- Hypothesis that is believed (or needs to be supported) by the researcher –a research hypothesis.

# Level of Significance

- The critical probability in choosing between the null & alternative hypotheses.
- The probability of making a Type I error.
- The higher the significance level, the higher the probability of rejecting a null hypothesis when its true.
- Risk that a researcher is willing to take of rejecting the null hypotheses when it happens to be true.

- Confidence level:

  A percentage or decimal value that tells how confident a researcher can be about being correct.

# Critical Region

- The rejection region.
- If the value of mean falls within this region, the null hypothesis is rejected.
- Critical value

  The value of a test statistic beyond which the null hypothesis can be rejected.

# Decision Rule(Test of Hypothesis)

- The rule according to which we accept or reject null hypothesis.

# Type I Error & Type II Error

- A Type I error is the mistake of rejecting the null hypothesis when it is true.
- $\alpha$- Type I error.

- A Type II error is the mistake of failing to reject the null hypothesis when it is false.
- $\beta$- Type II error.

# Type I Error & Type II Error

- Probability  of Type I error is determined in advance.
- Level of significance of testing the hypothesis.

# Type I Error & Type II Error

| | Accept Ho | Reject Ho |
|---|---|---|
| Ho (True) | Correct Decision | Type I Error |
| Ho (False) | Type II Error | Correct Decision |

# Type I Error & Type II Error

Suppose there is a test for a particular disease.

- If the disease really exists and is diagnosed early, it can be successfully treated
- If it is not diagnosed and treated, the person will become severely disabled
- If a person is erroneously diagnosed as having the disease and treated, no physical damage is done

# Type I Error & Type II Error

| Decision | No disease | Disease |
|---|---|---|
| Not diagnosed | OK | Type I error |
| Diagnosed | Type II error | OK |

treated but not harmed
by the treatment

irreparable damage
would be done

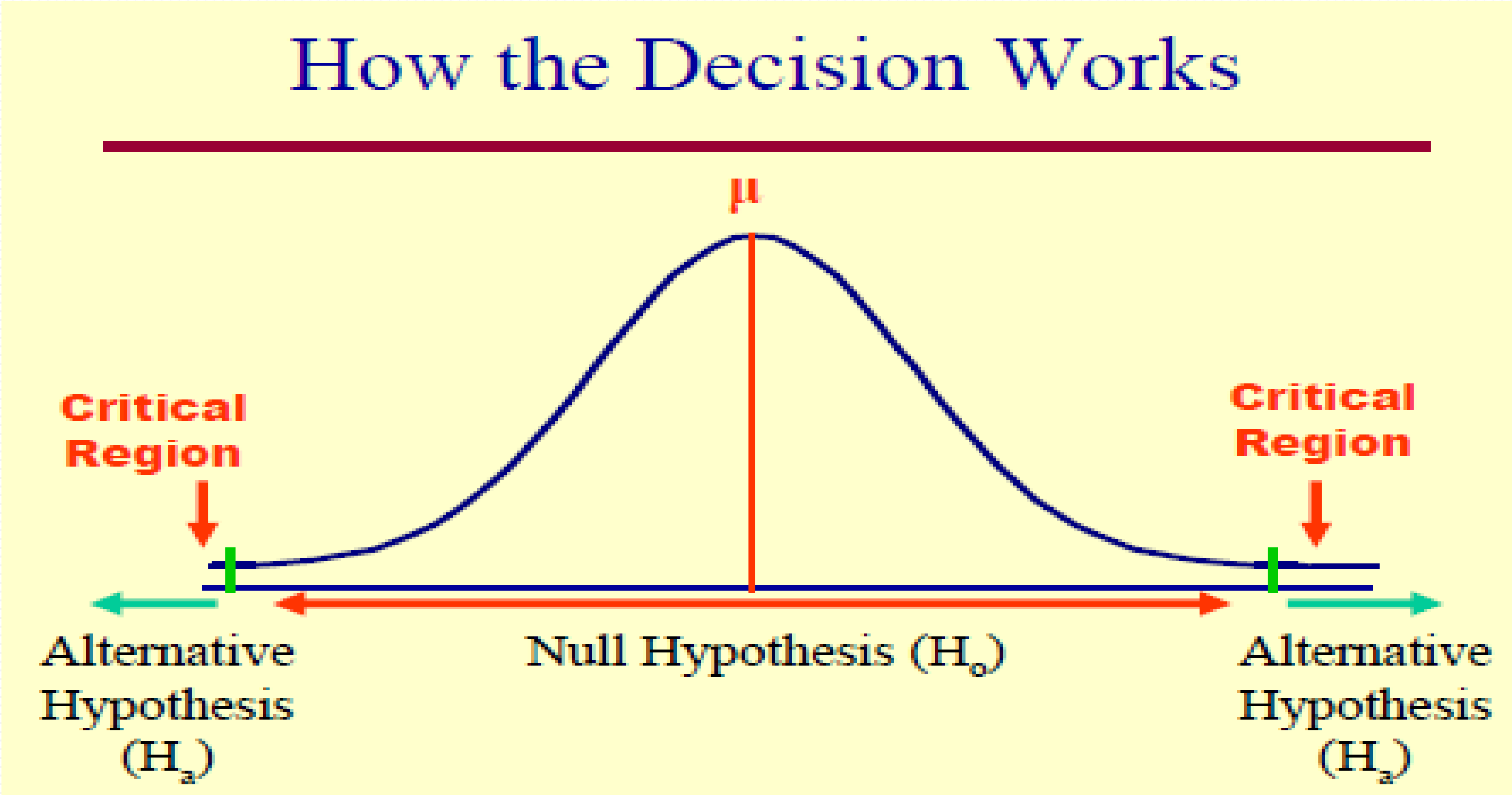
# Power of Test

- The ability of a test to reject a false null hypothesis.
- The probability of supporting an alternative hypothesis that is true.
- Power = 1- $\beta$
- High value of 1- $\beta$(near 1) means test is working fine, it is rejecting a null hypothesis when it is false.

# One Tailed & Two Tailed Test

- Two-Tailed Tests
- If the null hypothesis is rejected for values of the test statistic falling into either tail of its sampling distribution.
- A deviation in either direction would reject the null hypothesis
- Normally $\alpha$ is divided into $\alpha/2$ *on one* side and $\alpha/2$ on the other.

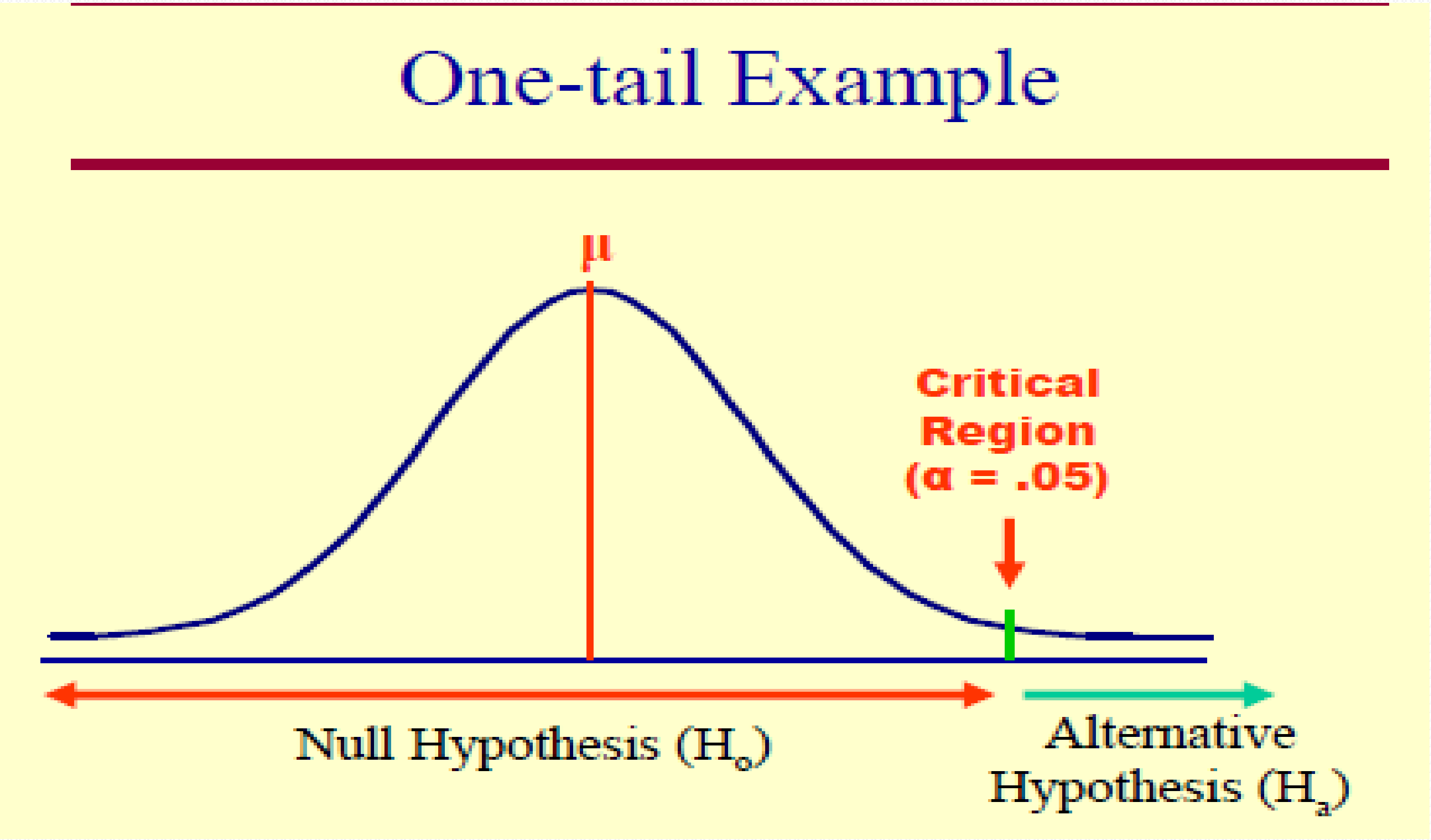
# One Tailed & Two Tailed Test



# One Tailed & Two Tailed Test

- <u>One-Tailed Tests</u>
- Only used when the other tail is nonsensical.
- If null hypothesis is rejected only for values of the test statistic falling into one specified tail of its sampling distribution.

# One Tailed & Two Tailed Test



# One Tailed & Two Tailed Test

- A manufacturer of a light bulb wants to produce bulbs with a mean life of 1000 hours. If the lifetime is shorter, he will lose customers to the competitors; if the lifetime is longer, he will have a very high production cost because the filaments will be very thick. Determine the type of test.
- The wholesaler buys bulbs in large lots & does not want to accept bulbs unless their mean life is at least 1000 hours. Determine the type of test.

# One Tailed & Two Tailed Test

- A highway safety engineer, decides to test the load bearing capacity of a bridge that is 20 yrs old. Considerable data is available from similar tests on the same type of bridge. Which type of test is appropriate? If the maximum load bearing capacity of this bridge must be 10 tons, what are the null & alternative hypotheses?

# One Sample & Two Sample Tests

- <u>One Sample Test</u>

  When we want to draw inferences about the population on the basis of given sample.

- <u>Two Sample Test</u>

  When we want to compare and draw inferences about 2 populations on the basis of given samples.

# Independent & Paired Samples

- Independent Samples
- Drawn randomly from different populations.

- Paired Samples
- When the data for the two samples relate to the same group of respondents.

# Types of Hypotheses

- Research hypotheses.
- Logical hypotheses.
  - Null hypothesis (Ho).
  - Alternative hypothesis (Ha).
- Statistical hypotheses.

# Research Hypotheses.

- Statement in words as to what the investigator expects to find.

- Example.

  Students who drink caffeine will be able to memorise information faster than students who do not drink caffeine.

# Logical Hypotheses

- Stated in terms of null & alternate hypotheses.
- Null Hypothesis (Ho).

  Students who drink caffeine will be not be able to memorise information faster than students who do not drink caffeine.
- Alternative Hypothesis (Ha).

  Students who drink caffeine will be able to memorise information faster than students who do not drink caffeine.

# Statistical Hypotheses

- Statement in statistical terms as to what would be found if the research hypothesis is true.

- A sales manager has asked her salespeople to observe a limit on travelling expenses. The manager hope to keep expenses to an average of $ 100 per salesman per day. What will be Ho and Ha?

# Steps in Hypotheses Testing

1. Formulation of the null and alternate hypothesis
2. Definition of a test statistic
3. Determination of the distribution of the test statistic
4. Definition of critical region of the test statistic
5. Testing whether the calculated value of the test statistic falls within the acceptance region.

# 1: Formulation of $H_0$

- The Null hypothesis assumes a certain specific value for the unknown population parameter.
- Defined as an inequality – greater than or less than.
- For example, if the mean of a population is considered, then
  - Ho: $\mu \leq \mu_o$
  - Ho: $\mu = \mu_o$
  - Ho: $\mu \geq \mu_o$

# 2: Formulation of $H_a$

- The alternate hypothesis assigns the values to the population parameter that is not contained in the null hypothesis.
- For example,
  - Ha: $\mu > \mu_o$
  - Ha: $\mu \neq \mu_o$
  - Ha: $\mu < \mu_o$
- The null hypothesis is accepted or rejected on the basis of the information provided by the sample.

# 3: Definition of a Test Statistic

- A test statistic must be defined to test the validity of the hypothesis.
- The test statistic is computed from sample information.
- A number calculated to represent the match between a set of data and the expectation under the null hypothesis

*A test that uses the z-score as a test* statistic is called a *z-test.*

# 4: Determination of the distribution of the test statistic

- The probability distribution of the test statistic depends on the null hypothesis assumed, the parameter to be tested, and the sample size. Commonly used ones are the Normal, Student's "t", Chi-square and F-distributions.

# 5: Definition of the critical region for the test statistic

- The set of values of the test statistic that leads to the rejection of $H_o$ in favour of $H_a$ is called the rejection region or critical region.
- Depends upon whether the testing is one-sided or two-sided.

# 6: Decision rule

- A decision rule is used to accept or reject the null hypothesis.
- P- value

  $P < \alpha$

  Reject the null hypothesis

  Statistically significant

- Test statistic

  Test statistic (calculated value) < Table value of $\alpha$

  Accept $H_o$

  Statistically insignificant

# 7: Outcome

- The acceptance or rejection of the hypothesis will lead to the following possible outcomes:

- To accept $H_o$ when $H_o$ is true- Correct decision
- To reject $H_o$ when $H_o$ is false- Correct decision
- To reject $H_o$ when $H_o$ is true- Type I error
- To accept $H_o$ when $H_o$ is false- Type II error

# 8: Error probabilities

- The inferences made on the basis of the sample information would always have some degree of error.

  $\alpha$ = Type I error = Rejecting $H_o$ when $H_o$ is true

  $\beta$ = Type II error = Accepting $H_o$ when $H_o$ is false

- Rejecting a null hypothesis (Type I error) is more serious than accepting it when it is false. Therefore, the error probability $\alpha$ is referred to as the significance level.

# Parametric & Non Parametric Tests

- Parametric Test
- Statistical procedures that use interval or ratio scaled data and assume populations or sampling distributions with normal distributions.

- Non Parametric Test
- Statistical procedures that use nominal or ordinal scaled data and make no assumptions about the distribution of the population.

# Parametric Tests

- z-test
- t-test
- F-test
- Chi square test

# Conditions for using Tests

| | Population standard deviation **known** | Population standard deviation **unknown** |
|---|---|---|
| N larger than 30 | **z- Test** | **z- Test** |
| N smaller than 30 | **z- Test** | **t-Test** |

# t-Test

- Used when sample size is ≤ 30.
- Given by W.S. Gosset (pen name *Student*)
- Also called Student's *t* distribution.
- Based on t distribution.
- The relevant test statistic is t.

# t-Test

- Conditions
- Sample should be small.
- Population standard deviation must be unknown.

- Assumption
- Normal or approximately normal population.

# t Distribution

- Characteristics
- Flatter than normal distribution.
- Lower at mean, higher at tails than normal distribution.
- As degrees of freedom increase, t-distribution approaches the standard normal distribution (df=8 or more)
- Degrees of Freedom: No. of observations minus the no. of constraints or assumptions needed to calculate a statistical term.

# t-Test

- Confidence Interval $\overline{x} \pm t\sigma_{\overline{X}}$

$$t = \frac{\overline{x} - \mu_{H0}}{\sigma_{\overline{x}}}$$

$$\sigma_{\overline{X}} = \frac{\sigma_p}{\sqrt{n}}$$

$t - Distribution$

$$\hat{\sigma}_{\overline{X}} = \frac{\hat{\sigma}}{\sqrt{n}}$$

Degrees of Freedom: ***n-1***

# t-Test

- The specimen of copper wires drawn from a large lot to have the following breaking strength (in Kg weight)

578, 572, 570, 568, 572, 578, 570, 572, 596, 544

- Test whether the mean breaking strength of the lot may be taken to be 578 kg, at 5% significance level.

$$S = \sqrt{\frac{\sum\left(X - \overline{X}\right)^2}{n-1}}$$

# t-Test

- Given a sample mean of **83**, a sample standard deviation of **12.5**, & a sample size of **22**, test the hypothesis that the value of population mean is **70** against the alternative that is more than 70. Use **0.025** significance level.

# z -Test

- Based on the normal distribution.
- Mostly used for judging the significance level of mean.
- The relevant test statistic is z.
- The value of z is calculated & compared with its probable value.

- If calculated value is less than table value- accept $H_o$

$$z = \frac{\overline{x} - \mu_{H0}}{\sigma_{\overline{x}}} \qquad \sigma_{\overline{X}} = \frac{\sigma_p}{\sqrt{n}}$$

# z -Test

- The mean of a certain production process is known to be **50** with a standard deviation of **2.5**. The production manager may welcome any change in mean value towards higher side but would like to safeguard against decreasing values of mean. He takes a sample of **12** items that gives a mean value of **48.5**. What inference should the manager take for the production process on the basis of sample results? Use **5%** significance value.

# z -Test

- A sample of **400** male students is found to have a mean height **67.47** inches. Can it be reasonably regarded as a sample from a large population with mean height **67.39** inches & standard deviation **1.30** inches? Test at **5%** level of significance.

# z -Test

- Hinton press hypothesizes that the average life of its largest web press is **14,500** hours. They know that the standard deviation of press life is **2100** hours. From a sample of **36** presses , the company finds a sample mean of **13,000** hours. At a **0.01** significance level, should the company conclude that the average life of the presses is less than the hypothesized **14,500** hours?

# F-Test

- Based on F-Distribution
- Used to compare variance of 2 independent samples.
- Relevant Test statistic is F.
- Larger the F value, greater the possibility of having statistically significant results.

# F-Test

- <u>Characteristics</u>

- Family of distributions .
- Has 2 degrees of freedom.

$$F = \frac{\sigma_{s1}^2}{\sigma_{s2}^2}$$

$$\sigma^2 = \frac{\sum\left(X - \overline{X}\right)^2}{n-1}$$

# F-Test

- Two random samples drawn from two normal populations are

| Sample 1 | 20 | 16 | 26 | 27 | 23 | 22 | 18 | 24 | 25 | 19 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample 2 | 27 | 33 | 42 | 35 | 32 | 34 | 38 | 28 | 41 | 43 | 30 | 37 |

- Test using variance ratio of 5% and 1% level of significance whether the two populations have the same variance.