CHAPTER 4 :
SIMPLE & COMPOUND INTEREST

4.0 Introduction

4.1 Simple Interest

4.2 Compound Interest

4.0 INTRODUCTION

Interest – Definition

- Interest is money earned when money is invested.
- Interest is charged incurred when a load or credit is obtained.
- 2 types of interest:
  - Simple interest
  - Compound interest

4.1 SIMPLE INTEREST

Interest calculated on original principle for the entire period it is borrowed / invested

Simple Interest – Formula

- The formula to calculate the *simple interest* is given by

$$I = PRT$$

where :

I = Simple Interest
P = Principal / Investment
R = rate per annum (year)
T = Time in years

4.1 SIMPLE INTEREST

The sum of the original principal & the interest earned

Simple Interest – Simple amount

- The *simple amount* formula is given as

$$S = \text{Principal} + \text{Interest earned}$$
$$= P + PRT$$
$$= P(1 + RT)$$

where :
S = Simple amount
P = Principal / Investment
R = Rate per annum (year)
T = Time in years

EXAMPLE 1

1. RM 1000 is invested at two years in a bank, earning a simple interest rate of 8% per annum. Determine the simple interest earned.

Solution

$P = 1000$

$R = 8\% = 0.08$

$T = 2$

$I = PRT$

$= 1000 \times 0.08 \times 2$

$= 160$

EXAMPLE 1

2. RM 10, 000 is invested for 4 years 9 months in a bank earning a simple interest rate of 10% per annum. Calculate the simple amount at the end of the investment period.

Solution

$P = 10{,}000$

$R = 10\% = 0.10$

$T = 4 \text{ years } 9 \text{ months}$

$= 4\frac{9}{12} = 4.75$

$$S = P(1 + RT)$$
$$= 10{,}000(1 + 0.1 \times 4.75)$$
$$= 14{,}750$$

EXAMPLE 1

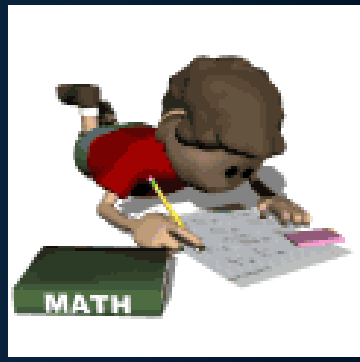
3. Hasif invests RM 5000 in an investment fund for 3 years. At the end of the investment period, his investment will be worth RM 6125. Determine the simple interest rate that is offered.

4. How long does it take a sum of money to triple itself at a simple interest rate of 5 % per annum?

5. Haris invested RM 10,000 in two accounts, some at 10% per annum and the rest at 7 % per annum. His total interest for one year was RM 820. Calculate the amount invested at each rate.

Solution

PRACTICE 1



1. RM 500 is invested at 12 % per annum. Calculate:
   i. The interest at the end of a year.
   ii. The interest earned at the end of 5 years.

2. A person borrows RM 4000 for a period of 6 years at 20 % simple interest per annum. Calculate:
   i. The amount of interest payable on the loan.
   ii. The total amount to be repaid.

3. RM 700 is invested for 5 years. At the end of this time the simple interest amounts to RM 420. Work out the rate of simple interest.

PRACTICE 1

1. RM 500 is invested at 12 % per annum. Calculate:
   i. The interest at the end of a year. **RM60**
   ii. The interest earned at the end of 5 years. **RM300**

2. A person borrows RM 4000 for a period of 6 years at 20 % simple interest per annum. Calculate:
   i. The amount of interest payable on the loan. **RM4800**
   ii. The simple amount to be repaid. **RM8800**

3. RM 700 is invested for 5 years. At the end of this time the simple interest amounts to RM 420. Work out the rate of simple interest. **12%**

4.1 SIMPLE INTEREST

Simple Interest – Present Value

The present value of an investment (or debt) due on some future date is the value now when invested today would have become the same value as the investment (or debt) due on some future date.

- The formula to calculate the *present value* is given by

$$P = \frac{S}{(1+RT)} = S(1+RT)^{-1}$$

where :

P = Present Value
S = Simple amount
R = rate per annum (year)
T = Time in years

EXAMPLE 2

Find the present value at 8% simple interest of a debt RM3000 due in ten months.

Solution

$S = 3000$

$R = 8\% = 0.08$

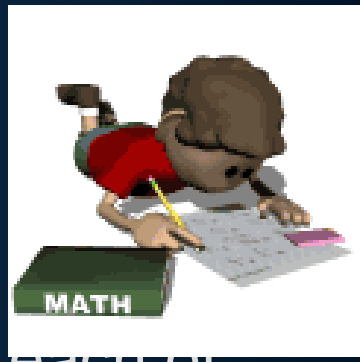
$T = \frac{10}{12}$

$$P = \frac{S}{(1+RT)} = \frac{3000}{\left(1+0.08\left(\frac{10}{12}\right)\right)}$$

$$= 3000 \times \frac{15}{16}$$

$$= 2812\frac{1}{2} @ \text{RM}\,2812.50$$

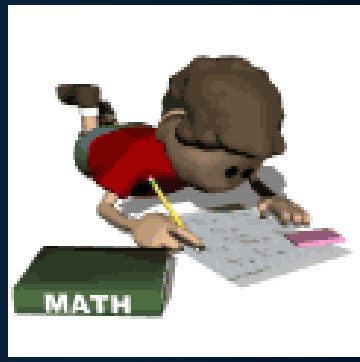
PRACTICE 2



1. Work out the amount of simple interest for each of the following:
   i. RM 800 invested for 1 year at 6 % per annum.
   ii. RM 1,500 borrowed for 6 years at 11 % p.a.
   iii. RM 2,000 invested for 7 years at 14 % p.a.

2. Find the length of time for
   i. RM 1,000 to be the interest on RM 5,000 invested at 5 % p.a.
   ii. RM 480 to be the interest on RM 2,000 invested at 8 % p.a.
   iii. RM 1,200 to be the interest on RM 3,000 invested at 10 % p.a.

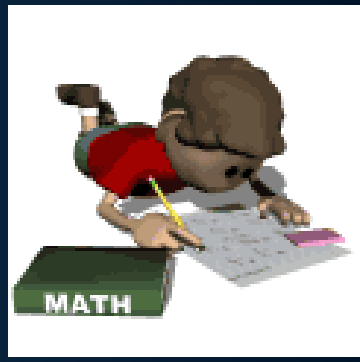
PRACTICE 2



3. Find the rate per annum on simple interest for:
   i. RM 420 to be the interest on RM 1,200 invested for 5 years
   ii. RM 72 to be the interest on RM 200 invested for 3 years.
   iii. RM 1,200 to be the interest on RM 3,000 invested for 4 years.

4. Find the principal required for:
   i. The simple interest to be RM 600 on money invested for 3 year at 5 % p.a.
   ii. The simple interest to be RM 40 on money invested for 2 years at 10 % p.a.
   iii. The simple interest to be RM 1,500 for 4 years at 9 % p.a.?

PRACTICE 2



5. RM 500 is invested for 7 years at 10 % p.a. simple interest. How much will the investment be worth after this period?

6. RM 2,000 was invested at 12% p.a. simple interest. The investment is now worth $2,960. For how long was the money invested?

4.2 COMPOUND INTEREST

Compound Interest – Definition

- Compound interest computation is based on the principal which changes from time to time.
- Interest that is earned is compounded / converted into principal & earns interest thereafter.
- The principal increases from time to time.

4.2 COMPOUND INTEREST

Compound Interest – Definition

- Differences between simple interest & compound interest:

	Simple interest	*Compound interest*
Based on	*original principal*	*principal that grows from 1 interest interval to another*
Function	*Linear*	*Exponential*

4.2 COMPOUND INTEREST

Compound Interest – Terms

Terms	*Symbols*
Original principal	***P***
Nominal interest rate (per year)	***r***
Frequency of conversions	***m***
Periodic interest rate	***i = r/m***
investment period/ term (years)	***t***
Number of conversion periods in the investment period	***n = mt***

EXAMPLE 3

Suppose RM 9000 is invested for seven years at 12% compounded quarterly.

Terms used

$P = 9000$

$r = 12\% \rightarrow$ interest calculated 4 times a year

$m = 4$

$t = 7$

$$i = \frac{r}{m} = \frac{12\%}{4} = 3\%$$

$$n = mt = 4(7) = 28$$

4.2 COMPOUND INTEREST

Compound amount / future value is ***S*** after ***n*** interest periods

Compound Interest – Formula

- The formula to calculate the *compound amount* is given by

$$S = P(1+i)^{n}$$

$n = mt$

$i = \frac{r}{m}$

where : S = Compound amount
P = Original principal
m = frequency of conversions
r = nominal interest rate (per year)
t = investment period

- The *compound interest* is $I = S - P$

EXAMPLE 4

Determine the future value of RM 1000 which was invested for :

a) 4 years at 4% compounded annually
b) 5 years 6 months at 14%compounded semi – annually
c) 2 years 3 months at 4% compounded quarterly
d) 5 years 7 months at 5% compounded monthly
e) 2 years 8 months at 9% compounded every 2 months
f) 250 days at 10% compounded daily

EXAMPLE 4

Solution

$P = 1{,}000$

$$S = P(1+i)^n$$

a) 4 years at 4% compounded annually

$r = 4\%;\ m = 1;\ t = 4\ years;$

$$i = \frac{4\%}{1} = 4\%;\ n = 1(4) = 4$$

$$S = 1000(1+4\%)^4 = \text{RM}1{,}169.86$$

b) 5 years 6 months at 14% compounded semi – annually

$$r = 14\%;\ m = 2;\ t = 5\ years\ 6\ months = 5\frac{6}{12} = 5.5$$

$$i = \frac{14\%}{2} = 7\%;\ n = 2(5.5) = 11$$

$$S = 1000(1+7\%)^{11} = \text{RM}2{,}104.85$$

EXAMPLE 4

Solution

$P = 1{,}000$

$$S = P(1+i)^n$$

c) 2 years 3 months at 4% compounded quarterly

d) 5 years 7 months at 5% compounded monthly

EXAMPLE 4

Solution

$P = 1{,}000$

$$S = P(1+i)^n$$

e) 2 years 8 months at 9% compounded every 2 months

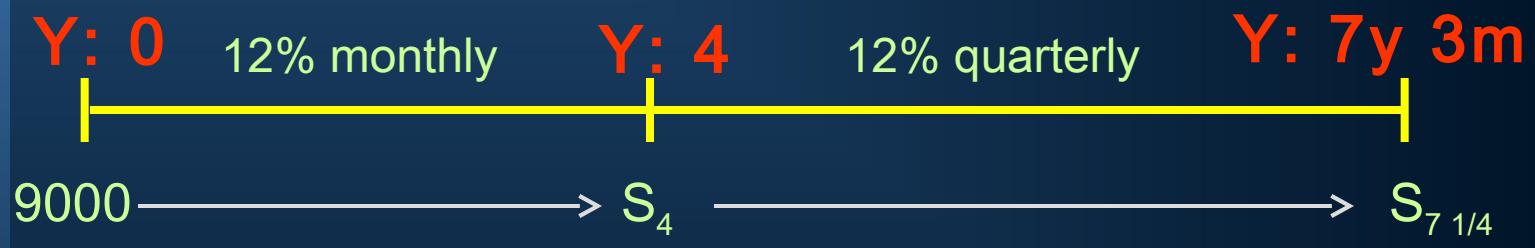
f) 250 days at 10% compounded daily

1 year = 360 days

EXAMPLE 5

RM 9000 is invested for 7 years 3 months. This investment is offered 12% compounded monthly for the first 4 years & 12% compounded quarterly for the rest of the period. Calculate the future value of this investment.

Solution



$P = 9000$

$r_1 = 12\%;\ m_1 = 12;\ t_1 = 4\ years$

$i = \dfrac{12\%}{12} = 1\%;\ n = 12(4) = 48$

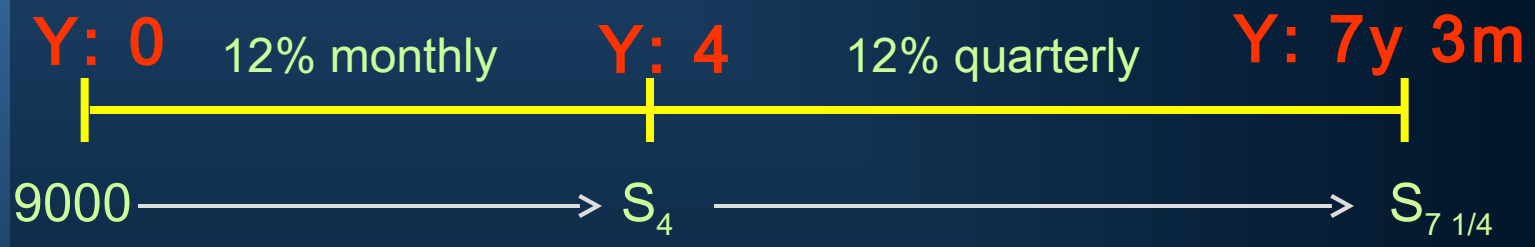
Amount of investment at the end of 4 years:

$$S_4 = 9000(1+1\%)^{48}$$
$$= \text{RM}14{,}510.03$$

EXAMPLE 5

RM 9000 is invested for 7 years 3 months. This investment is offered 12% compounded monthly for the first 4 years & 12% compounded quarterly for the rest of the period. Calculate the future value of this investment.

Solution



$P = S_4 = 14{,}510.03$

$r_2 = 12\%;\ m_2 = 4;$

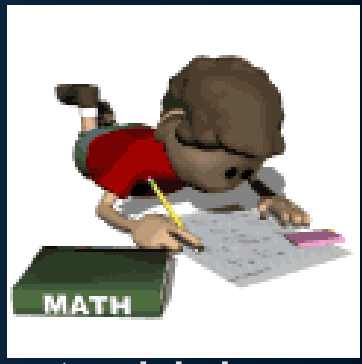
$t_1 = 3\ years\ 3\ months = 3.25$

$i =$

$n =$

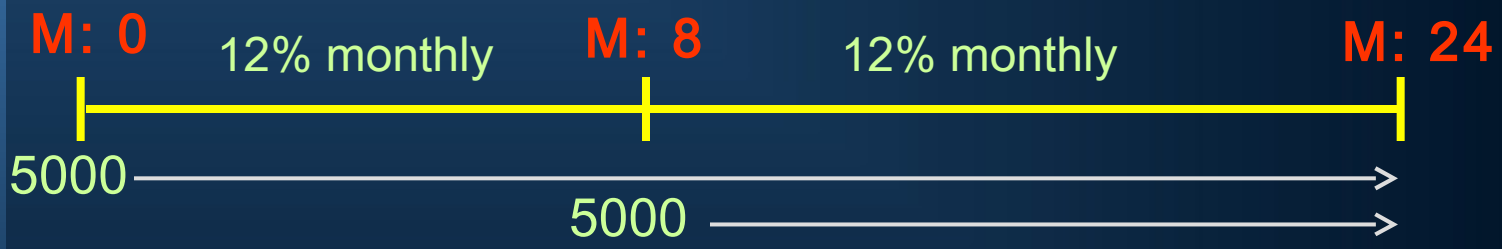
Amount of investment at the end of 7 ¼ years:

EXAMPLE 6



Julia saved RM 5,000 in a savings account which pays 12% compounded monthly. 8 months later she saved another RM 5,000. Determine the amount in the account two years after her first saving.

Solution



EXAMPLE 7

What is the nominal rate compounded monthly that will make RM 1,000 become RM 2,000 in five years?

Solution

$P = 1{,}000;\ S = 2{,}000;$

$m = 12;\ t = 5\ years;$

$i = \frac{r}{12};\ n = 12(5) = 60$

$$2{,}000 = 1{,}000\left(1 + \frac{r}{12}\right)^{60}$$

$$2 = \left(1 + \frac{r}{12}\right)^{60}$$

$$2^{\frac{1}{60}} = 1 + \frac{r}{12}$$

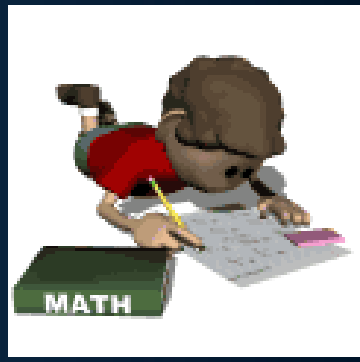
$$1.0116 = 1 + \frac{r}{12}$$

$$\frac{r}{12} = 0.0116$$

$$r = 0.1394$$

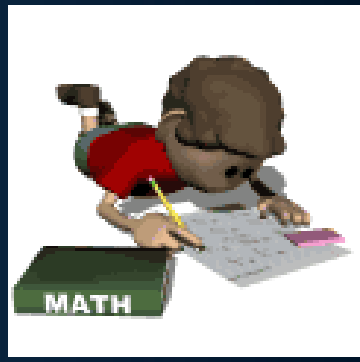
r is 13.94%

PRACTICE 3



1. Calculate the future values for the following investment:
   i. RM 1000 at 7% compounded annually for 8 years
   ii. RM 2500 at 9% compounded semi – annually for 10 years
   iii. RM 42000 at 7.75% compounded quarterly for 8 years
   iv. RM 180,000 at 9% compounded monthly for 6 years and 3 months.
   v. RM 150,000 at 12% compounded daily for 3 years.
2. At what rate compounded semi – annually will RM 2000 become RM3500 in five years?
3. Shima invested a certain sum of money in an account that pays 5% compounded quarterly. The account will amount to RM 1000 in 27 months' time. Calculate the original principal that was invested.

PRACTICE 3



4. How long will it take for RM 5000 to grow to RM 6000 if the investment earns interest at the rate of 12% compounded monthly?
5. How long will it take an investment of RM 2000 to double if the investment earns interest at the rate of 9% compounded monthly?
6. Calculate the amount to be invested now at 6% compounded monthly so as accumulate RM 8888 in three years.
7. Five years ago, Adib had saved RM 10,000 in an account that pays 6% compounded monthly. Now he intends to add another X ringgit into the account. Determine the value of X if the account will amount to RM 30,000 in 10 years' time.

4.2 COMPOUND INTEREST

Effective rate
s useful when an investor wants to compare investments with different compounding periods but he needs to put them on a common basis

Compound Interest – Effective Rate

- 2 rates are **equivalent** if they yield the same future value at the end of 1 year.
- **A nominal rate** is interest that is calculated more than once a year.
- **An effective rate** is the actual rate that is earned in a year. It can also be defined as the simple interest that would produced the same accumulated amount in 1 year as the nominal rate compounded *m* times a year.

EXAMPLE 8

RM 800 is invested for one year. If the interest rate is

a) 9.04% compounded annually

b) 8.75% compounded quarterly

Determine the amount after one year.

Solution

$P = 800$

$r_1 = 9.04\%;\ m_1 = 1;\ t = 1 \text{ years}$

$$i = \frac{9.04\%}{1} = 9.04\%;\ n = 1(1) = 1$$

$$S = 800(1 + 9.04\%)^1$$

$$= \text{RM}872.32$$

$P = 800$

$r_2 = 8.75\%;\ m_2 = 4;\ t = 1 \text{ years}$

$$i = \frac{8.75\%}{4} = 2.19\%;\ n = 4(1) = 4$$

$$S = 800(1 + 2.19\%)^4$$

$$= \text{RM}872.42$$

9.04% compounded annually is an effective rate

8.75% compounded quarterly is a nominal rate.

} **Equivalent**

4.2 COMPOUND INTEREST

Effective rate is ***simple interest*** that would produced the same accumulated amount in 1 year as the nominal rate compounded ***m*** times a year.

Compound Interest – Effective Rate

- The formula to calculate the ***effective rate of interest*** is given by

Effective annual yield

$$r_{eff} = \left(1 + \frac{r}{m}\right)^{m} - 1$$

where : r_{eff} = Effective rate of interest

m = frequency of conversions

r = nominal interest rate (per year)

EXAMPLE 9

Determine the effective rate of interest corresponding to a nominal rate of 8% per year compounded

a) annually
b) semi – annually
c) quarterly
d) monthly
e) daily

Solution

a) annually

$$r = 8\%;\ m = 1;$$

$$r_{eff} = (1 + 0.08) - 1 = 0.08$$

Effective rate if 8% per year

b) Semi – annually

$$r = 8\%;\ m = 2;$$

$$r_{eff} = \left(1 + \frac{0.08}{2}\right)^2 - 1 = (1.04)^2 - 1 = 0.0816$$

Effective rate if 8.16% per year

EXAMPLE 9

Determine the effective rate of interest corresponding to a nominal rate of 8% per year compounded

a) annually
b) semi – annually
c) quarterly
d) monthly
e) daily

Solution

c) quarterly

$r = 8\%;\ m = 4;$

$r_{eff} =$

d) monthly

$r = 8\%;\ m = 12;$

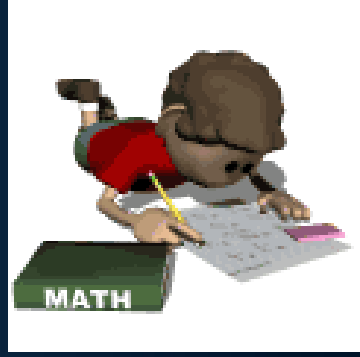
$r_{eff} =$

e) daily

$r = 8\%;\ m = 360;$

$r_{eff} =$

EXAMPLE 10



1. Determine the effective rate which is equivalent to 16% compounded semi – annually.
2. Calculate the nominal rate, compounded monthly which is equivalent to 9% effective rate.
3. Ah Meng wishes to borrow some money to finance some business expansion. He has received two different quotes:

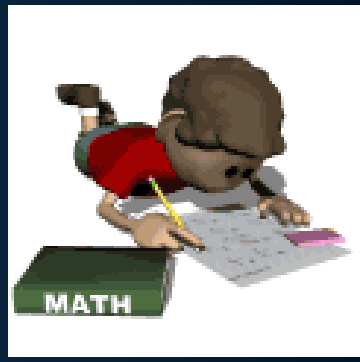
   Bank A: charges 15.2% compounded annually

   Bank B: charges 14.5% compounded monthly

   Which bank provides a better deal?

Solution

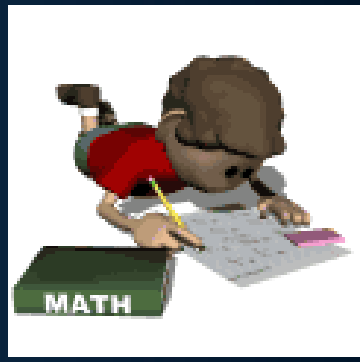
PRACTICE 4



1. Calculate the effective rate that is equivalent to
   i. 4 ½% compounded semi – annually
   ii. 16% compounded quarterly
   iii. 15% compounded monthly
   iv. 8% compounded weekly
   v. 12% compounded daily.

2. RM 1000 was invested for two years at 10% simple interest. Calculate the effective rate that was earned?
3. What is the nominal rate compounded monthly that is equivalent to 12% effective rate?

PRACTICE 4



4. Bank Bersatu offers two interest rates for fixed deposits:

   Offer 1: 12% compounded quarterly

   Offer 2: 13.55% simple interest

   If you wish to invest for one year, which offer would you choose?

5. Martha invested $40,000 in a boutique 5 years ago. Her investment is worth $70,000 today. What is the effective rate of her investment?

6. In the last 5 year, MAA Mutual Fund grew at the rate of 10.4% per year compounded quarterly. Over the same period, Public Mutual Fund grew at the rate of 10.6% per year compounded semi – annually. Which mutual fund has a better rate of return?

4.2 COMPOUND INTEREST

The principal, P is often referred *the present value (or discounted value)* at i% per interest period of an amount S.

Compound Interest – Present Value

- The formula to calculate the *present value* is given by

$$P = \frac{S}{(1+i)^n} = S(1+i)^{-n}$$

$n = mt$

$i = \frac{r}{m}$

where : S = Compound amount
P = Original principal
m = frequency of conversions
r = nominal interest rate (per year)
t = investment period

EXAMPLE 11

How much money should be deposited in a bank paying interest at the rate 6% per year compounded monthly so that at the end of 3 years the accumulated amount will be RM 20,000?

Solution

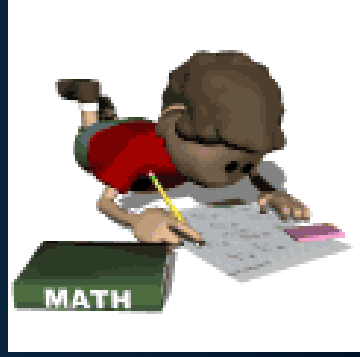
$S = 20{,}000;\ r = 6\%$

$m = 12;\ t = 3\ years;$

$i = \dfrac{6\%}{12} = 0.5\%;\ n = 12(3) = 36$

$$P = 20000(1 + 0.5\%)^{-36}$$
$$= \text{RM}\,16{,}712.90$$
$$\approx \text{RM}\,16{,}713$$

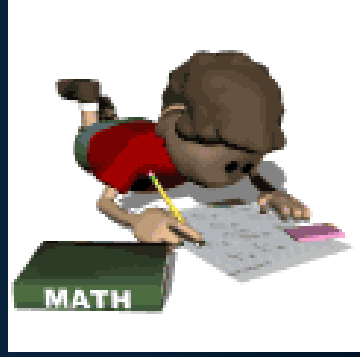
EXAMPLE 12



Determine the present value of RM 49,158.60 due in 5 years at an interest rate of 10% per year compounded quarterly?

Solution

EXAMPLE 13



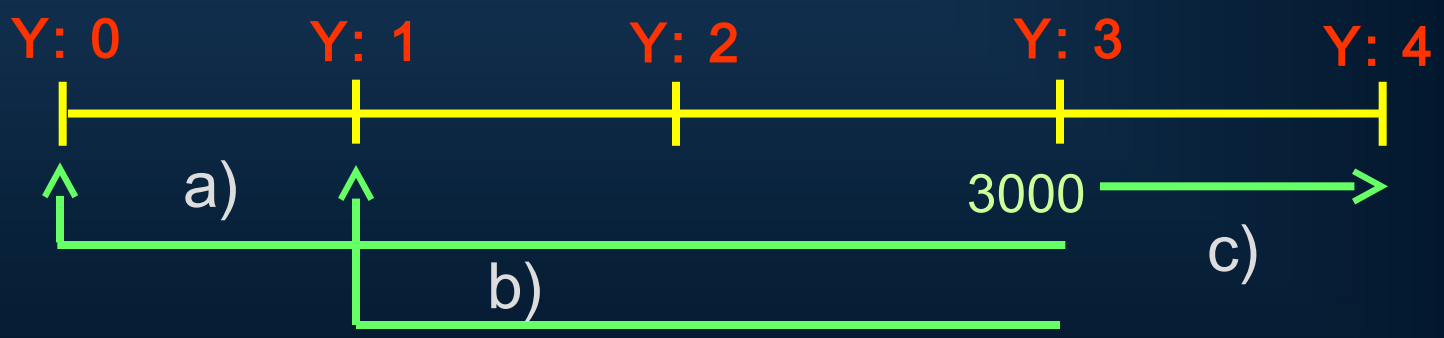
A debt of RM 3000 will mature in three years' time. By assuming that the money is worth 14% compounded semi – annually, calculate

a) the present value of this debt

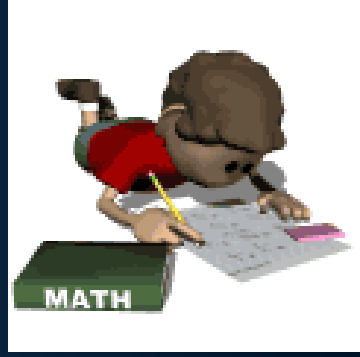
b) the value of this debt at the end of the first year

c) the value of this debt at the end of the four years

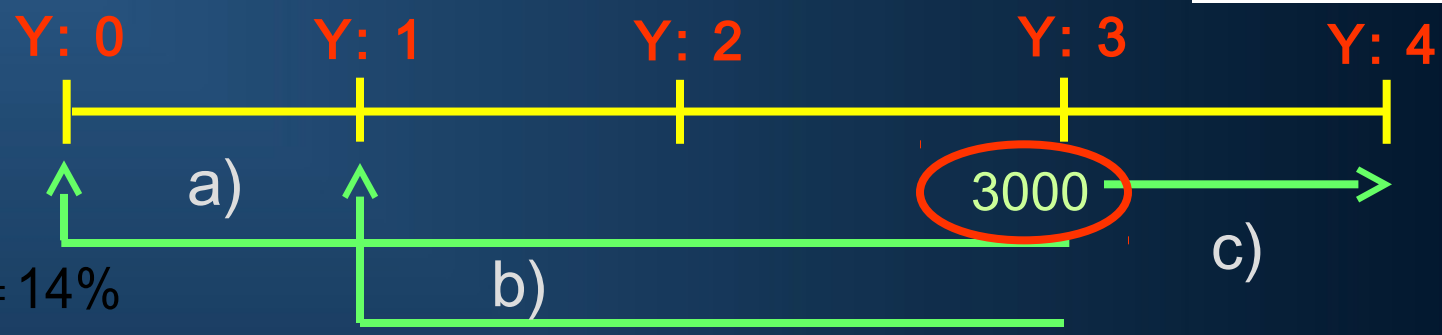
Solution



EXAMPLE 13



Solution



$S = 3000;\ r = 14\%$

$m = 2;\ t = 3\ years$

$i = \dfrac{14\%}{2} = 7\%;\ n = 2(3) = 6$

a) $P = 3000(1+7\%)^{-6}$

$= \text{RM}1999.03$

$S = 3000;\ r = 14\%$

$m = 2;\ t = 2\ years$

$i = \dfrac{14\%}{2} = 7\%;\ n = 2(2) = 4$

b) $P = 3000(1+7\%)^{-4}$

$= \text{RM}$

$P = 3000;\ r = 14\%$

$m = 2;$

$t = 1\ year$

$i = \dfrac{14\%}{2} = 7\%$

$n = 2(1) = 2$

c) $S = 3000(1+7\%)^{2}$

$= \text{RM}$

Value of the debt is on the **right side** of the original debt

4.2 COMPOUND INTEREST

Compound Interest – Continuous compound

- The future value (or the accumulated amount) of sum money compounded continuously is given by

$$A = Pe^{it}$$

where : A = Accumulated amount (future value)
P = Original principal
i = continuous compounding rate
t = time in years

EXAMPLE 14

1. Determine the accumulated amount of RM 2500 for six months at 10% compounded continuously.
2. Calculate the amount to be deposited now so as to accumulate RM 9800 in 6 years at 7% compounded continuously.

Solution

1. $P = 2500;\ i = 10\%$

$$t = \frac{6}{12}\,year = 0.5$$

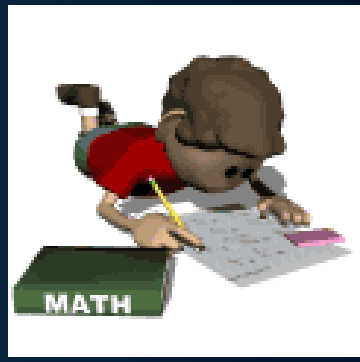
$$A = 2500e^{(0.1\times 0.5)}$$

$$= \text{RM}\,2628.18$$

2. $A = 9800;\ i = 7\%$

$$t = 6\,years$$

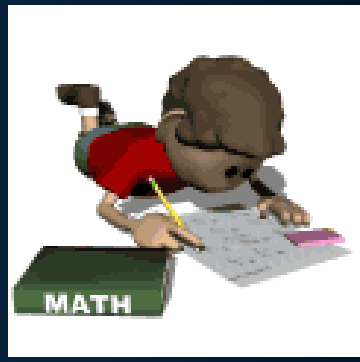
PRACTICE 5



1. Calculate the present values of RM 40,000 due in 4 years at the given rate of interest
   i. 6% compounded semi – annually
   ii. 8% compounded quarterly
   iii. 7% compounded monthly
   iv. 9% compounded daily.

2. A debt of RM8000 will mature in four years' time. By assuming that the money is worth 9% compounded quarterly, calculate
   a) the present value of this debt
   b) the value of this debt at the end of the two years
   c) the value of this debt at the end of the five years

PRACTICE 5



3. Aiman invested a sum of money 5 years ago in a savings account that has since paid interest at the rate of 8% per year compounded quarterly. His investment is now worth RM22,289.22. How much did he originally invest?
4. What is the future value of RM2999 that was invested for 30 months at 6% compounded continuously?
5. Calculate the amount to be deposited now so as to accumulate RM 9888 in 24 months at 9.5% compounded continuously.