H.C.F. AND L.C.M.

Definitions

1. **Multiples :** All numbers are said to be multiples which are completely divisible by a given number.

   Example : Find the multiples of 4 and 6.

   Soln. Multiples of 4 = 4, 8, 12, 16, 20….

   Multiples of 6 = 6, 12, 18, 24, 30….

2. **Factors :** One number is said to be a factor of another when it divides the other exactly.

   Example : Find the factors of 12 and 16.

   Soln. Factors of 12 = 1, 2, 3, 4, 6, 12

   Factors of 16 = 1, 2, 4, 8, 16

3. **Common Multiple :** A common multiple of two or more numbers is a number which is exactly divisible by each of them. Thus 12 is a common multiple of 3, 4 and 6.
4. **Lowest Common Multiples (L.C.M.) :** The L.C.M. of two or more numbers is the smallest number which is a multiple of all the given numbers. When the given have been resolved into prime factors, the L.C.M. is given by the product of the factors of the resolved expressions, each considered once one with the maximum exponent which appears in it.

   Example : (i) Find the L.C.M. of 3, 4 and 6.

   Soln. Multiples of 3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30…..

   Multiples of 4 = 4, 8, 12, 16, 20, 24, 28, 32….

   Multiples of 6 = 6, 12, 18, 24, 30….

   ∴ Common multiples of 3, 4 and 6 are = 12, 24, 36….

   ∴ Least common multiples = 12 i.e. L.C.M. = 12 **Ans.**

   Example : (ii) The L.C.M. to 480, 360 and 600 can be found as under :

   $480 = 2^5 \times 3 \times 5$;

$360 = 2^3 \times 3^2 \times 5$;

$600 = 2^3 \times 3 \times 5^2$

L.C.M. $= 2^5 \times 3^2 \times 5^2 = 7200$ **Ans.**

5. **Common Factor :** A common factor of two or more numbers is a number that divides each to them exactly. Thus 3 is a common factor of 9, 18, 21 and 33.
6. **Highest Common Factor (HCF) :** HCF of two or more numbers is the greatest number that divides each of them exactly. Thus 6 is the HCF of 18 and 24 because there is no number greater than 6 that divides both 18 and 24.

   Note : The terms highest common divisor and greatest common measure are often used in the sense of highest common factor (HCF).

7. **To find the HCF of two or more numbers :**

   Rule : Change the given numbers into prime factors and then find the product of all the prime factors common to all the numbers. The product will be the product HCF.

   Example : Find the HCF of 8 and 12

   Soln. $8 = 2 \times 2 \times 2$, $12 = 2 \times 2 \times 3$

   $\therefore$ HCF $= 2 \times 2 = 4$ **Ans.**

   Trick : If the given numbers are of equal base and unequal power then their HCF will be least power of the number.

   Example : Find the HCF of 3^6, 3^{11} and 3^{17}.

   Soln. HCF $= 3^6$ **Ans.**

Memorable Terms

I.	G.C.D. or g.c.d. : greatest common divisor.
II.	L.C.D. or l.c.d. : least common denominator.
III.	L.C.M. or l.c.m. : least common multiple.
IV.	(a, b) : The G.C.D. of a and b; the open interval from a to b.
V.	(a, b) : The L.C.M. of a and b; the closed interval from a to b.

8. **Divisible Numbers :** The numbers which have more than two factors are called divisible numbers.

Example : 4, 6, 8, 9, 10, 12, 14, 15 etc. are called divisible numbers.

9. **Prime Factor :** Any number which represents the product of the prime numbers is called prime factor.

   Example : Prime factor of 36 = 2 × 2 × 3 × 3

Important Formulae

1. LCM of a fraction = $\frac{LCM\ of\ Numerators}{HCF\ of\ Denominators}$
2. HCF of a fraction = $\frac{HCF\ of\ Numerators}{LCM\ of\ Denominators}$
3. Product of the two numbers = HCF × LCM
4. LCM = $\frac{Product\ of\ the\ two\ numbers}{HCF}$
5. HCF = $\frac{Product\ of\ the\ two\ numbers}{LCM}$

Memorable Facts

1. Two or more than two numbers cannot be larger than the LCM of those numbers.
2. HCF of co-prime numbers is always 1.
3. If one number is a factor of other number the smaller number is HCF of itself.
4. More than one factor of 1 cannot be possible so it is not a prime number.
5. The number 1 is neither a prime number nor an even number.
6. Least common multiple is equal to the LCM of two or more than two numbers.
7. If one number is a multiple of the other then the LCM is always the larger number.

Rules to find L.C.M. : There are two methods to find L.C.M. : (A) Division method (B)Factor method.

(A) Find LCM by division method :

Example : (i) Find LCM of 12, 18, 27, and 36.

Soln.

2	12,	18,	27,	36
2	6,	9,	27,	18
3	3,	9,	27,	9
3	1,	3,	9,	3
3	1,	1,	3,	1
	1,	1,	1,	1

∴ LCM = 2 × 2 × 3 × 3 × 3 = 108 **Ans.**

Example : (ii) Find LCM of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$

Soln. $\frac{LCM\ of\ 2,\ 3\ and\ 4}{HCF\ of\ 3,\ 4\ and\ 5} = \frac{12}{1} = 12$ **Ans.**

(B) Find LCM by Factor Method :

Example : (i) Find LCM of 8 and 20

Soln. Factors of 8 = 2 × 2 × 2

Factors of 20 = 2 × 2 × 5

∴ LCM = 2 × 2 × 2 × 5 = 40 **Ans.**

Rule to find HCF : There are two methods to find HCF : (1) Division method (2) Factor method.

(1) Find HCF by Division method :

Example : Find HCF of 24 and 36

Soln.

```
24 ) 36 ( 1
    -24
    ---
12 ) 24 ( 2
      24
      --
       0
```

∴ HCF = 12 **Ans.**

(2) Find HCF by Factor Method :

Example : Find HCF of 8 and 36.

Soln. Factors of 8 = 2 × 2 × 2

Factors of 36 = 2 × 2 × 3 × 3

⇒ Common factors of 8 and 36 = 2 × 2

∴ HCF = 4 **Ans.**