Ampere's Law or Ampere's Circuital law :

Statement :

It states that the line integral of the magnetic field for any closed curve is equal to μ_0 times the net current I through the area bounded by the curve.

$$\text{i.e. } \int \vec{B}.\vec{dl} = \mu_0 I.$$

This law plays the same role in magnetostatics as Gauss's law does in electrostatics.

Proof :

Consider a long straight conductor carrying a current I kept perpendicular to the plane of the paper so that the current flows inwards.

The magnetic field intensity at a distance r is given by

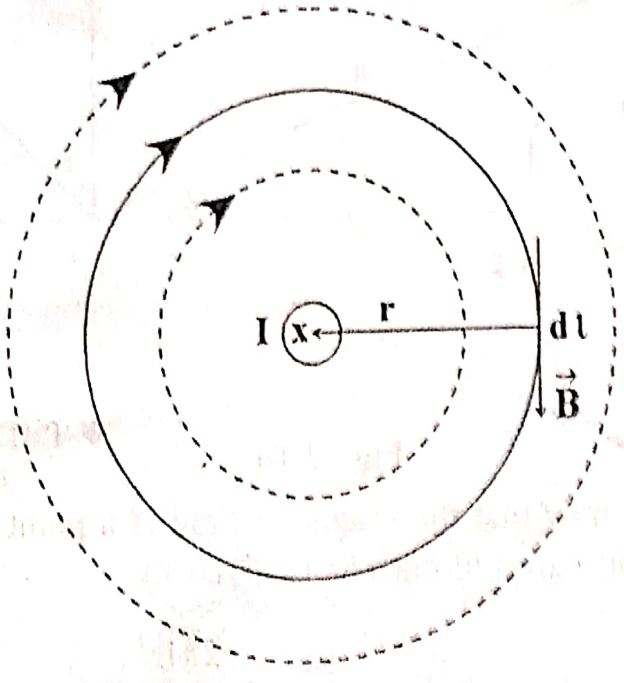


Fig. 2.9

$$B = \frac{\mu_0 I}{2\pi r}$$

and its direction is tangent to the circle of radius r.

The field is constant at every point on the circle and parallel to the current element $\vec{dl}$.

The line integral is given by

$$\oint \vec{B}.\vec{dl} = \oint \frac{\mu_0 I}{2\pi r}\, dl = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

$$\therefore \oint \vec{\mathbf{B}}.\vec{\mathbf{dl}} = \mu_0 \mathbf{I} \qquad \because \oint dl = 2\pi r$$

which is independent of the radius r.

Ampere's law is true for any assembly of current and for any closed curve.

$\oint \vec{B}.\vec{dl}$ is μ_0 times the current through the area bounded by the circle. It is known as **Ampere's circuital law.**

Scanned by CamScanner

Additional Information

Maxwell's Field Equations :

There are four fundamental equations of electro magnetism and corresponds to a generalisation of certain experimental observations regarding electricity and magnetism. The following four laws are the *differential* form of Maxwell's equation.

i) $\vec{\nabla}.\vec{D} = \rho$ or div $\vec{D} = \rho$

It corresponds to the *Gauss law for the electric field.* where $\vec{D}$ is the electric displacement and ρ is the free charge density.

ii) $\vec{\nabla}.\vec{B} = 0$ or div $\vec{B} = 0$

It corresponds to *Gauss law for magnetic field.*

where B is the magnetic induction.

iii) $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ or curl H $= \vec{J} + \frac{\partial \vec{D}}{\partial t}$

It corresponds to the Ampere's *law* in circuital form for magnetic field and accumpanging current. where $\vec{H}$ is the magnetic field intensity and $\vec{J}$ is the current density.

iv) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ or curl $\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

It represents *Faraday's law in circuital form for the induced* emf produced by the rate of change of magnetic flux linked with circuit. where $\vec{E}$ is the electric intensity.

Derivations of Maxwells Equations :

I. Gauss' Law for Electric Fields :

Consider a surface S bounding a volume V within a dielectric medium. Initially the volume does not contain any charge but let us allow the dielectric to be polarised by keeping it in an electric field. Now let us also place some charge on the dielectric body.

So we have two type of charges :

a) real charge of density ρ

b) bound charge density ρ'.

Gauss' law then can be written as

$$\oint_s \vec{E}\,d\vec{S} = \frac{1}{\varepsilon_0}\int(\rho+\rho')dV$$

$$\varepsilon_0\oint_s \vec{E}.d\vec{S} = \int\rho\, dV + V\int\rho'\, dV \qquad .....(1)$$

But as the bound charge density ρ' is defined as $\rho' = -\text{div } P$.

where P is the induced dipole moment per unit volume polarisation of the medium.

$$\oint_s \vec{E}.d\vec{S} = \int_V \text{div } E\, dV$$

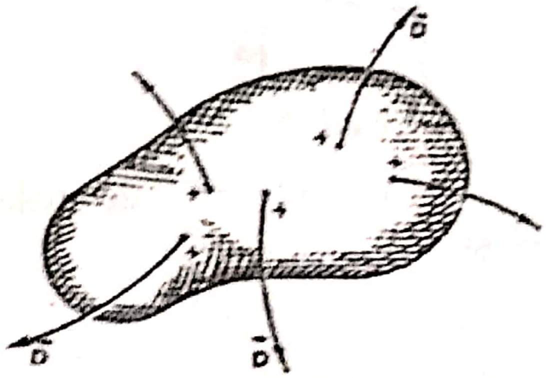


Fig.2.14

So equation (1) becomes

$$\varepsilon_0 \int_V \text{div } E\, dV = \int_V \rho\, dV - \int_V \text{div } P\, dV$$

i.e. $\int_V \text{div}(\varepsilon_0\vec{E}+\vec{P})\, dV = \int_V \rho\, dV$

or $\int_V \text{div } D\, dV = \int_V \rho\, dV$ as $\vec{D} = \varepsilon_0\vec{E}+\vec{P}$

or $\int_V (\text{div } D-\rho)\, dV) = 0.$

Since this equation is true for all volumes, the integand must vanish. Thus we have

$$\text{div } \vec{D} = \nabla.\vec{D} = \rho \qquad .....(2)$$

Hence the result.

ii) **Gauss' Law for Magnetic Field :**

Experiments show that the number of magnetic lines of force entering any arbitrary closed surface is exactly the same leaving it. Therefore the flux of magnetic induction B across any closed surface is always zero i.e.

$$\oint_S \vec{B}.\vec{dS} = 0$$

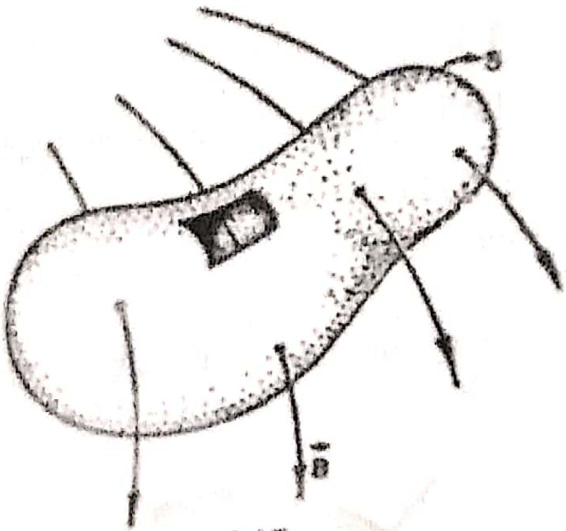


Fig. 2.15

Converting this surface integral into volume integral by Gauss' theorem, we get

$$\int_V \text{div}\, B\, dV = 0.$$

But as the surface bounding the volume is quite arbitrary the above equation will be true only when the integrand vanishes i.e.

$$\text{div}\, \vec{B} = \nabla.\vec{B} = 0 \textit{ (Hence the law)} \quad .....(3)$$

iii) **Ampere's Circuital Law :**

From this law the work done in carrying unit magnetic pole once round a closed arbitrary path linked with the current I is given by

$$\oint_C \vec{H}.\vec{dl} = I$$

$$\oint_C \vec{H}.\vec{dl} = \int_S \vec{J}.\vec{dS} \qquad \left(\text{as } I = \int \vec{J}.\vec{dS}\right)$$

where S is the surface bounded by the closed path C.

By changing the line integral into the surface integral by Stoke's theorem, we get.

$$\int_S \text{curl } \vec{H}.d\vec{S} = \int_S \vec{J}.d\vec{S}$$

i.e. $$\text{curl } \vec{H} = \vec{J} \quad \text{.....(4)}$$

But Maxwell found it to be incomplete for changing electric fields and assumed that a quantity.

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

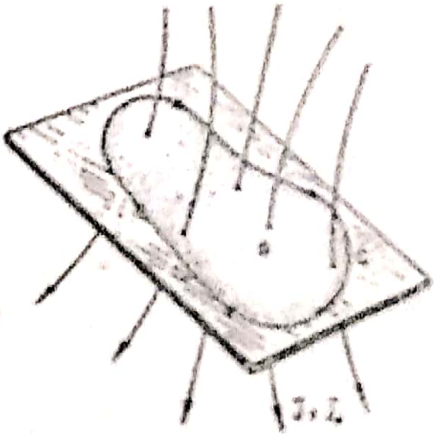


Fig.2.16

called displacement current must also be included in it so that it may satisfy the continuity equation i.e. $\vec{J}$ must be replaced in equation(4) by $\vec{J} + \vec{J}_d$ so that the law becomes

$$\text{curl } \vec{H} = \vec{J} + \vec{J}_d$$

i.e. $$\text{curl } \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \quad \textit{Hence the law} \quad \text{.....(5)}$$

ii) **Faraday's law :**

It *states that the induced emf in a circuit is proportional to negative time rate of change of magnetic flux linked with the circuit.*

.....(6)

i.e. $$\epsilon = -\frac{d\phi_B}{dt}$$

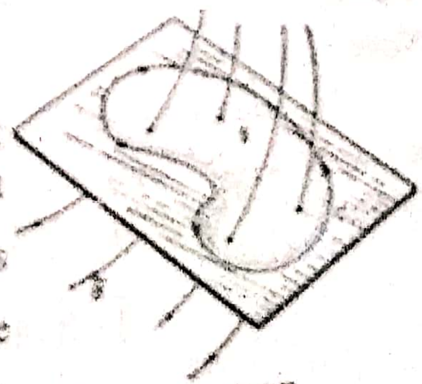


Fig.2.17

Now if $\vec{E}$ be the electric intensity at a point the work done in moving a unit charge through a small distance $\vec{dl}$ is $\vec{E}.\vec{dl}$. So the work done in moving the unit charge once round the circuit is

$\oint_C \vec{E}.\vec{dl}$. Now as **e.m.f.** is *defined as the amount of work done in moving a unit charge once round the electric circuit.*

$$\in \; = \; \oint_C \vec{E}.\vec{dl} \qquad .....(7)$$

So comparing equation (6) and (7), we get

$$\oint_C E.dl \; = \; -\frac{d\phi_B}{dt} \qquad .....(8)$$

But as

$$\phi_B \; = \; \int_s \vec{B}.\vec{dS}$$

So

$$\oint_C \vec{E}.\vec{dl} \; = \; -\frac{d}{dt}\int \vec{B}.\vec{dS}$$

Transformating the line integral by Stoke's theorem into surface integral we get

$$\int_S \text{curl } \vec{E}.\vec{dS} \; = \; \frac{d}{dt}\int \vec{B}.\vec{dS}$$

Assuming that surface S is fixed in space and only B changes with time, above equation yields.

$$\int_S \left(\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} \right).\vec{dS} \; = \; 0$$

As the above integral is true for any arbitrary surface the integrand must vanish.

i.e.

$$\text{curl } \vec{E} \; = \; -\frac{\partial \vec{B}}{\partial t}. \quad \textbf{(Hence the law)} \qquad .....(9)$$

Note :

The equation(9) shows that a changing magnetic field produces an electric field and vice versa. The magnatic field produced by changing electric field is given by

$$\oint_C B.dl \; = \; \mu_0 \, \varepsilon_0 \frac{d\phi_E}{dt} \; = \; \frac{1}{c^2}\frac{d\phi_E}{dt}$$

Discussion :

1. These equations are based on experimental observations. The equations (2) and (5) correspond to electricity while (3) and (9) to magnetism.
2. These equations are general and apply to all electromagnetic phenomena in media which are at rest w.r.t. the coordinate system.
3. These equation are not independent of each other as from equation (9) we can derive (3) and from (5), (2). This is why equations (3) and (9) are called the first pair of Maxwell's equations while (2) and (5) are called the second pair.
4. The equation (2) represents Coulomb's law while (5) the law of conservation of charge i.e. continuity equation.
5. Comparing the equans (2) with (3) and (5) with (9) we find that left hand sides are identical while right hand sides are not. This in turn implies that electric and magnetic phenomena are assymmetric and this asymmetry arises due to the non-existance of monopoles.

$$\therefore \quad n = \sqrt{\mu_r \varepsilon_r}$$

It is always greater than 1 $\because \varepsilon_r$ and ε_r are always greater than 1

Poynting Theorem and Poynting Vector :

It states that, in a plane electromagnetic wave, the rate of flow of energy through unit area is proportional to the product of electric and magnetic intensities.

Proof :

A travelling wave carries energy with it. For example, a radio wave carries energy from the transmitter to receiver. This energy is electromagnetic i.e. due to electric and magnetic fields. At any instant, the total energy W per unit volume of the space (or density of electromagnetic energy) of permeability μ and permittivity ε is given by

$$W = \frac{1}{2}(\mu \vec{H}^2 + \varepsilon \vec{E}^2)$$

$$= \frac{1}{2}(\vec{H}.\vec{B} + \vec{E}.\vec{D}) \quad \because \mu\vec{H} = \vec{B} \text{ and } \varepsilon\vec{E} = \vec{D}$$

where $\vec{B}$ is the magnetic induction and $\vec{D}$ is the electric displacement.

Let dv be the small volume enclosed by the surface. Since the electromagnetic field changes, the rate of decrease of energy is given by

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial t}\left[\int \frac{1}{2}(\vec{H}.\vec{B} + \vec{E}.\vec{D})\right] dv$$

$$= -\frac{\partial}{\partial t}\int \frac{1}{2}(\mu\vec{H}^2 + \varepsilon\vec{E}^2)dv$$

$$= -\frac{\partial}{\partial t}\int_v \frac{1}{2}(\mu\vec{H}^2 + \varepsilon\vec{E}^2)dv$$

$$= \int_v \left(\mu\vec{H}\frac{\partial\vec{H}}{\partial t} + \varepsilon\vec{E}\frac{\partial\vec{E}}{\partial t}\right)dv$$

In an isotropic dielectric medium, we have by Maxwell's equations.

$$-\mu \frac{d\vec{H}}{dt} = \text{curl } \vec{E} \text{ and } \varepsilon \frac{dE}{dt} = \text{curl } \vec{H}$$

As A curl B – B curl A = div (B × A) = –div (A × B)

$$\therefore \quad \frac{\partial W}{\partial t} = -\int \text{div } (\vec{E} \times \vec{H})\, dv$$

By Gauss's theorem we know that if a surface encloses a volume v, then $\int \text{div } \vec{A}.\vec{dv} = -\int \vec{A}\, \hat{n}\, \vec{ds}$ if the unit vector $\hat{n}$ has the direction of the inward, normal to the surface element ds

$$\frac{\partial W}{\partial t} = \int (\vec{E} \times \vec{H})\, \vec{ds}$$

This gives the **Poyting Theorem**

The right hand side of the above equation is the integral over the surface S of the normal component of the vector.

$$\therefore \quad \frac{\partial W}{\partial t} = \oint (\vec{E} \times \vec{H})\, \vec{ds}$$

i.e. $$P = (\vec{E} \times \vec{H})$$

wher P is the **Poynting vector.**

Physical Significance :

1. P represents an amount of energy flowing per second across unit area which is perpendicular to the direction of flow.
2. The vector $\vec{E} \times \vec{H}$ is normal to the plane containing $\vec{E}$ and $\vec{H}$ so that in an electromagnetic disturbance, the energy travels simultaneously in this direction.
3. Though $\vec{E}$ and $\vec{H}$ being oscillatory, change in direction, P remains unidirectional.
4. The magnitude of P varies. It is maximum when $\vec{E}$ and $\vec{H}$ are maximum and zero when $\vec{E}$ and $\vec{H}$ are zero.

steady currents

stationary charges produces electric fields that are constants in time. Hence the term Electrostatics. steady currents produce mag fields that are constant in time. the theory of steady currents is called magnetostatics

The steady current I means a continuous flow that has been going on with out any change & without any charge piling up anywhere.

When a steady current flows into a wire its magnitude I must be small all along the line, otherwise charge would be piling up somewhere & it wouldn't be steady current.

Displacement current

Displacement current is a quantity appearing in Maxwell's eqns that is defined in terms of the rate of change of electric displacement field

or

It is a current which produces in the region in which the electric field & hence the electric flux changes with time.

Maxwell changed the defn of total current density to adapt the eqn of continuity to time dependent fields.

Ampere's circuital law is

$$\oint B.dl = \mu_0 I$$

$$\& \quad \oint_S H.dl = \int_S J.ds$$

changing line integral into surface integral by Stokes th

$$\int_S \text{curl} H \, ds = \int_S J \, ds$$

$$\text{curl} H = J \rightarrow (1)$$

let us put in eqn of continuity which is

$$\text{div} J = -\frac{\partial \rho}{\partial t}$$

$$\text{div}(\text{curl} H) = -\frac{\partial \rho}{\partial t}$$

$$0 = -\frac{\partial \rho}{\partial t}$$

eqn (1) leads to steady conditions in which charge density is not changing. Therefore for time dependent fields eqn (1) should be modified. Maxwell suggested that the defn of total current density is incomplete & add something to J let it be J'

$$(1) \Rightarrow \quad \text{curl} H = J + J' \rightarrow (2)$$

we take div of eqn (2)

$$\text{div}(\text{curl} H) = \text{div}(J + J')$$

$$0 = \text{div} J + \text{div} J'$$

$$\text{div} J' = -\text{div} J = \frac{\partial \rho}{\partial t} \rightarrow (3)$$

WKT $\rho = \nabla \cdot D$

(3) $\Rightarrow$ $\operatorname{div} J' = \frac{\partial \rho}{\partial t}$

$\operatorname{div} J' = \frac{\partial}{\partial t} (\nabla \cdot D)$

$\operatorname{div} J' = \nabla \frac{\partial D}{\partial t}$

$\operatorname{div} J' - \operatorname{div} \frac{\partial D}{\partial t} = 0$

$\operatorname{div} \left(J' - \frac{\partial D}{\partial t} \right) = 0 \rightarrow (4)$

(4) is true for all arbitrary ~~values~~ volume

$J = \frac{\partial D}{\partial t}$

$\operatorname{curl} H = J' + \frac{\partial D}{\partial t}$ //

Scanned by CamScanner

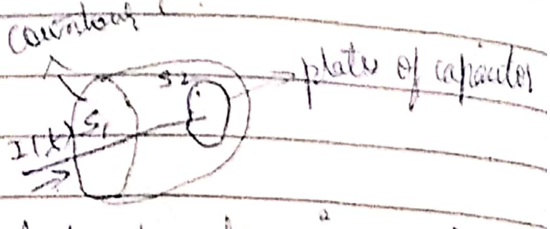
Generalisation of Ampere's law

WK the Ampere's circuital law

$$\oint H.dl = \int J.n\, da \rightarrow (1)$$

Find a generalisation that is always valid.

Consider a ckt which consists of a small parallel plate capacitor being charged by a constant current I.



If Ampere's law is applied to the contour C & the surface S_1 we find

$$\oint_C H.dl = \int_{S_1} J.n da = I \rightarrow (2)$$

On the other hand, Ampere's law is applied to contour C & surface S_2 then J is zero at all points on S_2 &

$$\oint_C H.dl = \int_{S_2} J.n da = 0 \rightarrow (3)$$

(2) & (3) contradict each other & thus cannot both be correct. If C is imagined to be a great distance from capacitor, it is clear that the situation is not different from the standard Ampere law cases. Since the eqn is given by curl ie

$$\nabla \times H = J \rightarrow (4)$$

this also requires modification

The proper modification is because the integrals on the RHS are different

$$\int_{S_2} J.n_2 da - \int_{S_1} J.n_1 da \neq 0 \rightarrow (5)$$

S_1 & S_2 together form a closed surface ; n_2 is outward drawn & n_1 is inward drawn.
If this fact is taken then

$$\oint_{S_1+S_2} J \cdot n \, da \neq 0 .$$

The net transport current through S_1+S_2 closed surface does not vanish because charge is piling up on the plates of the condenser enclosed by the surface ;
charge conservation requires &

$$\oint_{S_1+S_2} J \cdot n \, da = -\int \frac{\partial \rho}{\partial t} \, dV .$$