B.Sc. I
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Unit 4: Fourier Series.

- ✓ Introduction to periodic functions
- ✓ Definition of fourier series
- ✓ Evaluation of the coefficients of fourier series
- ✓ Cosine series
- ✓ Sine series
- ✓ Dirichilet's condition
- ✓ Graphical representation of even and odd functions
- ✓ Advantages of fourier series.
- ✓ Physical applications of fourier series analysis : 1) square wave
  2) half wave rectifier.

Important questions
(from previous years exams)

- Evaluate the coefficient a_0 of fourier series (Nov-14, oct-16, oct-18)
- Discuss on physical application of fourier series in square wave (Nov-14)
- Explain sine series in fourier series (Mar-15, oct-16)
- Explain full wave rectifier using fourier series (mar-15)
- Graphical representation of even and odd functions (oct-16)
- physical application of fourier series analysis for square wave (oct-17)
- coefficients of fourier series (oct-18).

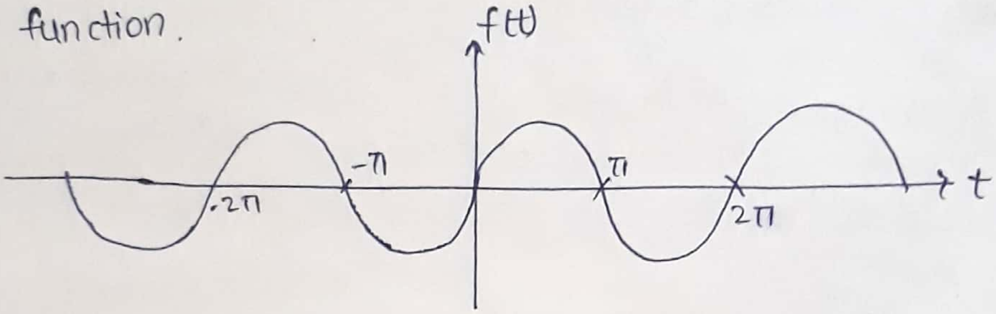
Periodic functions -

If the value of each ordinate f(t) repeats itself at equal interval in the abscissa, then f(t) is said to be a periodic function.

If $f(t) = f(t+T) = f(t+2T) = \ldots$ then T is called the period of the function f(t).

Ex. $\sin x = \sin(x+2\pi) = \sin(x+3\pi) \neq \ldots$

so sin x is a periodic function with the period of 2π. This is also called as ~~sinst~~ sinusoidal periodic function.



Definition of fourier series -

A series of cosines and sines of an angle and its multiples of the form

$$= \frac{a_0}{2} + a_1\cos x + a_2\cos 2x + \ldots + a_n\cos nx + \ldots$$
$$+ b_1\sin x + b_2\sin 2x + \ldots + b_n\sin nx + \ldots$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n\sin nx$$

is called as the fourier series.

where $a_0, a_1, a_2, \ldots a_n, b_1, b_2, \ldots b_n$ are constants (or fourier coefficients).

A fourier series f(x) can be expanded in a fourier series.

The series consists of the following

1. A constant term (a_0) (called as dc component in electrical work)
2. a component at the fundamental freq^n determined by a_1, b_1.
3. component of the harmonics (multiples of a_1 & b_1) determined by $a_2, a_3, \ldots b_2, b_3 \ldots$

Some useful Integrals-

$$\int_0^{2\pi} \sin nx \, dx = 0 \qquad \int_0^{2\pi} \cos nx \, dx = 0$$

$$\int_0^{2\pi} \sin^2 nx \, dx = \pi \qquad \int_0^{2\pi} \cos^2 nx \, dx = \pi$$

$$\int_0^{2\pi} \sin nx \cdot \sin mx \, dx = 0 \qquad \int_0^{2\pi} \cos nx \cdot \cos mx = 0$$

$$\int_0^{2\pi} \sin nx \cdot \cos mx \, dx = 0$$

$$\int u \cdot v \, dx = u \int v \cdot dx - \int \left[\frac{du}{dx} \cdot \int v \, dx \right] dx$$

Advantages of fourier series

1. Discontinuous functions can be represented by fourier series. although the derivatives of discontinuous functions does not exists.
2. The fourier series is useful in expanding the periodic functions
3. Expansion of an oscillating functions by fourier series gives all modes of oscillation (fundamental and overtones) which are useful in physics.
4. fourier series of a discontinuous function is not uniformly convergent at all points.

	0 0°	π/6 30°	π/4 45°	π/3 60	π/2 90	π 180	3π/2 270	2π 360°
sinθ	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	-1	0
cosθ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1	0	1
tanθ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined	0	not defined	0

$$\cos n\pi = (-1)^n$$

$$\sin n\pi = 0$$

Evaluation of coefficients of fourier series: (a_0, a_n, b_n)

$$f(x) = \frac{a_0}{2} + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \cdots + a_n\cos nx + \cdots$$
$$+ b_1\sin x + b_2\sin 2x + b_3\sin 3x + \cdots + b_n\sin nx + \cdots$$

To find a_0 :-

Integrate both sides of ① from $x=0$ to $x=2\pi$

$$\int_0^{2\pi} f(x)\,dx = \frac{a_0}{2}\int_0^{2\pi} dx + a_1\int_0^{2\pi}\cos x\,dx + a_2\int_0^{2\pi}\cos 2x\,dx + \cdots + a_n\int_0^{2\pi}\cos nx\,dx$$
$$\cdots + b_1\int_0^{2\pi}\sin x\,dx + b_2\int_0^{2\pi}\sin 2x\,dx + \cdots + b_n\int_0^{2\pi}\sin nx\,dx + \cdots$$

$$= \frac{a_0}{2}\int_0^{2\pi} dx$$

as other integrals are zero.

$$\int_0^{2\pi}\cos x\,dx = [\sin x]_0^{2\pi} = [\sin(2\pi) - \sin(0)] = 0$$

$$\int_0^{2\pi}\sin x\,dx = [-\cos x]_0^{2\pi} = -[\cos 2\pi - \cos 0] = -[1-1] = 0.$$

$$= \frac{a_0}{2}[x]_0^{2\pi}$$

$$= \frac{a_0}{2}[2\pi - 0]$$

$$\int_0^{2\pi} f(x)\,dx = a_0\pi$$

$$\boxed{\therefore\ a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx}$$

To find a_n

multiply each side of equation ① by $\cos nx$ and integrate from $x=0$ to $x=2\pi$.

$$\int_0^{2\pi} f(x)\cos nx\,dx = \frac{a_0}{2}\int_0^{2\pi}\cos nx\,dx + a_1\int_0^{2\pi}\cos x\cdot\cos nx\,dx + \cdots + a_n\int_0^{2\pi}\cos^2 nx\,dx +$$
$$+ b_1\int_0^{2\pi}\sin x\cdot\cos nx\,dx + b_2\int_0^{2\pi}\sin 2x\cos nx\,dx + \cdots$$

$$= a_n\int_0^{2\pi}\cos^2 nx\,dx = a_n\pi.$$

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$$\therefore a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx. \qquad \text{where } n = 1, 2, \cdots$$

<u>To find b_n</u>

Multiply each side of eqⁿ ① by $\sin nx$ and integrate from $x = 0$ to 2π.

$$\int_0^{2\pi} f(x)\sin nx\,dx = \frac{a_0}{2}\int_0^{2\pi}\sin nx\,dx + a_1\int_0^{2\pi}\cos x\sin nx\,dx + \cdots a_n\int_0^{2\pi}\cos nx\sin nx\,dx$$

$$\cdots + b_1\int_0^{2\pi}\sin x\sin nx\,dx + \cdots + b_n\int_0^{2\pi}\sin^2 nx\,dx + \cdots$$

$$= b_n\int_0^{2\pi}\sin^2 nx\,dx$$

$$= b_n\pi.$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx.$$

<u>Ex</u>. Find the fourier series representating

$$f(x) = x \qquad 0 < x < 2\pi.$$

<u>solⁿ</u> →

let $f(x) = x$

The fourier series is given by

$$f(x) = \frac{a_0}{2} + a_1\cos x + a_2\cos 2x + \cdots + a_n\cos nx$$
$$+ b_1\sin x + b_2\sin 2x + \cdots + b_n\sin nx$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[x^2\right]_0^{2\pi} = \frac{1}{2\pi}\left[(2\pi)^2 - 0\right]$$

$$= \frac{1}{2\pi}\cdot 4\pi^2 = 2\pi.$$

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$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x \cos nx\, dx$$

$$= \frac{1}{\pi}\left[x \cdot \int \cos nx\, dx - \int\left[\frac{d}{dx}\cdot x \cdot \int \cos nx\, dx\right] dx\right]$$

$$= \frac{1}{\pi}\left[x\left[\frac{\sin nx}{n}\right]_0^{2\pi} - \int_0^{2\pi} 1 \cdot \frac{\sin nx}{n}\, dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x \sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left\{\left(\frac{2\pi \sin 2n\pi}{n} + \frac{\cos 2\pi n}{n^2}\right) - \left(\frac{0 \sin 0}{n} + \frac{\cos 0}{n^2}\right)\right\}$$

$$= \frac{1}{\pi}\left(\frac{\cos 2\pi n}{n^2} - \frac{1}{n^2}\right)$$

$$a_1 = \frac{1}{\pi}\left(\frac{\cos 2\pi}{1} - \frac{1}{1}\right) = \frac{1}{\pi}(1-1) = 0. \quad \text{when } n=1.$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x \cdot \sin nx\, dx$$

$$= \frac{1}{\pi}\left[x \cdot \int_0^{2\pi} \sin nx\, dx - \int_0^{2\pi}\left[\frac{d}{dx}\cdot x \cdot \int_0^{2\pi} \sin nx\, dx\right] dx\right]$$

$$= \frac{1}{\pi}\left[-x\frac{\cos nx}{n} + \int_0^{2\pi} \frac{1 \cdot \cos nx}{n}\, dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{x \cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\left(\frac{-2\pi \cos 2\pi n}{n} + \frac{\sin 2\pi n}{n^2}\right) - \left(\frac{-0 \cos 0}{n} + \frac{\sin 0}{n^2}\right)\right]$$

$$= \frac{1}{\cancel{\pi}}\left[\frac{-2\cancel{\pi}}{n}\right] = -\frac{2}{n}\cos 2\pi n = -2n \text{ for } n \text{ is even}$$

$$= 2n \text{ for } n \text{ is odd}$$

$$\therefore \cancel{f(x) = \pi/-} \quad f(x) = \frac{2\pi}{2} + (-2n)\cos 2\pi n \sum_{n=1}^{\infty} \sin nx)$$

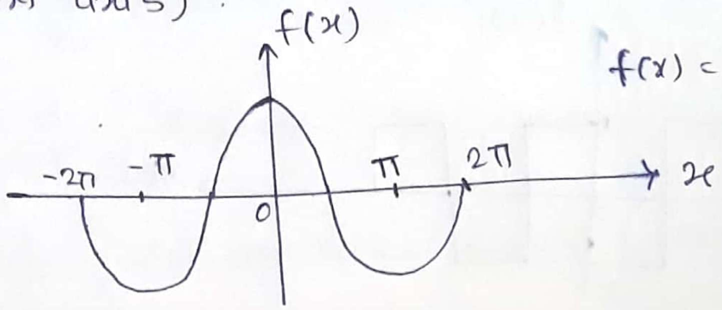
$$= \pi - 2n \cos 2\pi n \sum_{n=1}^{\infty} \sin nx$$

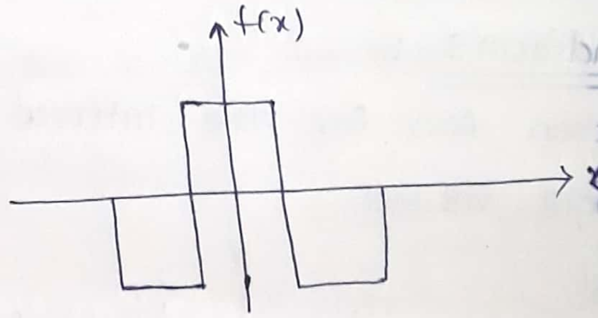
Even and odd functions : Graphical Representation

Even function :

A function $f(x)$ is said to be even function (symmetric function) if $f(-x) = f(x)$.

The graph of such function is symmetrical with respect to y-axis ($f(x)$ axis).





Here y axis is a mirror for reflection of the curve.

- The area under the curve from $-\pi$ to π is double the area from 0 to π.

$$\int_{-\pi}^{\pi} f(x)\,dx = 2\int_{0}^{\pi} f(x)\,dx$$

odd function

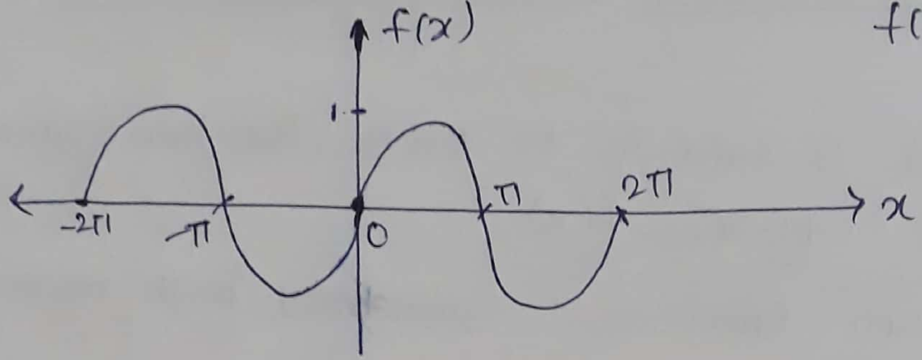
A function $f(x)$ is called as the odd function if $f(-x) = -f(x)$

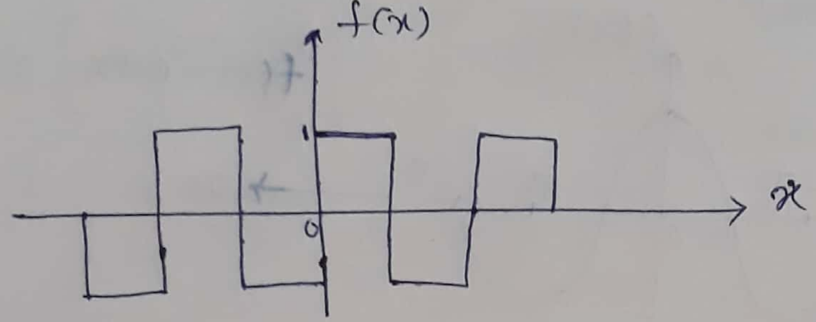
Here the area under the curve from $-\pi$ to π is zero.

$$\int_{-\pi}^{\pi} f(x)\,dx = 0.$$

graphs of odd functions.

$f(x) = \sin x$





*** Dirichilet's conditions :**

If the function $f(x)$ for the interval $(-\pi, \pi)$

① is a single valued

② is bounded

③ has at most a finite number of maxima & minima

④ has only a finite number of discontinuous points

⑤ is $f(x+2\pi) = f(x)$ for values of x outside $[-\pi, \pi]$

then,

$$S_p(x) = \frac{a_0}{2} + \sum_{n=1}^{p} a_n \cos nx + \sum_{n=1}^{p} b_n \sin nx$$

converges to $f(x)$ as $p \to \infty$ at values of x for which $f(x)$ is continuous and to

$\frac{1}{2}[f(x+0) + f(x-0)]$ at the points of discontinuity.

Sine Series

As fourier series expansion is given by

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \cdots + a_n \cos nx + \cdots$$

$$+ b_1 \sin x + b_2 \sin 2x + \cdots + b_n \sin nx + \cdots$$

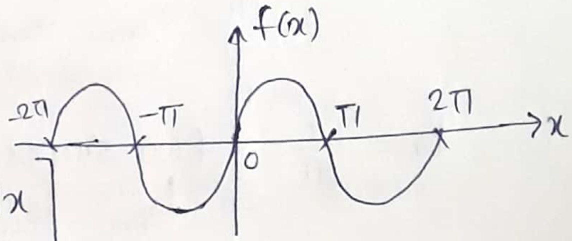
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{—} ①$$

where $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$ & $b_n = \int_{-\pi}^{\pi} f(x) \sin nx\, dx$

Let if $f(x)$ is ~~even~~ a odd function ie $f(-x) = -f(x)$

$$\therefore a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$



$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right]$$

$$= 0.$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos nx\, dx$$

$\underset{\text{odd}}{f(x)} \cdot \underset{\text{even}}{\cos nx} \Rightarrow$ odd.

$$= 0.$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx.$$

$\underset{\text{odd}}{f(x)} \cdot \underset{\text{odd}}{\sin nx} \Rightarrow$ even.

$$= \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx\, dx$$

$\therefore$ eqn ① becomes

$$f(x) = 0 + 0 + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \sin nx}$$

As the fourier series expansion contains sine terms therefore it is known as sine series.

* Cosine series

As the fourier series expansion is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx .$$

where $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\,dx \quad \& \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\,dx .$$

If $f(x)$ is even ie $f(-x) = f(x)$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi} 2 \int_{0}^{\pi} f(x)\,dx .$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{f(x)}_{\text{even}} \underbrace{\cos nx}_{\text{even}}\,dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx\,dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{f(x)}_{\text{even}} \underbrace{\sin nx}_{\text{odd}}\,dx$$

product of $f(x)$ & $\sin nx$ is odd

$\because$ $\sin nx$ is a odd function

$\therefore$ $b_n = 0$.

$\therefore$ The fourier series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + 0$$

as the fourier series contains only cosine terms and a constant term.

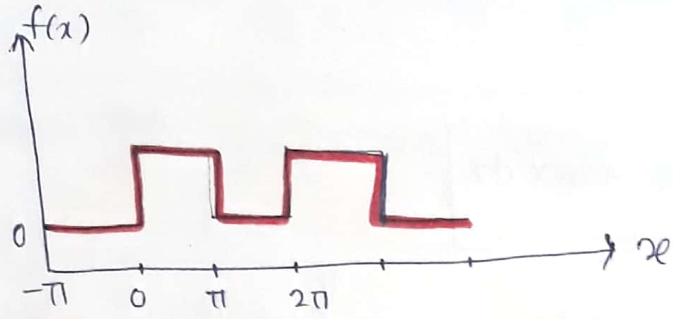
$\therefore$ it is known as a cosine series.

Physical application of fourier series Analysis.

① Square wave ② Half wave Rectifier

① Square wave.

$$f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ h & 0 < x \leq \pi \end{cases}$$



A square is as shown

A fourier series expansion of the f(x) is given by,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

To find out coefficients (a_0, a_n, b_n) of the fourier series.

we know $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx.$$

$$\therefore a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} h\,dx\right]$$

$$\therefore a_0 = \frac{1}{\pi}\int_0^{\pi} h\, dx$$

$$= \frac{h}{\pi}[x]_0^{\pi}$$

$$= \frac{h}{\pi}[\pi - 0] = h.$$

$$\boxed{\therefore a_0 = h}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{+\pi} f(x)\cos nx\, dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos nx\, dx + \int_0^{\pi} f(x)\cos nx\, dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\cdot\cos nx\, dx + \int_0^{\pi} h\cos nx\, dx\right]$$

$$= \frac{h}{\pi}\int_0^{\pi}\cos nx\, dx$$

$$= \frac{h}{\pi}\left[\frac{\sin nx}{n}\right]_0^{\pi} = \frac{h}{\pi}\left[\frac{\sin(n\pi)}{n} - \sin(0)\right]$$

$$= \frac{h}{\pi}(0-0) = 0.$$

$$\boxed{\therefore a_n = 0}$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx\, dx + \int_0^{\pi} f(x)\sin nx\, dx\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} h\sin nx\, dx\right]$$

$$= \frac{-h}{\pi}\left[\frac{\cos nx}{n}\right]_0^{\pi} = -\frac{h}{\pi n}[\cos n\pi - \cos 0]$$

$$= \frac{-h}{\pi n}[\cos n\pi - 1]$$

$= 0$ when n is even

$= 2h/\pi n$ when n is odd.

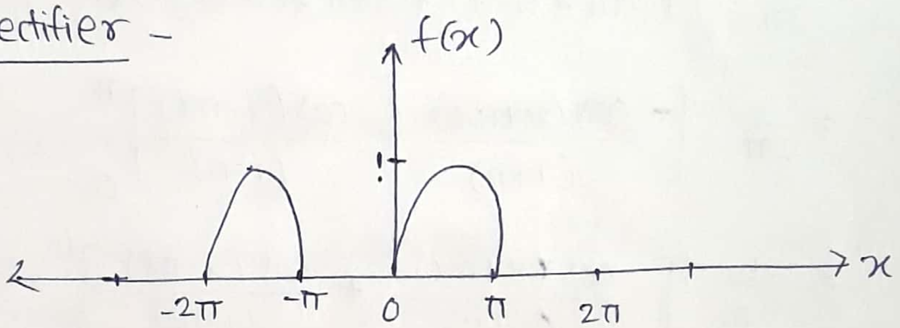
$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{a_0}{2} + 0 + \sum_{n=1,3,5\ldots}^{\infty} \left(\frac{2h}{\pi n}\right) \sin nx$$

$$= \frac{a_0}{2} + \frac{2h}{\pi}\left(\frac{1}{1}\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots\right)$$

$$\boxed{f(x) = \frac{h}{2} + \frac{2h}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots\right)}$$

Q) Half wave Rectifier -



$$f(x) = 0 \qquad -\pi \le x < 0$$
$$= \sin x \qquad 0 < x \le \pi$$

Output of the half wave rectifier is as shown

The fourier series expansion of the fun $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

The coefficients of fourier series expansion are given by,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$$

$$= \frac{1}{\pi}\left[0 + \int_{0}^{\pi} \sin x\,dx\right]$$

$$= \frac{1}{\pi}\left[\cos x\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[-\cos \pi + \cos 0\right] = \frac{1}{\pi}\left[1+1\right] = \frac{2}{\pi}.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} \sin x\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\int_{0}^{\pi} \frac{1}{2}.2\sin x\cos nx\,dx$$

$$= \frac{1}{\pi 2}\int_{0}^{\pi} 2\sin x\cos nx\,dx$$

$2\sin A\cos B$
$= \sin(A+B) + \sin(A-B)$

$$= \frac{1}{2\pi}\int_{0}^{\pi}[\sin(x+nx) + \sin(x-nx)]\,dx$$

$$= \frac{1}{2\pi}\left[-\frac{\cos(x+nx)}{(1+n)} - \frac{\cos(x-nx)}{(1-n)}\right]_0^{\pi}$$

$$= \frac{-1}{2\pi}\left[\frac{\cos(x+nx)}{(1+n)} + \frac{\cos(x-nx)}{(1-n)}\right]_0^{\pi}$$

$$= \frac{-1}{2\pi}\left[\frac{(1-n)\cos(x+nx) + (1+n)\cos(x-nx)}{(1-n^2)}\right]_0^{\pi}$$

$$= \frac{-1}{2\pi}\frac{1}{(1-n^2)}\left\{[(1-n)\cos(\pi+n\pi) + (1+n)\cos(\pi-n\pi)] - [(1-n)\cos(0+n0) + (1+n)\cos(0-0)]\right]$$

$$= \frac{-1}{2\pi}\frac{1}{(1-n^2)}\left\{[(1-n)[-\cos n\pi] + (1+n)[-\cos n\pi]] - [(1-n)\cos 0 + (1+n)\cos 0]\right]$$

$\cos(\pi+\theta) = -\cos\theta$
$\cos(\pi-\theta) = -\cos\theta$

$$= \frac{-1}{2\pi}\frac{1}{(1-n^2)}\left\{-\cos n\pi + n\cos n\pi - \cos n\pi - n\cos n\pi - [(1-n)+(1+n)]\right]$$

$$= \frac{-1}{2\pi}\left\{\frac{1}{(1-n^2)}\left[-2\cos n\pi - 2\right]\right.$$

$$= \frac{+1}{2\pi}\frac{2}{(1-n^2)}[\cos n\pi + 1]$$

$$= \frac{1}{\pi(1-n^2)}[\cos n\pi + 1] = \begin{cases} 0 & \text{odd} \\ \frac{2}{\pi(1-n^2)} & \text{even.}\end{cases}$$

b_n

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[0 + \int_{0}^{\pi} \sin x \cdot \sin nx\,dx\right]$$

$$= \frac{1}{\pi 2}\int_{0}^{\pi} 2\sin x\sin nx\,dx$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B).$$

$$= \frac{1}{2\pi}\cdot\int_{0}^{\pi}\left[\cos(x-nx) - \cos(x+nx)\right]dx$$

$$= \frac{1}{2\pi}\left[\int_{0}^{\pi}\cos(x-nx)\,dx - \int_{0}^{\pi}\cos(x+nx)\,dx\right]$$

$$= \frac{1}{2\pi}\left\{\left[\frac{\sin(x-nx)}{(1-n)}\right]_0^{\pi} - \left[\frac{\sin(x+nx)}{(1+n)}\right]_0^{\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{\sin(\pi-n\pi)}{(1-n)} - \frac{\sin(0-0)}{(1-n)}\right] - \left[\frac{\sin(\pi+n\pi)}{(1+n)} - \frac{\sin(0+0)}{(1+n)}\right]\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{\sin n\pi}{(1-n)} - 0\right] - \left[\frac{-\sin n\pi}{(1+n)} - 0\right]\right\}$$

$$\sin(\pi+\theta) = -\sin\theta.$$
$$\sin(\pi-\theta) = \sin\theta$$

$$= \frac{1}{2\pi}\left\{\frac{\sin n\pi}{(1-n)} + \frac{\sin n\pi}{(1+n)}\right]$$

$$= \frac{1}{2\pi}\frac{1}{(1-n^2)}\left[(1+n)\sin n\pi - (1-n)\sin n\pi\right]$$

$$= \frac{1}{2\pi}\frac{1}{(1-n^2)}\left[\sin n\pi + n\sin\pi - \sin n\pi + n\sin\pi\right]$$

$\cancel{\sin n\pi}$

$$= 0.$$

$\sin\pi = 0$

Some Important Points.

$\int \sin x \, dx = -\cos x + C$

$\int \sin nx \, dx = \frac{-\cos nx}{n} + C$

$\int \cos x \, dx = +\sin x + C$

$\int \cos nx \, dx = \frac{+\sin nx}{n} + C$

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C.$

$2 \sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

$2 \sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$

$2 \cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$

$2 \cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$