Mathematical Concept and SI Units

Mathematical concepts:

Logarithm

Logarithmic relation:
2.5933 in which 2 is characteristic & 0.5933 is mantissa.

Multiplication & division: Multiplication process may be turned into the much easier process of the addition by the use of logarithms.

i) 250 × 30 × 8

log 250 + log 30 + log 8

= 2.3979 + 1.4771 + 0.9031

= 4.7781 $\xrightarrow{\text{AI}}$ 60,000

In the same way division is turned into subtraction by expressing the nos. as logarithms.

i) $\frac{10000}{100} = \frac{10^4}{10^2} = 10^{4-2} = 10^2 = 100$

By using logarithm

log 10000 = 4.0

log 10^2 = 2.0 $\longrightarrow$ 2.0 $\xrightarrow{\text{AI}}$ 100

Powers & roots :

i) $(100)^3 = 3 \log 100$

$= 3(2.00)$

$= 3(2) = 6 \xrightarrow{AL} 1000000$

ii) $(3)^3 = 3 \log 3$

$= 3(0.4771) = 1.4313 \longrightarrow 27$

iii) $16^{2.3} = 2.3 \log 16$

$= 2.3 (1.2041)$

$= 2.7694 \xrightarrow{AL} 588$

iv) $\sqrt[3]{27} = (27)^{1/3}$

$= 1/3 \log 27$

$= \frac{1}{3} (1.4313)$

$= 0.4771 \xrightarrow{AL} 3$

Characteristic : It can be zero, positive or negative no.

Mantissa :- It is the decimal part of a log of given no.

pH :- The pH of soln is the -ve log of its H^+ ion conc^n

$$pH = -\log [H^+]$$

Si.

pOH :- The pOH of soln is the -ve log of its OH^- ion conc^n.

$$pOH = -\log [OH^-]$$

Since pure water ionises to $[H^+]$ and $[OH^-]$

$[H^+]$ conc^n $= 1 \times 10^{-7}$ moles/lit

$[OH^-]$ conc^n $= 1 \times 10^{-7}$ moles/lit

$pH + pOH = 14$

✓ i) Calculate pH of 0.001 M HCl

$[H^+] = 0.001$ M

~~$pH = -\log [H^+] = \log (0.001) = 0.001 = 1 \times 10^{-3}$~~

but $pH = -\log [H^+] = -\log (0.001) = -\log (1 \times 10^{-3})$

$= 3 \log 10 = 3$

(ii) Determine the pH of 0.1M NaOH soln

$[OH^-] = 0.1\ M$

$pOH = -\log [OH^-]$

$= -\log (0.1) = -\log (1 \times 10^{-1})$

$= 1 \log 10 = 1$

but $pH + pOH = 14$

$pH = 14 - pOH$

$= 14 - 1$

$pH = 13$

(iii) Calculate hydrogen ion conc of 4 pH soln

$pH = 4$

$pH = -\log [H^+]$

$4 = -\log [H^+]$

$\log [H^+] = -4$

$[H^+] = \frac{1}{\log} (-4)$

$=$ Anti $(-4) = 1 \times 10^{-4}$ mole/lit

(iv) Hydrogen ion concn of HCl soln is 3.2×10^{-2} M. Calculate its pH value

$[H^+] = 3.2 \times 10^{-2}\ M$

$pH = -\log [H^+]$

$= -\log (3.2 \times 10^{-2})$

$= -\log 3.2 + 2 \log 10$

$= -0.5051 + 2(1.0000)$

$= 2 - 0.5051 = 1.49$

v) Calculate pH value of 4.5×10^{-3} M soln of NaOH.

$[OH^-] = 4.5 \times 10^{-3}$

$pOH = -\log [OH^-]$

$= -\log (4.5 \times 10^{-3})$

$= -\log 4.5 + 3 \log 10$

$= -(0.6532) + 3(1.0000)$

$= 3 - 0.6532 = 2.3468$

but $pH + pOH = 14$

$pH + 2.3468 = 14$

$pH = 14 - 2.3468$

$= 11.65$

vi) pH of the soln is 3.6, calculate hydrogen ion conc^n

$pH = 3.6$

$-\log [H^+] = pH$

$-\log [H^+] = 3.6$

$\log [H^+] = -3.6$

$[H^+] = \bar{4}.4$

$\therefore [H^+] = 2.512 \times 10^{-4}$

-3.6

$-1+1$

-4.4

$\cdot 4 \times 10^{-4}$

$\downarrow H+1$

2.512×10^{-4}

or

$-3.6 + 4 - 4$

$\cdot 4 \times 10^{-4}$

Graphical representation of eqns :

Eqn of straight line is

$$y = mx + c$$

where m = slope of line

c = intercept

i) What is the eqn of the line which has a slope of $-1/2$ & intersect the ordinate at -10.

$$y = mx + c$$

$$= -\frac{1}{2}x - 10$$

$$= -0.5x - 10$$

ii) What is the eqn of st. line ? Give the eqn of a line with slope $1/3$ & intercept 7.

Eqn of st. line is $y = mx + c$

where m = slope of st. line

c = intercept

Here $m = 1/3$

$c = 7$

$\therefore y = 1/3 x + 7$

$y = 0.33x + 7$

iii) Find the eqn of st. Line passing thr' (2,4) & (4,6) having slope −3.

$x_1 = 2$, $y_1 = 4$ $\quad m = -3$

$x_2 = 4$ $\quad y_2 = 6$

$$\frac{y_2 - y_1}{y_1 - y} = m \frac{(x_2 - x_1)}{(x_1 - x)}$$

$$\frac{6-4}{4-y} = m \left(\frac{4-2}{2-x}\right)$$

$$\frac{6-4}{4-y} = -3 \left(\frac{4-2}{2-x}\right)$$

$$\frac{2}{4-y} = -3 \left(\frac{2}{2-x}\right)$$

$$\frac{2}{4-y} = \frac{-6}{2-x}$$

$$2(2-x) = -6(4-y)$$

$$4 - 2x = -24 + 6y$$

$$-2x = 6y - 24 - 4$$

$$-2x = 6y - 28$$

$$x = -3y + 14$$

$$-3y = x - 14$$

$$y = -\frac{x}{3} + \frac{14}{3}$$

$$= -0.33x + 4.66$$

$$y = -0.33x + 4.66$$

iv) Find the eqn of st. Line passing thr' points (4,2) & (6,6) having intercept $-1/5$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6-2}{6-4} = \frac{4}{2} = 2$$

$$y = mx + c$$

$$= 2x + (-1/5) = 2x - 1/5$$

$$y = 2x - 0.2$$

v) Find eqn of st. line passing thr' points (3,6) & (4,8) having slope (1/4)

$$\frac{y_2 - y_1}{y_1 - y} = m \frac{x_2 - x_1}{x_1 - x}$$

$$\frac{8-6}{6-y} = \frac{1}{4}\left(\frac{4-3}{3-x}\right)$$

$$\frac{2}{6-y} = \frac{1}{4}\left(\frac{1}{3-x}\right)$$

$$\frac{2}{6-y} = \frac{1}{}$$

$$\frac{8}{6-y} = \frac{1}{3-x}$$

$$24 - 8x = 6 - y$$

$$-y = 24 - 8x - 4$$

$$-y = -8x + 12$$

$$y = 8x - 12$$

Integration :

① $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

② $\int \frac{1}{x} dx = \log x + c$

③ $\int \sec^2 x \, dx = \tan x + c$

④ $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

⑤ $\int \sec x \tan x \, dx = \sec x + c$

⑥ $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

⑦ $\int e^x dx = e^x + c$

⑧ $\int \sin x \, dx = -\cos x + c$

⑨ $\int \cos x \, dx = \sin x + c$

(10) $\int a^x dx = \frac{a^x}{\log a} + C$

(11) $\int 1 dx = x + C$

(12) $\int (u+v) dx = \int u dx + \int v dx$

(1) Evaluate $\int_1^2 x^2 dx$

$$\int_1^2 x^2 dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{2^3}{3} - \frac{(1)^3}{3}$$

$$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

(2) Integrate the following fun. w.r.t. x and find within the limits $x=2$ to $x=4$

(1) x^3

$$\int_2^4 x^3 dx = \left[\frac{x^4}{4}\right]_2^4 = \frac{4^4}{4} - \frac{2^4}{4}$$

$$= \frac{4\times4\times4\times4}{4} - \frac{2\times2\times2\times2}{4}$$

$$= \frac{256}{4} - \frac{16}{4} = \frac{240}{4} = 60$$

* ii) $\int_2^4 \sqrt{x}\, dx$

$$\int_2^4 \sqrt{x}\, dx = \int_2^4 (x)^{1/2}\, dx$$

$$\left[\frac{x^{3/2}}{3/2}\right]_2^4 = \frac{4^{3/2}}{3/2} - \frac{2^{3/2}}{3/2}$$

$$= 0.2666$$

iii) $\int_2^4 2x^2\, dx$

$$\int_2^4 2x^2\, dx = \left[\frac{2x^3}{3}\right]_2^4$$

$$= \frac{2(4)^3}{3} - \frac{2(2)^3}{3}$$

$$= \frac{2(64)}{3} - \frac{2(8)}{3}$$

$$= \frac{128}{3} - \frac{16}{3} = \frac{240}{3} = 80\ \frac{1}{2}$$

iv) $\int_2^4 (x^3 + 3x^2)\, dx$

$$\int_2^4 (x^3+3x^2)\, dx = \int_2^4 x^3\, dx + \int_2^4 3x^2\, dx$$

$$\left[\frac{x^4}{4}\right]_2^4 + \left[\frac{3x^3}{3}\right]_2^4$$

$$\left[\frac{4^4}{4}\right] - \left[\frac{2^4}{4}\right] + \left(\frac{3(4)^3}{3} - \frac{3(2)^3}{3}\right)$$

$$\left(\frac{256}{4} - \frac{16}{4}\right) + \left(\frac{3(64)}{3} - \frac{8\times3}{3}\right)$$

$$\frac{240}{4} + \frac{192}{3} - \frac{24}{3}$$

$$60 + \frac{168}{3}$$

$$60 + 56$$

$$= 116$$

Integrate the following

* (i) $\int (3x^3 + 4x + 6)\, dx$

$$\int 3x^3\, dx + \int 4x\, dx + \int 6 \cdot dx$$

$$\frac{3x^4}{4} + \frac{4x^2}{2}$$

$$\frac{3x^4}{4} + 2x^2$$

ii) $\int_1^2 \frac{3}{2}x^2 + \frac{2}{3}x \, dx$

$\int \frac{3}{2}x^2 \, dx + \int \frac{2}{3}x \, dx$

$\frac{3}{2}\frac{x^3}{3} + \frac{2}{3}\frac{x^2}{2}$

$\int_1^2 \frac{x^3}{2} + \int_1^2 \frac{x^2}{3}$

$\left[\frac{2^3}{2} - \frac{1^2}{2}\right] + \left[\frac{2^2}{3} - \frac{1^2}{3}\right]$

$\left(\frac{8}{2} - \frac{1}{2}\right) + \frac{4}{3} - \frac{1}{3}$

$\frac{7}{2} + \frac{3}{3}$

$\frac{7}{2} + 1 = \frac{9}{2}$

Derivative :

① $\frac{d}{dx}(\sin x) = \cos x$

② $\frac{d}{dx}(\cos x) = -\sin x$

③ $\frac{d}{dx}(\tan x) = \sec^2 x$

④ $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

⑤ $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

⑥ $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

⑦ If $y = u + v$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

⑧ If $y = u - v$

$\therefore \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

(9) If $y = uv$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

(10) If $y = u/v$ & $v \neq 0$

$$\therefore \frac{dy}{dx} = \frac{v(du/dx) - u(dv/dx)}{v^2}$$

(11) $\frac{d}{dx}(c) = 0$ where c is constant

(12) $\frac{d}{dx}(x^n) = nx^{n-1}$

(13) If $y = x$ then $\frac{dy}{dx} = 1$

(14) If $y = \sqrt{x} = x^{1/2} = \frac{1}{2}x^{1/2-1} = 1/2\, x^{-1/2} = \frac{1}{2}\frac{1}{x^{1/2}}$

$$= \frac{1}{2\sqrt{x}}$$

(15) If $y = \log_e x$

$$\frac{dy}{dx} = 1/x$$

① If $f(x) = ax^2 + 2hxy + by^2$, find $f(x)$

$$f(x) = ax^2 + 2hxy + by^2$$

$$= 2ax + 2hy$$

$$= 2(ax + hy)$$

Permutation and combinations:

Permutation: The different arrangements that can be made with a given no. of things taking some or all of them at a time are called permutations.

$$^nP_r = \frac{n!}{(n-r)!}$$

① Find the value of $^{12}P_4$.

$$^{12}P_4 = \frac{12!}{(12-4)!} = \frac{12!}{8!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8!}{8!}$$

$$= 11880$$

② In how many ways can the letters of 'Lahore' be arranged?

→

Total no. of letters in the word Lahore = 6
These are all different ~~nos.~~ letters.

∴ No. of arrangements of these letters all at a time

$${}^{6}P_{6} = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

③ Find the value of ${}^{8}P_{2}$

$${}^{8}P_{2} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2!}{(8-2)!} = \frac{8 \times 7 \times 6!}{6!}$$

$$= 56$$

Combinations : The different selections or groups that can be made out of a given no. of things taking some or all of them at a time are called combinations.

$${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!} = \frac{{}^{n}P_{r}}{r!}$$

① Evaluate ${}^{50}C_{47}$

$${}^{50}C_{47} = \frac{50!}{47!\,(50-47)!} = \frac{50!}{47!\,(3)!}$$

$$= \frac{50 \times 49 \times 48 \times 47!}{47! \times 3!} = \frac{50 \times 49 \times 48}{3 \times 2 \times 1}$$

$$= 19600$$

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ii) Evaluate $^{51}C_{49}$

$$^{51}C_{49} = \frac{51!}{49!\,(51-49)!} = \frac{51!}{49!\,(2)!}$$

$$= \frac{51 \times 50 \times 49!}{49! \times 2!} = 1275$$

iii) Evaluate $^{18}C_{16}$

$$^{18}C_{16} = \frac{18!}{16!\,(18-16)!} = \frac{18!}{16!\,(2!)}$$

$$= \frac{18 \times 17 \times 16!}{16! \times 2!} = \frac{18 \times 17}{2 \times 1} = 153$$

iv) How many permutations are possible with seven diff colours taken two at a time. How many combinations ?

$$^{n}P_{r} = n(n-1)$$

$$= 7(7-1)$$

$$= 7(6) =$$

$$^{7}P_{2} = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{7 \times 6 \times 5!}{5!}$$

$$= 42$$

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${}^{n}C_{r}$

$${}^{7}C_{2} = \frac{7!}{2!\,(7-2)!} = \frac{7!}{2! \times 5!}$$

$$= \frac{7 \times 6 \times 5!}{2! \times 5!} = 21$$

PHYSICAL QUANTITIES AND THEIR DIMENSIONS :→

International systems of units

SI units are widely used but they have not been fully accepted by the scientific community.

SI base Unit

	Physical quantity	Unit	Symbol
①	Length	meter	m
②	Mass	Kilogram	kg
③	time	serond	sec
④	temp	kelvin	K
⑤	Electric current	ampere	A
⑥	no. of particles	mole	mol

Units of length

The SI unit of length is meter (m). Fractions & multiples of SI units are named by adding appropriate prefixes.

Unit	Symbol	Relⁿ
meter	m	
kilometer	km	$1km = 10^3 m$
decimeter	dm	$1dm = 10^{-1} m$
centimeter	cm	$1cm = 10^{-2} m$
millimeter	mm	$1mm = 10^{-3} m$
micrometer	μm	$1\mu m = 10^{-6} m$
nanometer	nm	$1nm = 1 \times 10^{-9} m$
picometer	pm	$1pm = 1 \times 10^{-12} m$
angstrom	A^0	$1A^0 = 10^{-8} cm = 10^{-10} m$

SI units of volume:

The derived SI unit of volume is cubic meter or m^3

The related units of volume are:

cubic centimeter or cm^3

cubic decimeter or dm^3

Another common measure of volume is liter. A liter is the volume occupied by a cube 10 cm on edge.

$$1 L = (10 cm)^3 = 1000 cm^3$$

$$1 l = 1000 ml$$

$$1000 ml = 1000 cm^3$$

$$1 ml = 1 cm^3$$

SI unit of temp :-

The temps are expressed in degree Celcius (°C). The SI system uses the Kelvin scale. Celcius & Kelvin temp are related as

$$K = {}^{\circ}C + 273$$

It may be noted that the unit for temp on the Kelvin scale is K and not °K.

Celcius & Fahrenheit scale is related by eqn

$${}^{\circ}F = \frac{9}{5}\,{}^{\circ}C + 32$$

Unit of mass & weight :

The mass (m) of an object is the amount of matter contained in that object. The weight (W) is the force & not mass. It is given by

$$W = m \times g$$

The basic unit of mass is gram.

Unit	Symbol	Reln
gram	g	
kilogram	kg	$1\,kg = 1 \times 10^{3}\,g$
milligram	mg	$1\,mg = 1 \times 10^{-3}\,g$
microgram	μg	$1\,\mu g = 1 \times 10^{-6}\,g$

Unit of force :

Force is defined as product of mass & acceleration (a)

$$\therefore F = m \times a$$

$$= kg \cdot m/s^2$$

$$= kg \cdot m\, s^{-2}$$

The derived SI unit for force is $kg\, ms^{-2}$. The unit is called newton & has symbol N,

$$\therefore 1N = 1\, kg \cdot ms^{-2}$$

Units of work & heat energy:-

$$w = f \times d$$

$$= N \cdot m$$

The SI unit of work & energy is newton-meter. It is also called Joule (J)

$$1J = 1Nm$$

Heat is energy that flows from one object to another because of temp difference bet^n the objects. The quantity of heat transferred is expressed in calories.

$$1\, cal = 4.184\, J$$

Unit of Pressure :

Pre is defined as force per unit area exerted on a surface

$$P = F/A = N/m^2 \text{ or } N\, m^{-2}$$

The SI unit $N\, m^{-2}$ is named Pascal hare

Symbol Pa.

The various units of pre are related as

$$1 atm = 760 \text{ Torr} = 76 mm Hg$$

$$= \cancel{1.10 \times}$$

$$= 1.01 \times 10^5 \text{ Pa}$$

Units of density:

Density is defined as mass per unit vol.

$$d = \frac{m}{V} = \frac{kg}{m^3} \text{ or } kg\,m^{-3} \text{ or } g\,cm^{-3}$$

The term specific gravity is the ratio of density of a sub to the density of a reference substance.

$$\text{Sp. gr} = \frac{\text{density of a sub}}{\text{density of reference sub}}$$

Sp. gravity being the ratio of two densities has no units.