Karnaugh Maps (K maps)

What are Karnaugh[1] maps?

- Karnaugh maps provide an alternative way of simplifying logic circuits.
- Instead of using Boolean algebra simplification techniques, you can transfer logic values from a Boolean statement or a truth table into a Karnaugh map.
- The arrangement of 0's and 1's within the map helps you to visualise the logic relationships between the variables and leads directly to a simplified Boolean statement.

[1]Named for the American electrical engineer Maurice Karnaugh.

Karnaugh maps

- Karnaugh maps, or K-maps, are often used to simplify logic problems with 2, 3 or 4 variables.

Cell = 2^n ,where n is a number of variables

For the case of 2 variables, we form a map consisting of 2^2=4 cells as shown in Figure

B \ A	0	1
0	$A+B$	$\overline{A}+B$
1	$A+\overline{B}$	$\overline{A}+\overline{B}$

Maxterm

B \ A	0	1
0	00 **0**	10 **2**
1	01 **1**	11 **3**

B \ A	0	1
0	$\overline{A}\overline{B}$	$A\overline{B}$
1	$\overline{A}B$	AB

Minterm

Karnaugh maps

- 3 variables Karnaugh map

Cell = 2^3=8

C \ AB	00	01	11	10
0	0 $\overline{A}\overline{B}\overline{C}$	2 $\overline{A}B\overline{C}$	6 $AB\overline{C}$	4 $A\overline{B}\overline{C}$
1	1 $\overline{A}\overline{B}C$	3 $\overline{A}BC$	7 ABC	5 $A\overline{B}C$

Karnaugh maps

- 4 variables Karnaugh map

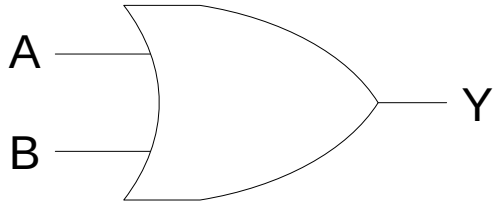
CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Karnaugh maps

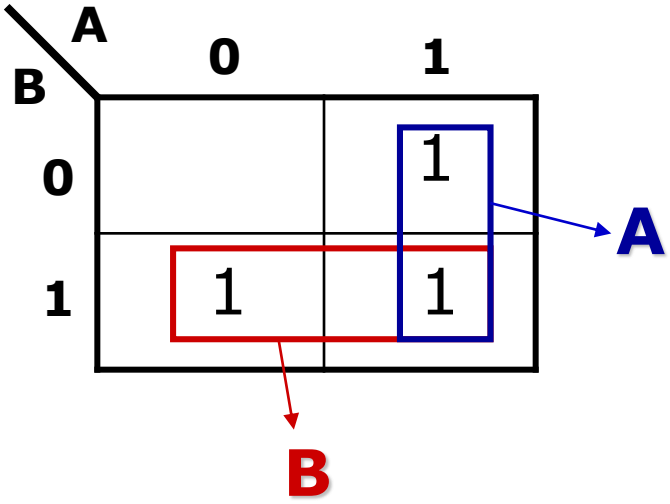
- The Karnaugh map is completed by entering a '1‘(or ‘0’) in each of the appropriate cells.
- Within the map, adjacent cells containing 1's (or 0’s) are grouped together in twos, fours, or eights.

Example

2-variable Karnaugh maps are trivial but can be used to introduce the methods you need to learn. The map for a 2-input OR gate looks like this:



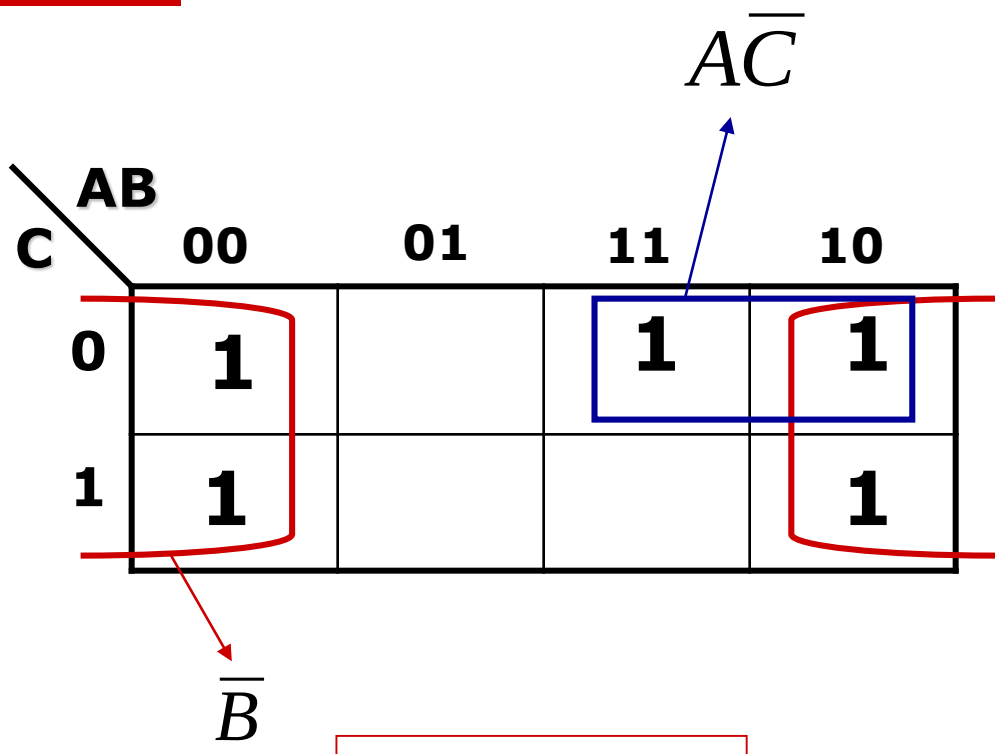
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



A+B

Example

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



$$\overline{B} + A\overline{C}$$

Exercise

- Let us use Karnaugh map to simplify the follow function.

  $F_1 = m_0+m_2+m_3+m_4+m_5+m_6+m_7$

  $F_2 = m_0+m_1+m_2+m_5+m_7$

- Answer

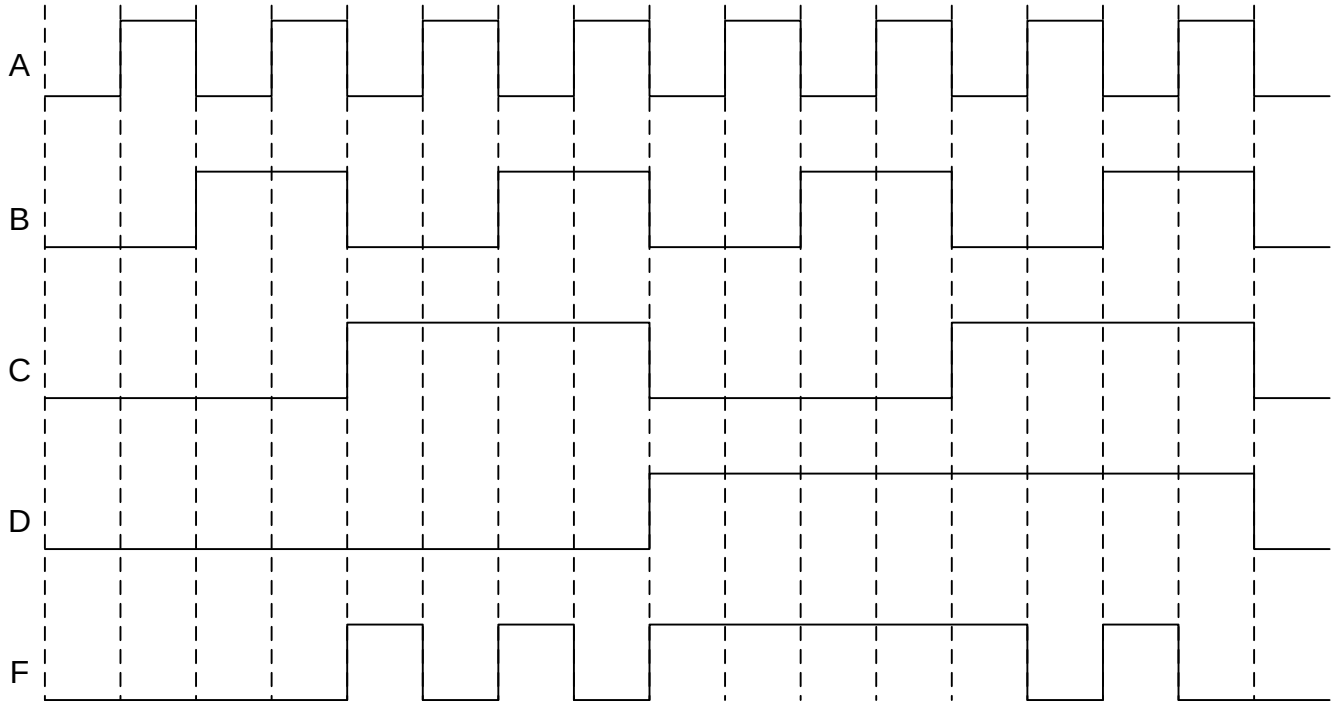
Exercise

Given the truth table, find the simplified SOP and POS form.

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

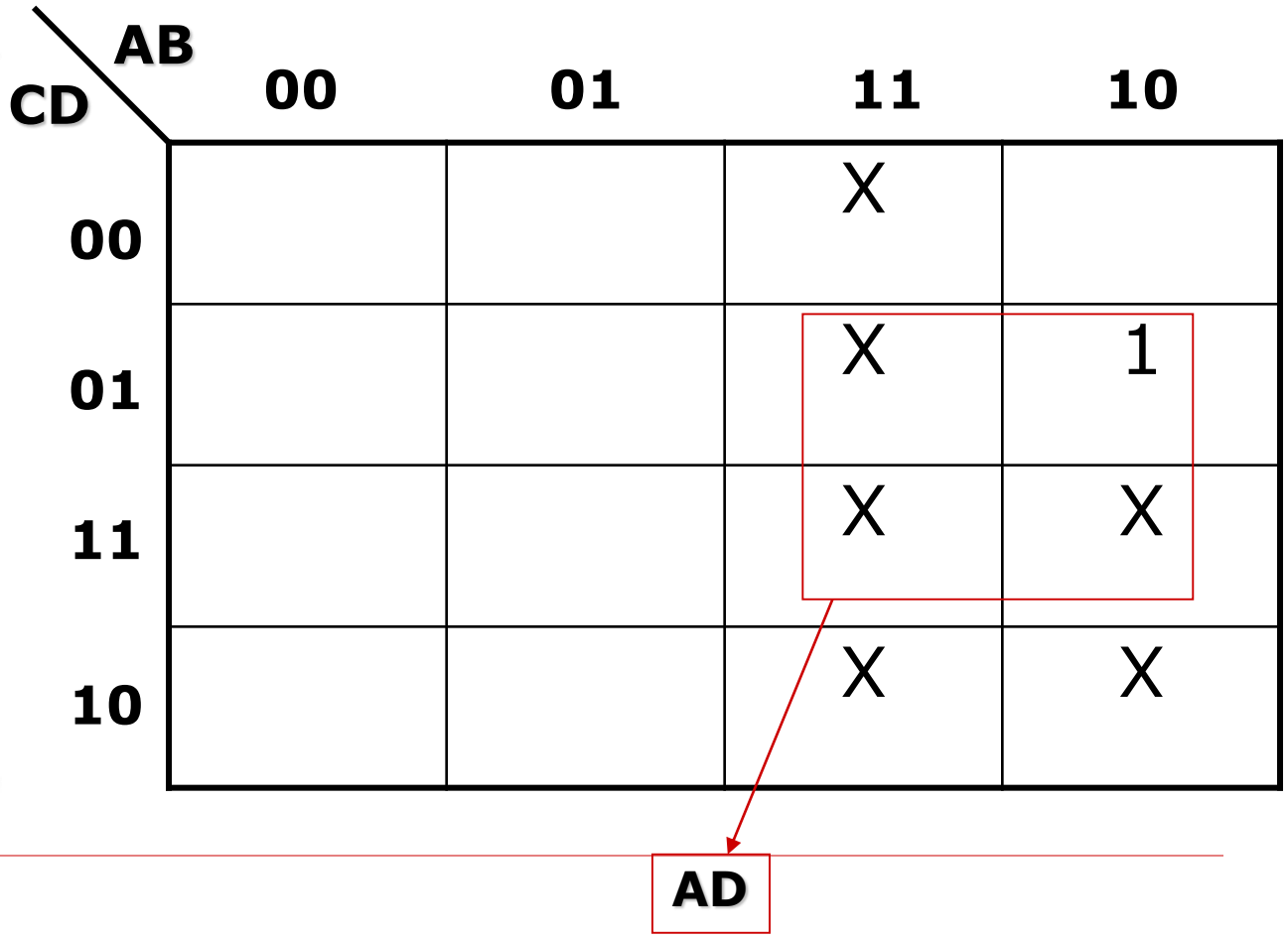
Exercise

- Design two-level NAND-gate logic circuit from the follow timing diagram.



Don’t care term

A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	x
1	0	1	1	x
1	1	0	0	x
1	1	0	1	x
1	1	1	0	x
1	1	1	1	x



Exercise

- **Design logic circuit that convert a 4-bits binary code to Excess-3 code**

A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0
1	0	1	0	x	x	x	x
1	0	1	1	x	x	x	x
1	1	0	0	x	x	x	x
1	1	0	1	x	x	x	x
1 1	1 1	1 1	0 1	x X	X X	X x	X x