B. Sc. III Year (Physics)

Sem - V Paper - XII

Name of Paper :- Quantum mechanics

PHY 301: (Compulsory Paper)

1) Unit - I : Particle Properties of Waves.

Introduction, Photo-electric effect, Quantum thory of light, the compton effect, de Broglie waves, Wave function, de Broglie Wave velocity, Wave & Group velocities, G. P.: Thomsons experiment, the Uncertainty principle and its applications, The wave particle Duality.

2) Unit-II: Schrodinger Equation:

Introduction, Schrodinger's Equation: time dependent form, Probability current, Expectation values, operators, Schrodinger's Equation: steady-state form or time independent form, Eigen values and Eigen functions, problems.

3) Unit - III : Applications of Quantum mechanics

Introduction, The particle in one dimensional box, energy quantization, the particle in a box: wave functions, the particle in a box: momentum quantization, the Harmonic oscillator, the Harmonic oscillator - Energy level, the particle in a three dimensional box.

4) Unit - IV : Quantum theory of the H-atom

Schrodinger's equation for the Hydrogen atom in spherical polar coordinate, separation of variables, quantum numbers, total quantum number, orbital quantum number, magnetic quantum number (Introduction only), Electron probability density.

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(2) Quantum theory of radiation or light & photon

The quantum concept give by planks in 1901, led to the important conclusion that radiation is not being emitted in continous fashion but in discrete budles of energy equal to $h\nu$. These budles or packets of radiant energy are called as quanta or photons. On the other hand it can be stated that exchanges of energy between radiation and matter can not take place continously but are limited to discreate set of values 0, $h\nu$, $2h\nu$, $3h\nu$, $4h\nu$ ...... It is also obvious that quanta of radiations of different frequencies have different sizes or energies.

Suppose photo regarded as a particle (of radiation) then to find out the characteristics of particle such as mass, momentum, energy, statistics etc.

1) Energy of photon :-

Energy of photon is only in multiples of $h\nu$,

$\therefore E = nh\nu$, where h is the planks constant and ν its frequency. Howeer, according to modern ideas that limiting value of photon energy $\frac{1}{2}h\nu$ and other energie have been found to differ by an integral multiple of $h\nu$.

If a photon undergoes interaction with matter, either it can be completely absorbed, transfering all its energy, or it may transfer part of its energy, and its frequency is adjust to a lower value and mainting particle characters. If many photon exist, they have more energy and intensity of radiation is also large. It means that intensity is not concerned with individual photon energies but simply gives their number. It is found that energy is only dependent on the intrinsic property of the photon. Thus energy of the photon is independent of its intensity, depending only on its frequency. This concept fail in classical mechanics & observed that radiation is purely wave and energy estimated by the intensity of wave disturbance, dependent on the physical properties of the medium.

ii) Constant 'h' of photon :-

It is denotes an elementary quantum or quantity of action. It is responsible for the discrete individuality of photon which makes radiation behave like a particle. The dimensions of h can be.

$$h = \frac{\text{Energy}}{\text{Frequency}} = \frac{ML^2T^{-2}}{T^{-1}}$$

$$= [M][LT^{-1}][L]$$

$$= \text{Angular momentum}$$

∴ h is defined as the smallest quantum of angular momentum of a particle. A momentum is always associated with motion. It means the h always represent motion. ($h/2\pi$)

iii) Mass and momentum of photon :-

As photon have energy and are in motion in velocity c. we know that

$$E = mc^2 \qquad m = \frac{E}{c^2} = \frac{h\nu}{c^2} \quad \text{--- (i)}$$

~~Now~~ we know that momentum is-

$$p = mv$$

$$= \frac{h\nu}{c^2} \times c = \frac{h\nu}{c} \quad \text{--- (ii)}$$

$$\therefore p = \frac{h\nu}{c}$$

$$\therefore E = h\nu = mc^2 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} c^2$$

$$m_0 = \sqrt{1-\frac{v^2}{c^2}} \cdot \frac{h\upsilon'}{c^2} \quad \text{--- (iii)}$$

As the photon always travels with the velocity of light 'c' while its energy content $h\upsilon$ is always finite. It means that the rest mass m_0 of the photon approaches zero, in such way that as v approaches 'c' and its energy always approaches the value $h\upsilon$. Thus photon is not a particle in complete because a material particle much have a rest mass. Hence photon is always having a wave-structure, some times in compact from like of photons.

iv) Non-electrical nature of photon's :-

The photons consistituting radiations are electrically neutral. They are not affected by electric or magnetic field and also they do not ionise directly them selves. However they can eject charged particles from matter when they imping on atoms.

v) Photon statistics :-

There is an important difference between an ordinary gas and the photon gas. The molecules of gas move about with different velocities and different energies while the photon all travel with the same velocity c through with different energies, $h\upsilon$.

$= \frac{nh\nu}{\nu}$

$E = mc^2$

$E\sqrt{1-\frac{v^2}{c^2}} = m_0c^2$

$= \frac{n \cdot h}{T}$

$E^2\left(1-\frac{v^2}{c^2}\right) = m_0c^4$

$E^2 = \frac{m_0^2c^4}{c^2-v^2}$

$h\upsilon$

E

$m_0 = 1-\frac{v^2}{c^2} \cdot \frac{h^2v^2}{c^2}$

$0 = 1-\frac{v^2}{c^2}$

$0 = 1-\frac{v^2}{c^2}$

$0 = c^2 - v^2$

$c^2 = v^2$

Photoelectric Effect :-

Defn :-

Photoelectric effect is the phenomenon of ejections of electron from a metal plate when light of a suitable-wavelength falls on it. The emitted electrons are called as photoelectrons and phenomenon is called as photoelectric effect.

The phenomenon was discovered by Hertz when he allowed ultraviolet light to fall on zinc plate. The phenomenon was experimentally verified by Hallwaches, Lenard, J. J. Thomson, R. A. Millikan and others.

Afterwards it was discovered that alkali metals like lithium, sodium, potassium, rubidium and cesium eject electrons when visible light falls on them. Millikan (1863-1913) investigated effect with a number of alkali metals over a wide range of light frequencies and was given Nobel prize in 1923.

Experimental Arrangement :-

A simple experimental arrangement to study the photoelectric effect is shown in figure.

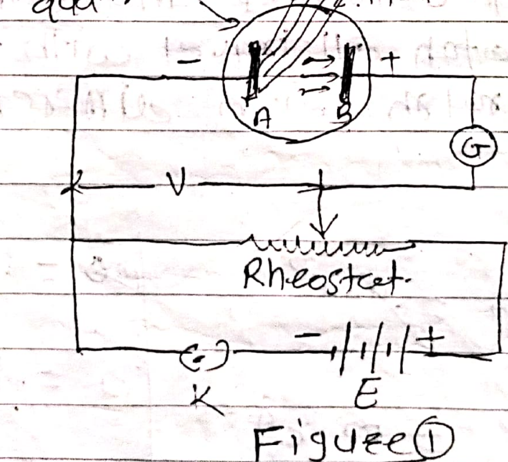


Figure ①

The apparatus consists of two photosensitive surfaces A and B enclosed in a evacuated quartz bulb. The plate A is connected to the negative terminal of a potential divider while

The plate B is connected to the positive terminals through a galvanometer or a micro ammeter. In the absence of any light, there is no flow of current and hence there is no deflection in the galvanometer or microammeter. But when monochromatic light is allowed to fall on plate A, currents starts flowing in the circuit which is indicated by galvanometer. The current is known as photocurrent. This show that when light falls on the metallic surface, electron are ejected.

The number of electrons emitted and their K.E. depends on the following factors.

i) The potential difference between the two electrodes or plates A & B.

ii) The intensity of incident radiation.

iii) The frequency of incident radiation.

iv) The photometal used.

② characteristic of photo-electrons

① The effect of potential difference :-

For a given photometallic surface A, keeping the intensity and frequency of the incident radiation fixed. Let us consider the effect of potential difference between the plates. When the +ve potential of the plate B is increased, photoelectric current is also increased. However, if the positive potential is further increased such that it is large enough to collect all the photoelectrons emitted from plate A, the photoelectric current reaches a certain maximum value. This value of the current is known as saturation current. Further increase in the potential hardly produces any appreciable increase in current. If the potential difference is kept zero, it is observed that photoelectric current still flows in the same direction. This shows that the incident radiation not only provides a conducting path but in addition an electromagnetic force to photo-electrons. If the negative or retarding potential is further increased, the photocurrent decreases and finally becomes zero at particular value.

"The negative potential of the plate B at which the photoelectric current becomes zero is called as cut off potential or stopping potential."

(2)

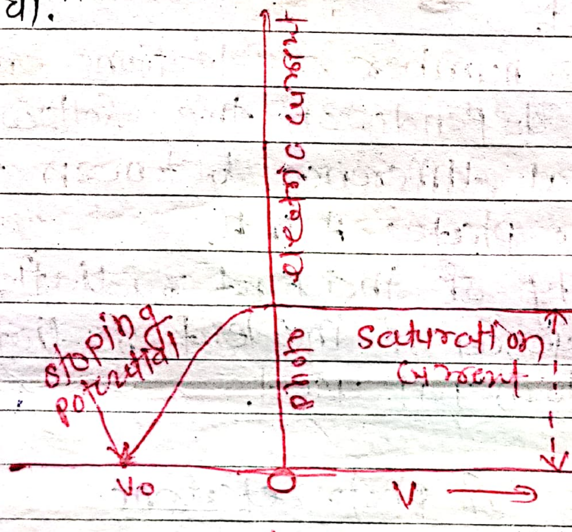


Fig (2)

(*) Effect of intensity of incident Radiation :-

The variation of photocurrent as a function of potential difference between the two plates for different intensities of incident radiation. If the intensity of incident radiation is increased from I to 2I and the experiment is repeated then the photoelectric current increases in the same ratio for all positive values of V. As V is made negative, the photoelectric current decreases sharply & reaches zero at the same value of the voltage V_0, the stopping potential. Hence we conclude that i) the stopping potential is independent of the intensity of incident radiation and ii) the saturation current is proportional to the intensity of incident radiation i.e. higher is the intensity of incident radiation higher is the saturation current.

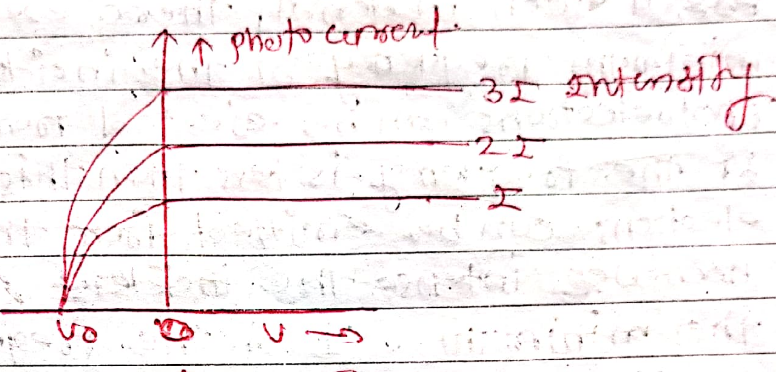


Figure (3)

(3) The Effect of the frequency of incident radiation :-

The effect of varying frequency of the incident radiation while keeping the same emitting surface and same intensity of incident radiation. The variation of stopping potential with frequency of incident light. Here stopping potentials are measured for different frequencies. The graph shows that at frequency ν_0, the stopping potential is zero. The frequency ν_0 is known as threshold frequency and wavelength corresponding to threshold frequency is called as threshold wavelength. The photoelectric effect occurs above this frequency while below this frequency. Hence, the threshold frequency is defined as the minimum frequency (ν_0), of the incident radiation which can cause photoelectric emission i.e. this frequency is just able to liberate electrons without giving them addition energy.

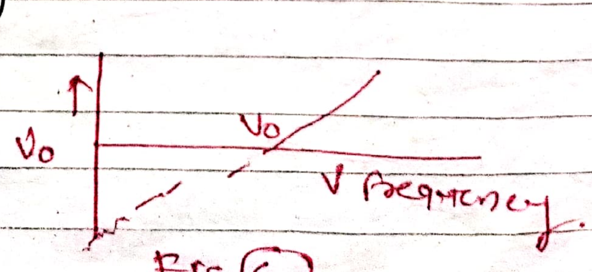


Fig (4)

Fundamental Laws of Photo-electric Emission:-

1) The number of electrons emitted per second, ie photoelectric current is proportional to the intensity of incident light.

2) For a given material, there exists a certain minimum frequency of the incident light so that photoelectrons can be ejected from metal surface. If the frequency is less than this frequency no electrons can be emitted from the metal surface however intense the incident light may be. The minimum value of the frequency is known as threshold frequency and the corresponding wavelength as threshold wavelength.

3) The maximum velocity or the K.E. of photo-electron depends on the frequency of radiation and not on intensity. The K.E. of photo-electrons increases with increase of frequency of incident light.

4). The rate at which the electrons are emitted from a photo cathode is independent of its temp. This shows that photoelectric phenomenon is entirely from thermionic emission.

5) Electron emission from a photosensitive surface is almost instantaneous and emission continues as long as the frequency of incident radiation is greater than the threshold frequency. The time lag betwn the incident radiations and the emission of electrons is less than 10^{-8} s.

6) For a given metal surface, stopping potential Vo is directly proportional to frequency but is independent of the intensity of incident light.

(*) Einstein's Photoelectric Equation

Following ~~idea~~ plank's idea that light consists of photons, Einstein proposed an explanation of photo electric effect as early as 1905. According to Einstein's explanation in photoelectric effect one photon is completely absorbed by one electron, which there by gains the quantum of energy and may be emitted from the metal. The photon's energy is used in the following two parts.

i) A part of its energy is used to free the e⁻ from the atom and away from the metal surface. This energy is known as photoelectric work function of the metal. This is denoted by W_0.

ii) The other part is used a giving kinetic E. ($\frac{1}{2}mv^2$) to the electron. Thus,

$$h\nu = W_0 + \frac{1}{2}mv^2 \quad \text{--- (1)}$$

Where

v - is the velocity of emitted electron.

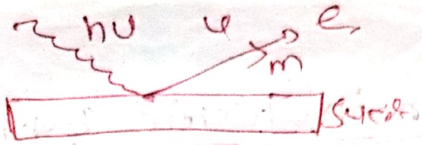
Equation (1) is known as Einstein's photoelectric equation.

When the photon's energy is of such a value that it can only liberate the electron from metal, then the K.E. of the electron will be zero. Eq (1) as.

$$h\nu_0 = W_0 \quad \text{--- (ii)}$$

Where ν_0 is called the threshold frequency.

Threshold frequency is defined as the minimum frequency which can cause photo-electron emission. If the frequency of the photon is below threshold frequency no emission of electron will take place. Corresponding to threshold frequency, long wavelength limit (λ_0). It represents the upper limit of wavelength for photo-electric effect. Its physical significance is that radiations having wave length longer than λ_0 would not be able to



eject electrons from a given metal surface whereas those having $\lambda < \lambda_0$ will. The value of λ_0 is -

$$\lambda_0 = \frac{c}{\nu_0} \quad \therefore \quad c = \nu_0 \lambda_0 \quad \text{--- (ii)}$$

we know that $W_0 = h\nu_0$ [$\because$ from eqn (ii)]

$$\nu_0 = \frac{W_0}{h} \quad \text{--- (iv)}$$

put this value in above eqn, we get,

$$\lambda_0 = \frac{c}{\nu_0} = \frac{c}{W_0/h} = \frac{ch}{W_0} \quad \text{--- (v)}$$

$$\therefore \lambda_0 = \frac{3\times10^{8} \times 6.625\times10^{-34}}{W_0} \quad \text{--- (vi)}$$

Here W_0 is in Joules. If W_0 in eV. Then

$$\lambda_0 = \frac{19.875\times10^{-26}}{1.602\times10^{-19} W_0} = \frac{12.4\times10^{-7}}{W_0} \text{ metre} \quad \text{(vii a)}$$

$$\lambda_0 = \frac{12400}{W_0} \text{ A}^0 \quad \text{--- (vii b)}$$

from eqn (vii b),

$$W_0 = \frac{12400}{\lambda_0} \text{ eV, where } \lambda_0 \text{ in A}^0.$$

Now, substituting the value of $W_0 = h\nu_0$ in eqn (i), we get,

$$h\nu = h\nu_0 + \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = h\nu - h\nu_0 = h(\nu - \nu_0)$$

$$K.E. = h(\nu - \nu_0) \quad \text{--- (viii)}$$

The eqn (viii) is the another form of Einstein's photoelectric equation.

we from eqn (i)

$$\frac{1}{2}mv^2 = h\nu - W_0$$

for particular emitter, work function w_0 = constant.

$$KE = \frac{1}{2}mv^2 \propto h\upsilon$$

$$v^2 \propto \upsilon \quad \text{--- (ix)}$$

Thus, the increase in frequency υ of incident light causes increases in velocity of photo-electrons, provided intensity of incident light is constant.

An increase in the intensity of radiation is equivalent to an increase in the number of photons falling on the emitting surface. If the frequency of incident radiation is above the threshold frequency ($\upsilon > \upsilon_0$) then the number of emitted electrons will increase. So that the intensity of emitted electron is directly proportional to the I of incident radiation.

$$\therefore \frac{1}{2}mv^2 = h\upsilon - h\upsilon_0$$

If V_0 is the stopping potential.

$$eV_0 = h\upsilon - h\upsilon_0$$

$$V_0 = \frac{h\upsilon}{e} - \frac{h}{e}\upsilon_0 \quad \text{--- (x)}$$

h/e are constant υ_0 is also const.
The eqn (x) shows that graph between V_0 & υ would be straight line of slope h/e.

⊛ Compton Effect :-

Classical theory of X-ray scattering :-

In 1925 prof. Compton discovered that when a monochromatic the phenomena of scattering. It may be due to an elastic collision of two particles i.e. photon of X-rays and scatter photon / electron. " When a photon of energy hυ collides with an electron, it will lose a part of its energy in the form of K.E. 'hυ'', and to the electron. Thus the scattered photon or radiations contains two components. one components having a lower same frequency or greater wavelength and other having same frequency or wavelength as that of incident radiations. " This observed change in radiation frequency or wave length of the scattered radiation is known as compton effect."

"This unchanged frequency scattered radiation is called as unmodified radiation while the radiation of lower frequency is called as modified radiation. This phenomena is also called as compton effect."

⊛ Working or construction :-

Suppose incident X-rays consist of photon each of energy hυ where h - is the plank's constant & υ - is the frequency. This energy hυ is much greater than that required to eject a free electron from the target or scatter electrons. Let the photon collide with an electron at rest in the target. A part of its energy is imported to the electron which is ejected with velocity v - in a direction, making an angle θ with that of the incident X-ray photon as shown in figure ①.

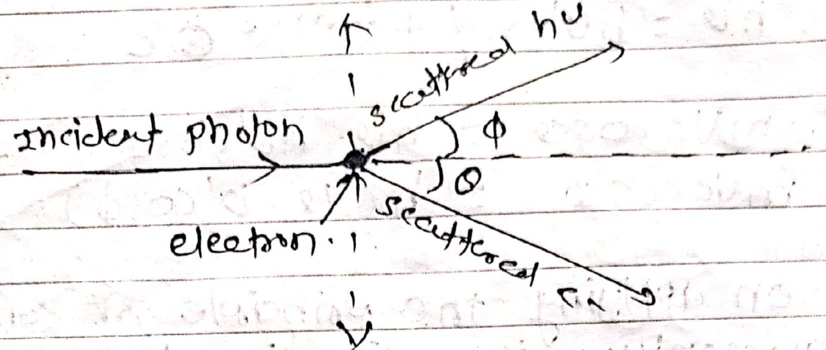


Fig ①: Compton Effect

The remaining energy is associated with X-ray photon, having lower frequency ν' and moving in direction making an angle φ with that of the incident photon. If m_0 is the rest mass of the electron, its mass, when moving with a velocity v, will be given by the theory of relativity as-

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{---(i)}$$

where c is the velocity of light.

on applying principle of conservation of energy, we get.

$$h\nu + m_0c^2 = h\nu' + mc^2 \quad \text{---(ii)}$$

$$mc^2 = h(\nu-\nu') + m_0c^2$$

squaring the above eqn, we get.

$$m^2c^4 = h^2[\nu^2 - 2\nu\nu' + \nu'^2] + m_0^2c^4 \quad \text{(iii)}$$

$$m^2c^4 = h^2 \quad + 2hc(\nu-\nu')m_0c^2 + m_0^2c^4$$

Now on appling principles of conservation of momentum in the direction of incident photo,

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + mV\cos\phi \quad \text{---(iv)}$$

multiplying by 'c' on both sides of the above eqn we gets:

$$h\nu = h\nu'\cos\phi + mv\cos\theta\, c$$

$$\therefore mvc\cos\theta = h\nu - h\nu'\cos\phi$$

$$mvc\cos\theta = h(\nu - \nu'\cos\phi) \quad \text{---} (V)$$

Now

on applying the principle of conservation of momentum in a direction $\perp$ to the direction of the incident photon,

$$0 = \frac{h\nu'}{c}\sin\phi - mv\sin\theta \quad \text{----} (iv)$$

$$\therefore \frac{h\nu'}{c}\sin\phi = mv\sin\theta$$

The above eqn multiplying by c on both sides we gets.

$$\therefore mvc\sin\theta = h\nu'\sin\phi \quad \text{---} (vi)$$

squaring the eqn (V) and (6) we gets.

$$m^2v^2c^2\cos^2\theta = h^2(\nu - \nu'\cos\phi)^2$$

$$= h^2(\nu^2 - 2\nu\nu'\cos\phi + \nu'^2\cos^2\phi)$$

$$\therefore m^2v^2c^2\cos^2\theta = h^2(\nu^2 - 2\nu\nu'\cos\phi + \nu'^2\cos^2\phi) \quad \text{---} (vii)$$

& -lly.

$$m^2v^2c^2\sin^2\theta = h^2\nu'^2\sin^2\phi \quad \text{--} (viii)$$

adding eqn (vii) & (viii), we gets:

$$\therefore m^2v^2c^2\cos^2\theta = h^2(\nu^2 + \nu'^2\cos^2\phi - 2\nu\nu'\cos\theta)$$

$$+ \quad m^2v^2c^2\sin^2\theta = h^2\nu'^2\sin^2\phi$$

$$m^2v^2c^2(\cos^2\theta + \sin^2\theta) = h^2(\nu^2 - 2\nu\nu'\cos\phi) + h^2\nu'^2(\sin^2\phi + \cos^2\phi)$$

$$\therefore m^2v^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\phi) + h^2\nu'^2 \quad \text{---} (ix)$$

$$\therefore m^2v^2c^2 = h^2[\upsilon^2+\upsilon'^2-2\upsilon\upsilon'\cos\phi] + h^2\upsilon'^2 \quad \text{---(ix)}$$

Here, substracting eqn (ix) in eqn (iii), one gets.

$$\therefore m^2v^2c^2 = h^2[\upsilon^2+\upsilon'^2-2\upsilon\upsilon'\cos\phi] + h^2\upsilon'^2$$

$$m^2c^4 = h^2[\upsilon^2+\upsilon'^2-2\upsilon\upsilon'] + 2h(\upsilon-\upsilon')m_oc^2 + m_o^2c^4$$

$$m^2c^2[c^2-v^2] = \cancel{h^2\upsilon^2} + \cancel{h^2\upsilon'^2} - 2\upsilon\upsilon'h^2 + 2h(\upsilon-\upsilon')m_oc^2 + m_o^2c^4 - \cancel{h^2\upsilon^2} - \cancel{h^2\upsilon'^2} + 2\upsilon\upsilon'\cos\phi$$

$$= -2\upsilon\upsilon'h^2 + 2h(\upsilon-\upsilon')m_oc^2 + m_o^2c^4 + 2\upsilon\upsilon'h^2\cos\phi$$

$$\therefore m^2c^2(c^2-v^2) = -2\upsilon\upsilon'h^2[1-\cos\phi] + 2h(\upsilon-\upsilon')m_oc^2 + m_o^2c^4 \quad \text{--- (x)}$$

Now, squaring calculating eqn one.

$$m^2 = \frac{m_o^2}{1-\frac{v^2}{c^2}} \quad \text{---- (xi)}$$

put this value in eqn (x), we gets.

$$\frac{m_o^2c^2}{1-\frac{v^2}{c^2}}(c^2-v^2) = -2\upsilon\upsilon'h^2(1-\cos\phi) + 2h(\upsilon-\upsilon')m_oc^2 + m_o^2c^4$$

$$\therefore \frac{m_o^2c^2}{\frac{\cancel{c^2-v^2}}{c^2}}(\cancel{c^2-v^2}) = -2\upsilon\upsilon'h^2(1-\cos\phi) + 2h(\upsilon-\upsilon')m_oc^2 + m_o^2c^4$$

$$\therefore \cancel{m_o^2c^4} = -2\upsilon\upsilon'h^2(1-\cos\phi) + 2h(\upsilon-\upsilon')m_oc^2 + \cancel{m_o^2c^4}$$

$$\therefore 2\cancel{h}^2\cancel{(\upsilon-\upsilon')} = \cancel{2h(\upsilon-\upsilon')m_oc^2}$$

$$2h(\upsilon-\upsilon')m_0c^2 = 2\upsilon\upsilon'h^2(1-\cos\phi)$$

$$\therefore (\upsilon-\upsilon') = \frac{\cancel{2}\upsilon\upsilon'h^{\cancel{2}}}{\cancel{2}\cancel{h}m_0c^2}(1-\cos\phi)$$

$$(\upsilon-\upsilon') = \frac{\upsilon\upsilon'h}{m_0c^2}(1-\cos\phi)$$

or $$\frac{\upsilon-\upsilon'}{\upsilon\upsilon'} = \frac{h}{m_0c^2}(1-\cos\phi)$$

$$\frac{1}{\upsilon'}-\frac{1}{\upsilon} = \frac{h}{m_0c^2}(1-\cos\phi)$$

multiplying by c on both sides of above eqn, we gets.

$$\frac{c}{\upsilon'}-\frac{c}{\upsilon} = \frac{h}{m_0c}(1-\cos\phi) \quad \text{---(xii)}$$

$$\therefore \lambda'-\lambda = \frac{h}{m_0c}(1-\cos\phi) \quad \left[\because \upsilon = \frac{c}{\lambda}\right]$$

$$\boxed{\Delta\lambda = \frac{h}{m_0c}(1-\cos\phi)} \quad \text{--- xiii}$$

From eqn (xiii) It is clear that the increase in wavelength $\Delta\lambda$ is independent of the wavelength of the incident radiation, and the nature of the scattering substance But depends upon the angle of scattering. The quantity h/m_0c is known as compton wavelength.

But $1-\cos\phi = 2\sin^2\frac{\phi}{2}$ $-\cos\phi = 2\sin^2\frac{\phi}{2}-1$

$\cos\phi = -2\sin^2\frac{\phi}{2}+1$

$\therefore$ put this value in eqn (xiii), we get

$$\Delta\lambda = \lambda'-\lambda = \frac{h}{m_0c}\left[\cancel{1}+2\sin^2\frac{\phi}{2}-\cancel{1}\right]$$

$$\Delta\lambda = \frac{2h}{m_0c}\sin^2\frac{\phi}{2}$$

$$\boxed{\therefore \lambda' = \lambda + \frac{2h}{m_0c}\sin^2\frac{\phi}{2}} \quad \text{——— (xiv)}$$

case - I When the scattering angle is zero, ϕ is zero and then.

$\sin\frac{\phi}{2} = \sin 0 = 0$

$$\therefore \boxed{\lambda' = \lambda} \quad \text{——— (xv)}$$

Thus there is no scattering.

case - II When $\phi = 90^\circ$, then $\sin^2\frac{\phi}{2} = 1/2$

$\therefore \lambda' = \lambda + \frac{2h}{m_0c} \times \frac{1}{2}$

$\lambda' = \lambda + \frac{h}{m_0c}$

$\therefore \lambda' - \lambda = h/m_0c \quad \therefore \boxed{\Delta\lambda = \frac{h}{m_0c}}$ —— (xvi a)

As h, m_0 & c are constants.

$\therefore \lambda' - \lambda = \frac{h}{m_0c} = \text{constant.}$

$$\boxed{\Delta\lambda = \text{constant} = \frac{h}{m_0c}} \quad \text{— (xvi b)}$$

case - III When $\phi = 180^\circ$, $\sin^2\frac{180}{2} = \sin 90^\circ = 1$

$\therefore \lambda' = \lambda + \frac{2h}{m_0c}$

$$\boxed{\therefore \Delta\lambda = \lambda' - \lambda = \frac{2h}{m_0c}} \quad \text{—— (xvii)}$$

8) # Here we conclude that radiation behaves like a particle.

* Wave and Particle duality of Radiation:-

A particle has mass and it can be located at some definite point. A particle can be specified by different quantities like mass, velocity, momentum and energy.

A wave is nothing but spread out disturbance. A wave is spread out over a relatively large region of space. In other word, a wave can not be confined a definate point. A wave can be specified by its frequency, wavelength, phase, amplitude and intensity. It has been observed that radiation some times behaves as a wave and at other time as a particle.

First, we see how radiation behaves as a wave. A radiation is nothing but visible light, infra red ray, ultra violet ray, X-ray etc. when radiation produces effects like interference, diffraction etc then it is said to behaves as a wave. e.g. interference phenomenon requires the presence of two waves at the same position at the same time, which can be achieved by wave only and not by particle. It is easily understood that two particles can not occupy the same position at the same time. Hence we conclude that radiation behaves like a wave.

Radiation behaves as a particle. consider the experiment of photoelectric effect, plank's quantum theory successfully explains this effect by showing that radiation interacts with the matter in the form of group of particles like a particle called photons or quanta. #

Thus radiation some times behaves like as a wave and at same other time as a particle. This is also called dual nature of radiation. Further is to be noted that radiation can not behave as a wave or particle.

De-Broglie's Hypothesis or De-Broglie's concept of Matter waves :-

In 1924, Louis De-Broglie's made a statement that there is a connection between wave and particle not only in case of radiation but also in case of matter. Matter is nothing but fundamental particle such as electron, proton, neutron etc. De-Broglie considered the fact that nature loves symmetry. If radiation like light can act as a wave some time and particle at other time. Then the material particles e.g. electron, proton etc should also acts as a wave at some other time.

According to De-Broglie's hypothesis, a moving particle always a wave associated with it, which is known as De-Broglie wave, matter wave or pilot wave, and the wavelength of matter wave is given by -

$$\lambda = \frac{h}{p} \quad \text{or}$$

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \quad [\because c = \nu\lambda]$$

$$\boxed{\therefore \lambda = \frac{h}{mv}} \quad \text{--- (i)}$$

$$\therefore E = h\nu \quad \text{--- (ii)}$$

According to Einstein theory

$$E = mc^2 \text{ or } m \cdot c \cdot c$$

$$E = pc \quad \text{--- (iii)}$$

$$\therefore h\nu = p \cdot c$$

$$\boxed{p = \frac{h\nu}{c}} \text{ or } \frac{h}{\lambda} \quad [\because \nu = \frac{c}{\lambda}]$$

De-Broglie's wavelength :-

On the basis of plank's theory of radiation, the energy of photon is given by.

$$E = h\nu = \frac{hc}{\lambda} \quad \text{--- (i)}$$

where c - velocity of light in vaccum
λ - wavelength of light.

According to Einstein's mass energy relⁿ.

$$E = mc^2 \quad \text{--- (ii)}$$

$\therefore$ From eqⁿ (i) & (ii), we get

$$mc^2 = \frac{hc}{\lambda} \quad \text{or} \quad \lambda = \frac{hc}{mc^2}$$

$$\therefore \boxed{\lambda = \frac{h}{m.c}} \quad \text{---- (iii)}$$

where $m.c = p =$ momentum associated with photon.

Consider a material particle having mass m and velocity v, we get momentum

$$p = m.v$$

$\therefore$ wavelength associated with this particle

$$\lambda = \frac{h}{m.v}$$

$$\boxed{\lambda = \frac{h}{p}} \quad \text{--- (iv)}$$

Let E = K.E. of the material particle

$$E = \frac{1}{2}mv^2$$

$$E = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$$

$$\therefore p = \sqrt{2mE}$$

$$\therefore \lambda = \frac{h}{\sqrt{2mE}} \quad \text{--- (v)}$$

This eqⁿ of De-Broglie wavelength

⑧ properties of matter waves :-

① Lighter is the particle, greater is the wavelength associated with it.

② smaller is the velocity of the particle, greater is the wavelength associated with it.

③ matter waves are generated by motion of particle irrespective of charge particle.

④ The velocity of matter waves depends on the velocity of matter particle.

⑤ The velocity of matter wave is greater than the velocity of light.

of electron. eqn (i) & eqn (viii) are same.

Wave velocity and Group velocity :-

Wave velocity or Phase velocity :-

"When a monochromatic wave i.e a wave of single frequency and wavelength travels through medium, its velocity of advancement in the medium is called as wave velocity.

Consider a wave whose displacement y is expressed as -

$$y = a \sin(\omega t - kx) \quad \text{---- (i)}$$

where a is the amplitude, ω is angular frequency $\omega = 2\pi n$ and $k = 2\pi/\lambda$ is the propagation constant of the wave.

The ratio of angular frequency ω to the propogation constant k is defined as wave velocity. This denoted by v_p. Hence.

$$v_p = \frac{\omega}{k} \quad \text{---- (ii)}$$

For the wave $(\omega t - kx)$ is the phase of wave motion. For the planes of constant phase (wavefronts), we have.

$$\omega t - kx = \text{constant} \quad \text{-- (iv)}$$

dif. w.r.t. time t:

$$\omega - k\frac{dx}{dt} = 0$$

$$k\frac{dx}{dt} = \omega \Rightarrow \frac{dx}{dt} = \frac{\omega}{k} = v_p \text{ - (}$$

Thus, the wave velocity is the velocity which the planes of constant phase advance through the medium. Due to this reason, the wave velocity is also called as phase velocity.

Group velocity :-

consider pulses rather than monochromatic wave. The pulse consists of a number of waves slightly differing in frequency from one another. The superposition of such waves is known as wave group (wave packet). When such a group travels in the medium, the phase velocities of different component are different. However, the observed velocity is the velocity with which the maximum amplitude of the group advances. This is called group velocity. Thus, the group velocity is the velocity with which the energy in the group is transmitted.

de-Broglie concept of wave velocity and group velocity :-

According to de-Broglie concept, matter has a dual nature i.e. some times it behaves like particle and sometimes like wave. How those contradictory statements are explained. First of all schroedinger explained the dual nature in terms of waves slightly differing in velocity and wavelength, with phases and amplitude such that they interfere constructively over a small region of space where the particle can be located and outside this space they interfere destructively so that the amplitude reduces to zero. Such a wave packet as shown in figure ① moves with its own velocity called the group velocity. The individual waves forming the wave packet have an average velocity which is called as phase velocity. It can be shown that the velocity of material particle v is the same as group velocity.

⊛ **Expression for group velocity :-**

Let us consider the case of two wave trains having same amplitude 'a' but slightly different angular frequencies (ω & ω') and phase velocities (v & v'). The waves can be represented

$$y_1 = a \sin(\omega t - kx) \quad \text{--- (i)}$$
$$y_2 = a \sin(\omega' t - k'x) \quad \text{--- (ii)}$$

where k & k' are propogation constants and defined as $(2\pi/\lambda)$. The resultant wave is -

$$y = y_1 + y_2$$
$$y = a \sin(\omega t - kx) + a \sin(\omega' t - k'x)$$
$$y = 2a \cos\left[\left(\frac{\omega - \omega'}{2}\right)t - \left(\frac{k-k'}{2}\right)x\right] \times$$
$$\sin\left[\left(\frac{\omega + \omega'}{2}\right)t - \left(\frac{k+k'}{2}\right)x\right]$$

$$\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)$$

$$= 2a\cos\left[\left(\frac{d\omega}{2}\right)t - \left(\frac{dk}{2}\right)x\right] \sin(\omega t + x)$$

$$\text{--- (iii)}$$

where $\frac{\omega + \omega'}{2} = \omega$ & $\frac{k+k'}{2} = k$.

eqn ① represents a wave of angular frequency ω and propagation constant k. The phase velocity v_p of the resultant wave is given by - $v_p = \frac{\omega}{k}$ = same as that of each composing wave. The amplified of the resultant wave modified. The amplitude is given by.

$$2a\cos\left[\left(\frac{d\omega}{2}\right)t - \left(\frac{dk}{2}\right)x\right]$$ or

$$2a\cos\frac{d\omega}{2}\left[t - \frac{dk}{d\omega}x\right] = 2a\cos\frac{d\omega}{2}\left[t - \frac{x}{v_g}\right]$$

~~where v_g is known as group velocity.~~

$$v_g = \frac{d\omega}{dk} = \frac{\omega - \omega'}{k - k'} \quad \text{--- (iv)}$$

where v_g - is the group velocity.

(*) Relation between group velocity and wave velocity :-

$$\omega = v_p k \quad \text{or} \quad d\omega = dv_p\, k + v_p\, dk.$$

$$\frac{d\omega}{dk} = v_p + \frac{dv_p}{dk}\cdot k \quad \text{--- (v)}$$

From eqns (iv) & (v), we get

$$v_g = v_p + k\frac{dv_p}{dk}$$

$$v_g = v_p + k\cdot\frac{dv_p}{d\lambda}\times\frac{d\lambda}{dk} \quad \text{--- (vi)}$$

since $k = \frac{2\pi}{\lambda}$ & $\frac{d\lambda}{dk} = -\frac{2\pi}{k^2}$

put this value in above eqn, we get

$$v_g = v_p - \cancel{k}\cdot\frac{dv_p}{d\lambda}\cdot\frac{2\pi}{k^{\cancel{2}}}$$

$$v_g = v_p - \frac{dv_p}{d\lambda}\cdot\frac{2\pi}{k}$$

$$v_g = v_p - \lambda\frac{dv_p}{d\lambda} \quad \text{--- (vii)} \quad \left[\because k = \frac{2\pi}{\lambda}\right]$$

Eqn (7) represents relationship between group velocity v_g and phase velocity v_p

$v_g = v_p$

Group velocity and Particle velocity :-

Eq. (viii) can be written as —

$$v_g = \lambda^2 \left[\frac{v_p}{\lambda^2} - \frac{1}{\lambda} \left(\frac{dv_p}{d\lambda} \right) \right]$$

$$= -\lambda^2 \frac{d}{d\lambda} \left(\frac{v_p}{\lambda} \right)$$

$$= -\lambda^2 \frac{d\upsilon}{d\lambda}$$

$$\frac{1}{v_g} = -\frac{1}{\lambda^2} \cdot \frac{d\lambda}{d\upsilon} = \frac{d}{d\upsilon} \left(\frac{1}{\lambda} \right) \quad \text{---- (viii)}$$

If E & V represent the total & Potential energy of the particle respectively, then K.E. of particle is -

$\frac{1}{2} m u^2 = E - V$, where u - is the particle velocity.

$$u = \left[\frac{2(E-V)}{m} \right]^{1/2}$$

According to de Broglie formula $\lambda = h/mu$

$$\frac{1}{\lambda} = \frac{mu}{h} = \left[\frac{2[E-V]}{m} \right]^{1/2} \quad \text{(ix)}$$

substituting the value of $(1/\lambda)$ from eqn (ix) in eqn (viii), we get.

$$\frac{1}{u} = \frac{d}{d\upsilon} \left[\frac{m}{h} \left\{ \frac{2(E-V)}{m} \right\}^{1/2} \right]$$

$$= \frac{d}{d\upsilon} \left[\frac{m}{h} \left\{ \frac{2(h\upsilon - V)}{m} \right\}^{1/2} \right]$$

$$= \frac{1}{h} \frac{d}{d\upsilon} \left[\{ 2m(h\upsilon - V) \}^{1/2} \right]$$

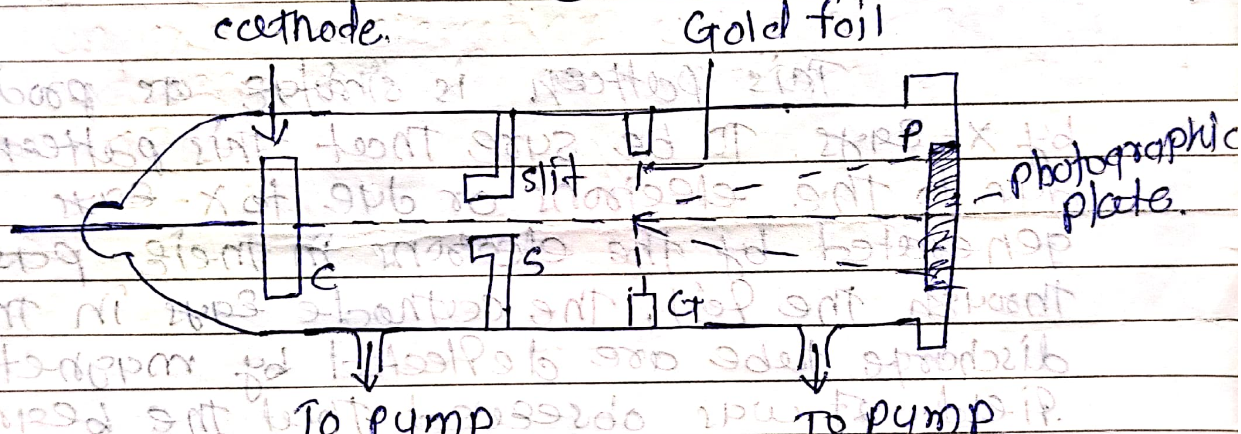
$$= \frac{1}{h} \frac{1}{2} (2m(h\upsilon - V))^{-1/2} \cdot 2mh$$

$$= \frac{m}{[2m(E-V)]^{1/2}} = \left[\frac{m}{2(E-V)} \right]^{1/2} \quad \text{(x)}$$

$$\frac{1}{v_g} = \frac{1}{v_p} \quad \text{-- (x')}$$

G.P. Thomson Experiments :-

In 1928. G.P. Thomson performed experiments with electrons accelerated from 10,000 to 50,000 volts. In this experiments, Thomson observed diffraction patterns exactly analogus to X-rays patterns, moreover he was able to determine the wavelengths associated with electrons. since diffraction patterns are only possible provided the waves are associated with material particle, hence his experiments supported the concept of matter waves. His experimental arrangement is shown in figure ①



The high energy electron beam is produced by the cathode C. The beam is excited with potential upto 50,000 volts. A fine pencil of this beam is obtained by passing it through the slit S. The accelerating fine beam of electrons now falls on a thin film G (of the order of 10^{-6} cm) of gold or aluminium. The photograph of the beam from the foil is recordered on a photographic plate P. Whole of the apparatus is exhausted to a high vaccum so that the electrons may not lose their energy in collisions with the molecules of the gas.

After developing the plate, a symmetrical pattern consisting of concentric rings about a central spot is obtained as shown in fig②.

Fig②: Electron diffraction pattern with a Gold foil.

This pattern is similar as produced by X-rays. To be sure that this pattern is due to the electrons or due to X-rays generated by the electrons in their passage through the foil, the cathode rays in the discharge tube are deflected by magnetic field. It was observed that the beam shifts correspondingly showing there by that the pattern is produced by electrons and not by X-rays. The observed rings can only be interpreted by considering that the diffraction pattern of the incoming beam is due to the diffraction of electrons by the foil. As the diffraction pattern can only be produced by waves and not by the particle so Thomson concluded that electrons behaves like waves.

The following conclusions are -

1. Thomson experiment clearly demonstrated the existence of matter waves because the diffraction pattern can be produced by the waves.
2. Thomson calculated the wavelength associated with the electron with the help of diffraction rings.
3. The calculated wavelengths obtained by diffraction pattern agree well with the wavelength obtained by de-broglie relation.
4. The associated wavelength depends only on the accelerating voltage and is independent of the material of the target.

Wave Function

State of a system can be described by a function. This function is called wave function or state function or state vector. The wave function is denoted by 'ψ'. The wave function is a function of generalized co-ordinates $q_1, q_2 \dots q_n$ and time 't'. The function ψ as well as its derivative in the space must be continuous finite and singlevalued over the region of the variables. The function ψ is one which may represents a state, where the total number of variables is equal to the number of degree of freedom of the system. At a given time, the state becomes a function of the space ~~co-ordinate~~ co-ordinates of the system only. All possible information about the system can be derived with the help of the wave function for the system. As such, the function ψ does not represent any of the observable quantities or states. $\psi^*\psi$ presents the probability density. Here ψ^* is the complex conjugate of ψ. obviously, the probability density is a real quantity.

The quantum theory will be useful in solving the different problems in measuring an observable is two restrictions are made selecting the function of state.

1) The function should possess an integrable square. It should not give a value which is Infinity.

$$\int \psi^* \psi \, dv < \infty$$

Here ψ is the function and ψ^* is its complex conjugate. dv is the ~~time~~ volume element of the configuration space.

2) The function ψ should be single-valued.
If ψ is a function of space coordinates, it should have a single value at a given point in space. For example let ψ be a function of angle θ. If the function ψ should be single-valued. It is necessary that the same value should be obtained if we add or substract a multiple of 2π radians to the angle θ. Thus

$$\psi(\theta) = \psi(\theta \pm 2\pi)$$

where $n = 1, 4, 3 \ldots$ an integer

Uncertainty Principle OR Heisenberg's Uncertainty Principle :-

1) Introduction :-

In 1927, Heisenberg's proposed a very interesting principle, which is a direct consequence of the dual nature of matter, known as uncertainty principle.

According to the classical mechanics, the position and momentum of moving electron can be determined great accuracy. When an electron is considered as a wave, it is however not possible to know the exact location of the electron on the wave as it is extending throughout a region of space. Thus-

"If an electron is exhibiting dual nature, is it possible to know the exact position of the electron in space at same given instant."

2) statements :-

According to Heisenberg's states that " It is impossible to determine simultaneously the position and momentum of the electron with any desired accuracy". The above defination is called as Heisenberg's uncertainty principle.

3) a) Mathematical Relation :- Heisenberg's principle can be stated mathematically as.

$$\Delta p . \Delta x \simeq \hbar \quad ----(1)$$

Where Δp - is the uncertainty in the direction of position. the momentum and Δx - the uncertainty in the direction of position.

The eqn ① is known as Heisenberg's eqn. which can be stated that. In words as, "the product of uncertainity in the simultaneous determination of the position and momentum of the a particle is equal to or greater than the $\hbar$. Here

$$\hbar = \frac{h}{2\pi}$$

where h - is the plank's constant.

$$\therefore \Delta p \cdot \Delta x \simeq \frac{h}{2\pi} \quad \text{--- (ii)}$$

Relation ① is fundamental because it sets a limit to the accurate and simultaneous measurement of position and momentum.

- The momentum in eqn ① should actually be the component of the momentum in the x-direction, and it is can be written as

$$\Delta p_x \times \Delta x \simeq \hbar \quad \text{--- (iii)}$$

b) Time - Energy uncertainity principle :-

Although the time-energy uncertainity relation can be obtained by considering a wave packet over limited time interval but the proof is beyond the level of this book. However, consider, the time energy uncertainity with the help of position momentum uncertainity. consider the case of a free particle with rest mass m_0 moving along x-direction with velocity v_x. The K.E is given by

$$E = \frac{1}{2} m_0 v_x^2 \quad \text{--- (i)}$$

$$E = \frac{p_x^2}{2m_0} \quad \text{--- (ii)}$$

If Δp_x and ΔE be the uncertainty in momentum and energy respectively then dim. eqn (ii), we have:

$$\Delta E = \frac{2p_x \cdot \Delta p_x}{2m_0}$$

$$p_x \cdot \Delta p_x = m_0 \Delta E$$

$$\Delta p_x = \frac{m_0}{p_x} \cdot \Delta E \qquad [\because p = m \cdot v,\ v = \tfrac{p}{m},\ \tfrac{1}{v} = \tfrac{m}{p}]$$

$$\Delta p_x = \frac{1}{v_x} \cdot \Delta E \quad \text{---} \text{ (iii)}$$

Further let the uncertainty in the time interval for measurement at point x is Δt, then uncertainty Δx in position is -

$$\Delta x = v_x \cdot \Delta t \quad \text{--} \text{ (iv)} \qquad v_x = \frac{\Delta x}{\Delta t}$$

From eqn (iii) & (iv), we get

$$\Delta x \cdot \Delta p_x = \frac{v_x \cdot \Delta t \cdot \Delta E}{v_x}$$

$$\therefore \Delta x \cdot \Delta p_x = \Delta t \cdot \Delta E$$

we know that $\Delta x \cdot \Delta p_x \simeq \hbar$ or $h/2\pi$.

$$\therefore \Delta t \cdot \Delta E \geq \frac{h}{2\pi} \quad \text{or} \quad \Delta t \cdot \Delta E \simeq \hbar \quad \text{-(v)}$$

According to ~~Bohr's~~ Bohr's orbits concept, an electron in an atom revolves in one of the quantized orbits. Each orbit has a sharply defined energy with no uncertainty whatever. This means that $\Delta E = 0$, Now, accordingly to uncertainty relation, Δt must be infinite. This shows that the energy states of the atom must have infinite life time. we know that the excited states of the atom from which radiation occours do not last indefinetly and life time is of the order of 10^{-8} sec. Thus the concept of Bohr orbits violates the uncertainity principle.

⊗ Exact statement of uncertainity principle

The product of the uncertainties in determining the position and momentum of the particle can never be smaller than the number of the order $\hbar$

Thus eqn ① takes the form,

$$\Delta p \times \Delta x \geq \frac{\hbar}{2}$$

-lly $\Delta E \times \Delta t \geq \frac{\hbar}{2}$

$$\Delta J \times \Delta \phi \geq \frac{\hbar}{2}$$

Experimental illustrations of consequence of uncertainity principle:-

① **Determination of the position of a particle by gama ray microscope:-**

Consider the case of the measuremen of the position of a particle say electron in the field of gama ray microscope. The resolving power i.e. the smallest distance between the two points that can be just resolved by the microscope is given by.

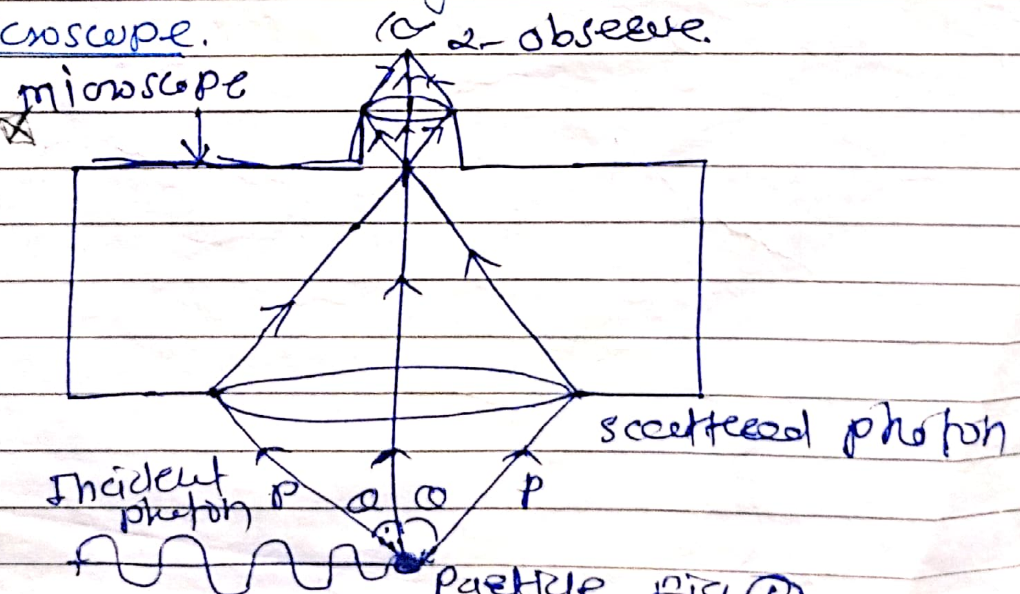
$$\Delta x \simeq \frac{\lambda}{2\sin\theta} \quad \text{---- (i)}$$

where λ - is the wavelength of light used, θ is the semivertical angle of the cone of light and Δx, the uncertainity in determining the position of the particle. as shown in figure.

In order to observe the electron it is necessary that atleast one photon must strike the electron and scatter inside the microscope. When a photon of initial momentum i.e

$$\lambda = h/p \quad \therefore p = \frac{h}{\lambda} \quad \text{--- (ii)}$$

After scattering enter the field of view microscope.



Particle fig ①

it may be anywhere within angle 2θ. Thus, if x component of momentum, i.e. P_x may lie between $p\sin\theta$ and $-p\sin\theta$. As the momentum is conserved in the collision, the uncertainity in the x - component of momentum is given by -

$$\Delta P_x = p\sin\theta - (-p\sin\theta)$$

$$= 2p\sin\theta$$

$$= 2\frac{h}{d}\sin\theta \quad \text{---} (iii) \quad [\because \text{eqn } ii]$$

from eqn (i) & (iii), we have -

$$\Delta x . \Delta P_x \approx \frac{d}{2\sin\theta} \times \frac{2h\sin\theta}{d}$$

$$\boxed{\Delta x . \Delta P_x \approx h} \quad \text{——} (iv)$$

This shows that the product of uncertainities in position and momentum is of the order of planck's constant.

$$\therefore \Delta x \cdot \Delta P_x \geq \frac{\hbar}{2}$$

2) Diffraction by a single slit :-

Suppose a narrow beam of electrons passes through a single narrow slit and produces a diffraction pattern on the screen as shown in figure.①

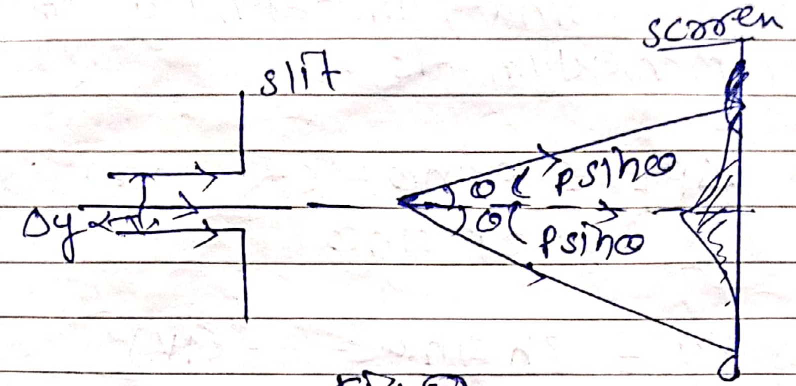


Fig ②

The first minimum of the pattern is obtained by putting n=1, the equation describing the behaviour of diffraction pattern due to a single slit. i.e -

$$n\lambda = 2d \sin\theta \quad --- (i)$$

& Hence

$$2\Delta y \cdot \sin\theta = \lambda \quad --- (ii)$$

where Δy is the width of the slit and θ is the angle of deviation corresponding to first minimum.

In producing the diffraction pattern on the screen all the electrons have passed through the slit but we cannot say definitely at what place of the slit. Hence the uncertainty in determining the position of the electron is equal to the width Δy of the slit. From eqn (ii), we have.

$$\Delta y = \frac{\lambda}{2 \sin\theta} \quad --- (iii)$$

Initially the electrons are moving along the x-axis and hence they have no component of momentum along y-axis. After diffraction at the slit, they are deviated from the initial path to form the pattern and have a component $p \sin\theta$. As y component of momentum may lie anywhere between $p\sin\theta$ and $-p\sin\theta$, uncertainity in y-component of momentum is –

$$\Delta p_y = p\sin\theta - (-p\sin\theta)$$
$$= 2p\sin\theta$$

$$\Delta p_y = \frac{2h\sin\theta}{d} \quad \text{---} (iv)$$

From eqn (iii) & (iv),

$$\Delta y \cdot \Delta p_y = \frac{d}{2\sin\theta} \times \frac{2h\sin\theta}{d}$$

$$\Delta y \cdot \Delta p_y = h$$

or

$$\Delta y \cdot \Delta p_y \geq \hbar$$

$(\hbar = h/2\pi$ $\quad h = \hbar \cdot 2\pi$ $\quad \hbar = \frac{h}{2\pi}$ $\quad h =$

This relation shows that the product of uncertainities in position and momentum is of the order of planck's constant.

The end

Applications of uncertainty principle :-

According to uncertainty principle the position and momentum of the particle cannot be measured with accuracy, while according to classical ideas it is possible to predict exactly the position and momentum of the particle at any instant when its initial position and momentum are known. According to new mechanics the term particle is something made up of a very deep & essential unison of both the corpuscle and the wave resulting in certain unavoidable uncertainty.

i) The non-existence of the electrons in the nucleus :-

The radius of the nucleus of any atom is of the order of 10^{-14} m, so that if an electron is confined within nucleus, the uncertainty in its position must not be greater than 10^{-14} m.

According to uncertainty principle :-

$$\Delta x \cdot \Delta p \geq \hbar \quad \text{--- (i)}$$

where Δx is the uncertainty in the position and Δp is the uncertainty in the momentum

$$\therefore \hbar = \frac{h}{2\pi} = \frac{6.6256 \times 10^{-34}}{2 \times 3.14}$$

$$\hbar = 1.055 \times 10^{-34} \text{ J.s} \quad \text{--- (ii)}$$

$$\therefore \text{Radius of nucleus} = 10^{-14} \text{ m}$$

$$\therefore (\Delta x)_{max} = 2 \times 10^{-14} \text{ m} \quad \text{--- (iii)}$$

substituting eqn (ii) & (iii), we get

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.055 \times 10^{-34}}{2 \times 10^{-14}}$$

$$\Delta p = 5.275 \times 10^{-21} \text{ kg.m/s} \quad \text{--- (iv)}$$

If this is the uncertainty in momentum of the electron, the momentum of the electron must be at least comparable with its magnitude.

i.e K.E. of the electron of mass m is –

$$KE = \frac{p^2}{2m}$$

$$= \frac{(5.275 \times 10^{-21})^2}{2 \times 9 \times 10^{-31} kg}$$

$$= \frac{(5.275)^2 \times 10^{-42} \times 10^{+31} \times 10^{19}}{2 \times 9 \times 1.6}$$

$$= 9.7 \times 10^{7} eV$$

$$= 9.7 MeV$$

This means that if the electron exist inside the nucleus, their K.E must be of the order of 9.7×10^{7} eV. But experimental observations show that no electron in the atom possesses energy greater than 4×10^{6} eV. Clearly the inference is that the electrons do not exist in the nucleus.

2) Radius of Bohr's 1st orbit :–

If Δx and Δp are the uncertainties in determining the position and momentum of electron in first orbit then,

$$\Delta x \cdot \Delta p \geq \hbar$$

$$\text{or } \Delta p = \frac{\hbar}{\Delta x}$$

The uncertainty in K.E. is –

$$E = \frac{p^2}{2m}$$

$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{1}{2m}\left[\frac{\hbar}{\Delta x}\right]^2$$

$$\therefore \Delta E = \frac{\hbar^2}{2m(\Delta x)^2} \quad \text{--- (1)}$$

The P.E. of electron.

$$V = \frac{1}{2\pi\varepsilon_0} \frac{Ze(-e)}{x}$$

$\therefore$ uncertainty in P.E. of same electron.

$$\Delta V = \frac{1}{4\pi\varepsilon_0} \frac{Ze(-e)}{\Delta x}$$

$$= -\frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{\Delta x}$$

$\therefore$ Total uncertainty Energy is -

$$\Delta E = \Delta T + \Delta V$$

$$= \frac{\hbar^2}{2m(\Delta x)^2} - \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{\Delta x}$$

$\therefore$ minimum energy $E_{min} = \Delta E$

$$= \frac{\hbar^2}{2m(\Delta x)^2} - \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{\Delta x}$$

The uncertainty in energy will be minimum if $\frac{d(E_{min})}{d(\Delta x)} = 0$

$$\therefore \frac{-\hbar^2}{m(\Delta x)^3} + \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{(\Delta x)^2} = 0$$

$$\therefore -\frac{\hbar^2}{m} + \Delta x . \frac{1}{4\pi\varepsilon_0} Ze^2 = 0$$

$$\therefore -\frac{\hbar^2}{m} + \frac{1}{4\pi\epsilon_0} ze^2 \Delta x = 0$$

$$\Delta x = + \frac{\hbar^2}{m} \cdot \frac{4\pi\epsilon_0}{ze^2}$$

$$\therefore \Delta x = 4\pi\epsilon_0 \frac{\hbar^2}{mze^2}$$

Radius of Ist orbit is $r = \Delta x$ i.e.

$$r = 4\pi\epsilon_0 \frac{h^2}{mze^2 \cdot 4\pi^2}$$

$$\boxed{r = \frac{\epsilon_0 h^2}{\pi m z e^2}}$$

This is just the radius of Bohr Ist orbit

3) Minimum energy of a harmonic oscillator