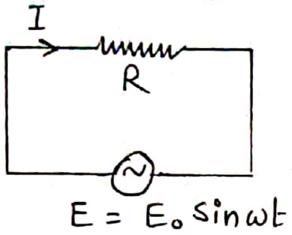
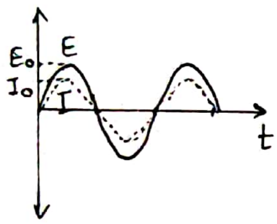
# Unit - IV Alternating Current Circuits



## ★ A.C. Through Resistor (R), Inductor (L) and Capacitor (C)

### a) A.C. through resistor (R):




Suppose that an alternating potential difference $E = E_0 \sin\omega t$ is applied to a resistor of resistance (R). Then the instantaneous value of the current I passing through this resistor is given by,

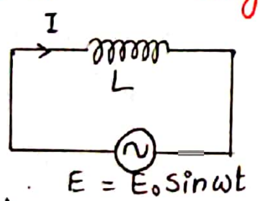
$$E = IR$$

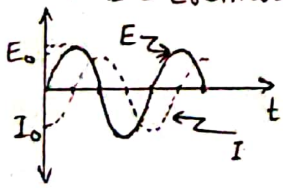
$$\Rightarrow I = \frac{E}{R} \Rightarrow I = \frac{E_0}{R}\sin\omega t$$

$$\Rightarrow I = I_0 \sin\omega t \ ; \ I_0 = \frac{E_0}{R}$$

The quantity $I_0 = \frac{E_0}{R}$, the maximum current, is same as obtained with direct current. Current I and voltage E both vary with $\sin\omega t$, hence are in phase.

### b) A.C. through pure inductor (L):




Suppose that an alternating potential difference $E = E_0 \sin\omega t$ is applied across pure inductor having self inductance L. If I is the instantaneous current at any instant t, then

$$E = L\frac{dI}{dt}$$

$$\Rightarrow E_0 \sin\omega t = L\frac{dI}{dt} \Rightarrow dI = \frac{E_0}{L}\sin\omega t\, dt$$

$$\Rightarrow I = \frac{E_0}{L}\int \sin\omega t\, dt$$

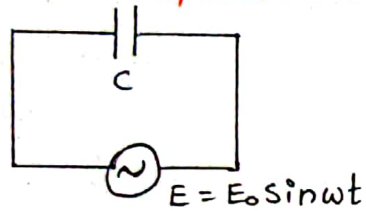
$$\Rightarrow I = \left(\frac{E_0}{L}\right)\frac{(-\cos\omega t)}{\omega}$$

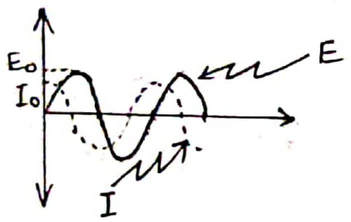
$$\Rightarrow I = \frac{-E_0}{L\omega}\cos\omega t \Rightarrow I = I_0 \sin(\omega t - \pi/2)$$

The quantity $I_0 = \frac{E_0}{\omega L}$ is equal to the maximum current. This relation shows that the effective AC resistance, i.e. the "inductive reactance" of inductor, $X_L = \omega L$ and the maximum current $I_0 = (E_0/X_L)$.

The unit of inductive reactance $X_L$ is also ohm. The reactance is greater for greater inductance and for the higher frequency $f$ $\left( f = \frac{\omega}{2\pi} \right)$. The current $I$, in the inductor lags voltage $E$, by $\pi/2$.

c) Pure Capacitor (C): If a capacitor of capacitance 'C' is connected across the alternating source, the instantaneous current $I$ passing through it is given by




$$I = \frac{dq}{dt} \text{ and } q = CE$$

$$\Rightarrow I = \frac{d(CE)}{dt} = C\frac{d(E_0 \sin \omega t)}{dt}$$

$$\Rightarrow I = CE_0 \cos \omega t \cdot \omega$$

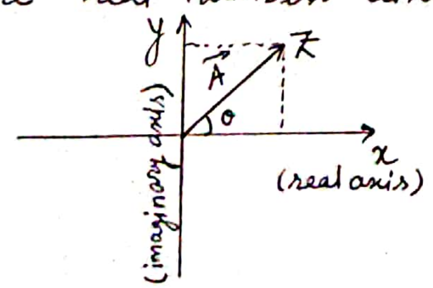
$$\Rightarrow I = \frac{E_0}{(1/\omega C)} \sin(\omega t + \pi/2)$$

$$\Rightarrow I = I_0 \sin(\omega t + \pi/2)$$

This relation shows that the quantity $\frac{1}{\omega C}$ is the effective AC resistance or the capacitive reactance $X_C$, it has unit as ohm. It is also clear that the current $I$ leads the voltage $E$ by $\pi/2$.

* Complex numbers, complex operator $-j$ & complex representation of voltage (E) and current (I).

A number $Z$, expressed in the form $Z = x + jy$, where $x$ & $y$ are real numbers and $j = \sqrt{-1}$ is known as complex number.




complex conjugate $\bar{Z}$ of a complex number is $\bar{Z} = x - jy$,

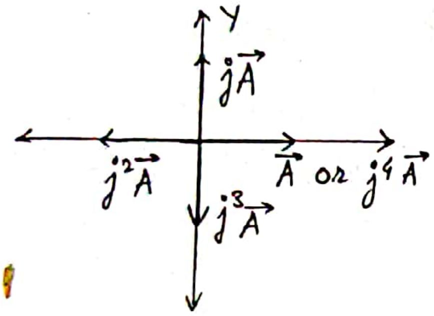
in polar form $Z = r e^{j\theta}$; where

$$r = \sqrt{x^2 + y^2} \ \& \ \theta = \tan^{-1}(y/x)$$

while $x = r\cos\theta$ & $y = r \sin\theta$.

A complex number $Z$, can be represented by a vector as shown in the figure.

In the context of electrical engineering, the vector - $\vec{A}$, which is representing the complex numbe is known as "phasor". When a given phasor $\vec{A} = a + jb$, the direction of which is along x-axis (or real axis), is multiplied by the "operator j", a new phasor $j\vec{A}$ is obtained which will be rotated by $\pi/2$ radians in counterclockwise sense from $\vec{A}$, i.e. along




Y-axis. If the operator j is applied now to this phasor $j\vec{A}$, a new phasor is obtained $j^2\vec{A}$ which will be along negative x-axis, having same magnitude as of $\vec{A}$. Thus $j^2\vec{A} = -\vec{A}$.

The action or operation of operator -j is, thus, rotating a given phasor $\vec{A}$ (vector $\vec{A}$) by $\pi/2$ radians in counter-clockwise sense irrespective of the orientation of $\vec{A}$.

As, $A = a + jb$, can also be represented as $A = |A|e^{j\phi}$, the imaginary part of A : imaginary $(A) = b$ or imaginary of $(|A|e^{j\phi}) = b$. Therefore, the alternating voltage E and current I, $E = E_0 \sin\omega t$ & $I = I_0 \sin(\omega t + \phi)$ are represented by $E = E_0 e^{j\omega t}$ ; $I = I_0 e^{j(\omega t + \phi)}$ and we can take the imaginary part to get the original alternating voltage and current.

## ★ Impedance and Reactance :

Alternating current in a circuit may be controlled by resistance R, inductance L, and capacitance C. In an AC circuit the ratio of potential difference across the circuit and the current flowing therein is termed as impedance and denoted by Z.

Reactance X : The hindrance offered by inductance and capacitance to the flow of alternating current ~~may be~~ in an AC circuit is called the reactance.

The reactance due to inductance is called inductive reactance $X_L$, while the reactance due to capacitance is called capacitive reactance $X_C$.

★ Complex Impedance : The ratio of the complex representation of a sinusoidal voltage to the complex representation of current through the circuit is known as complex impedance, and is given by $Z = \frac{E}{I}$

$$\Rightarrow \quad Z = R + jX$$

★ Using complex notation for AC through R, L & C :

a) Alternating voltage across a resistance R

Suppose an alternating voltage $E = E_0 e^{j\omega t}$ is applied across a pure resistance R. Then the current through the resistance at an instant t is,

$$I = \frac{E}{R} = \frac{E_0 e^{j\omega t}}{R} = I_0 e^{j\omega t}$$

where, $I_0 = \frac{E_0}{R}$

hence the actual value of the current is

$$I = \text{Imaginary}\left(I_0 e^{j\omega t}\right)$$

$$I = I_0 \sin \omega t.$$

Hence, current and voltage are in phase for a pure resistance.

b) Alternating voltage across an inductance L :

Let an alternating voltage $E = E_0 e^{j\omega t}$ be applied across an inductance L. If I be the instantaneous current and $dI/dt$ be the rate of change of current through the inductance, then the voltage across inductance is $L(dI/dt)$.

$$\Rightarrow L\frac{dI}{dt} = E \Rightarrow L\frac{dI}{dt} = E_0 e^{j\omega t}$$

$$\Rightarrow dI = \frac{E_0}{L} e^{j\omega t}$$ , on integration so, we get

$$I = \frac{E_0}{L} \cdot \frac{1}{j\omega} e^{j\omega t}$$

$$I = \frac{-j}{\omega L} \cdot E_0 e^{j\omega t}$$

but as $-j = e^{-j\pi/2}$ $\left(-j = \cos\pi/2 - j\sin\pi/2\right)$

$$\Rightarrow I = \frac{E_0}{\omega L} e^{j\omega t} \cdot e^{-j\pi/2}$$

$$\Rightarrow I = I_0 e^{j(\omega t - \pi/2)}$$ ; where $I_0 = \frac{E_0}{\omega L}$

the actual current is $I = I_0 \sin(\omega t - \pi/2)$

We can see that the current $I$ lags behind $E$ by $\pi/2$.

As, can be seen that the inductive reactance $X_L$ is

$$X_L = j\omega L$$

c) Alternating voltage across a capacitance 'C':

Let an alternating voltage, $E = E_0 e^{j\omega t}$ be applied to a pure capacitor $C$. If $q$ be the charge at any instant across the plates of capacitor, then $E = \frac{q}{C}$

$$\Rightarrow E_0 e^{j\omega t} = \frac{q}{C} \Rightarrow C E_0 e^{j\omega t} = q$$

$$\Rightarrow I = \frac{dq}{dt} = \frac{d}{dt}\left(C E_0 e^{j\omega t}\right) \Rightarrow I = j\omega C E_0 e^{j\omega t}$$

this can be written as,

$$I = \frac{E_0}{(1/\omega C)} \cdot j e^{j\omega t}$$ but $j = e^{j\pi/2}$

$$\Rightarrow I = I_0 e^{j(\omega t + \pi/2)} \quad ; \quad I_0 = \frac{E_0}{(1/\omega C)}$$

actual current is $I = I_0 \sin(\omega t + \pi/2)$

From this equation we see that the current through the capacitor is also alternating and sinusoidal in nature and its frequency is same as that of the applied voltage, but it leads the applied voltage by $\pi/2$, and the capacitance has a reactance

$$X_c = \frac{E}{I} \Rightarrow X_c = \frac{1}{j\omega C} \Rightarrow X_c = \frac{-j}{\omega C}$$