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Unit-III

Nuclear fissions and nuclear reactions

Nuclear fission, the fission products, energy released in fissions, nuclear transmutation reaction, Conservation laws, nuclear reaction Kinematics.

Nuclear fission:-

The discovery of nuclear fission stems basically from the experimental work done by Fermi (1934), who attempted to produce some transuronic elements (the elements having atomic numbers greater than 92, the atomic number for uranium). It was observed by Fermi that when a nucleus of ${}_{92}U^{235}$ was bombarded by slow moving neutrons, the end products did not contain an element of atomic number 93 i.e. one unit higher than that of the Uranium. Instead one of the end products showed the properties of barium. A series of careful experiments in 1939 by Otto Hahn and F. Strassman showed that when uranium is bombarded by neutrons, it splits into two radioactive nuclei which were identified as isotopes of barium (Z=56) and krypton (Z=36).

Meitner and Frisch interpretted the results by suggesting that the uranium nuclei absorb the neutrons and hence become excited. The excited nuclei then split into two fragments of nearly equal mass.

The phenomena, accompanied by the emission of three neutrons and an enormous amount of energy, was named by Meitner and Frisch as nuclear fission.

"Thus, nuclear fission is the process in which a heavy nucleus, after capturing a neutron, breaks up into two lighter nuclei of comparable masses and an enormous amount of energy is released in the process."

The nuclear reaction involved in nuclear fission can be expressed as,

$$ {}_{92}U^{235} + {}_{0}n^{1} \longrightarrow \underset{\text{Unstable}}{\left({}_{92}U^{236}\right)} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_{0}n^{1} + Q \quad \text{--- (1)} $$

where Q is the energy released in the reaction.

This reaction is one of the many possible modes of deformation. In fact, we can have a number of combinations of different isotopes of different elements in the above fission process as the end products.

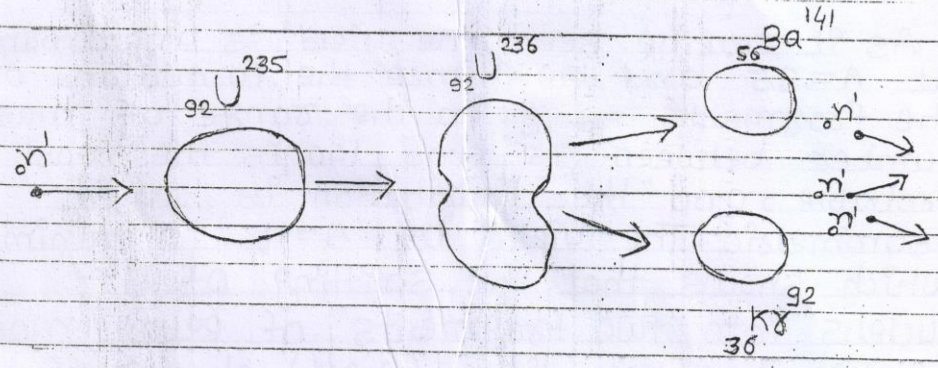
For ex. we have some other decay modes as represented by the following equations.

$$ {}_{92}U^{235} + {}_{0}n^{1} \longrightarrow \left({}_{92}U^{236}\right) \longrightarrow {}_{40}Zr^{98} + {}_{52}Te^{135} + 3\,{}_{0}n^{1} + Q \quad \text{--- (2)} $$

Also,

$$ {}_{92}U^{235} + {}_{0}n^{1} \longrightarrow \left({}_{92}U^{236}\right) \longrightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_{0}n^{1} + Q \quad \text{--- (3)} $$

The process of nuclear fission described by equation (1) can be illustrated as shown in fig.

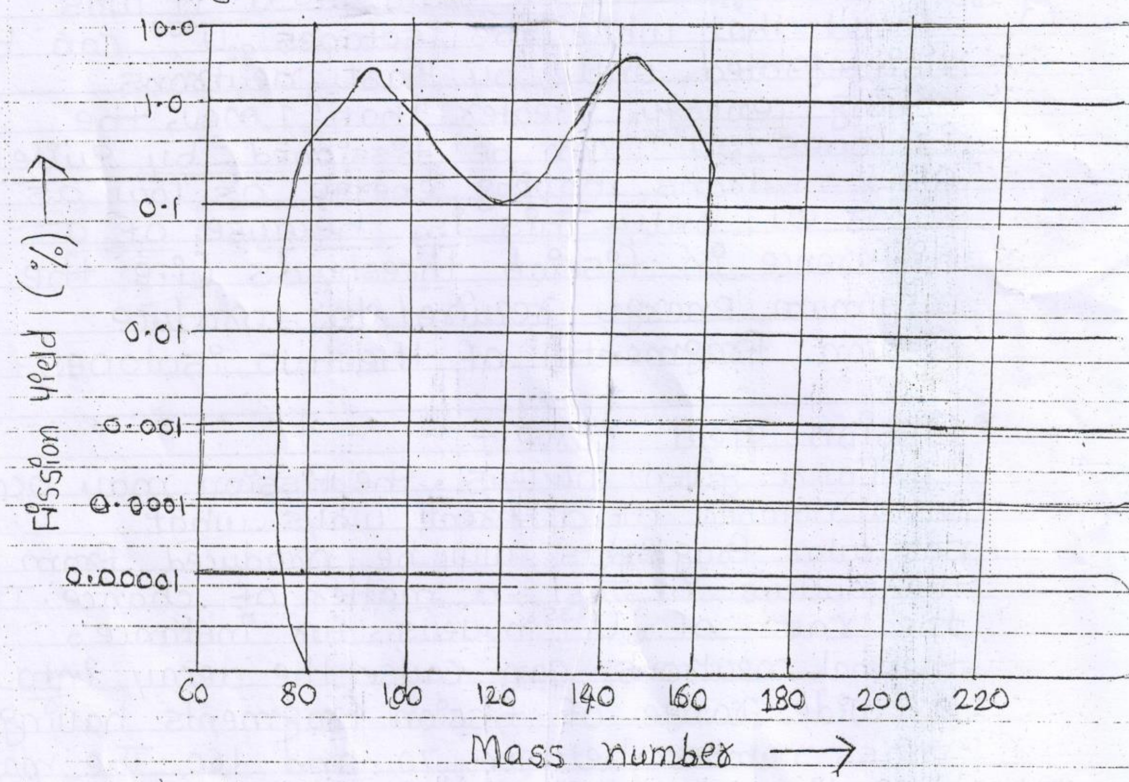


Q. Natural Uranium is mixture of two isotopes $_{92}U^{238}$ and $_{92}U^{235}$ in the ratio 145 : 1. It was found that while the isotopes $_{92}U^{238}$ can be disintegrated only by fast neutrons having energy greater than 1 MeV, the isotopes $_{92}U^{235}$ can be fissioned by quite slow neutrons having energy as low as 0.03 eV only. This is because of a difference in fission thresholds (i.e. the minimum energy required to produce fission fragments) of the two isotopes.

* Fission yield curve :-

In a given nucleus, the fission may occur in a number of different ways. What particular fragments will be produced from the nucleus in just a matter of chance. In the case of $_{92}U^{235}$ nucleus, for instance, thermal neutrons can cause the decay into a wide range of fission fragments having mass number between 70 and 160. The mass distribution of the fission fragments is shown in fig 9.10 by a fission yield curve in which the percentage yield of the different fission fragments are plotted against their mass number.

As it can be seen the yield is maximum at A=95 and 140 Hence the formation of the fragments lying in the range of mass number between 95 and 140 is the most probable, and the distribution is called asymmetric The yield at A=118 is minimum. Which shows that the splitting of U^{235} nucleus into two fragments of equal mass (A=118) has very little (0.01%) change of occuring This corresponds to a symmetric splitting of the U^{235} nucleus.



The fission products do not have any unique values of mass number and atomic number. The products fall into two groups

① a lighter group with mass number from 85 to 104

The excitation energy imparted to the nucleus, mostly by the binding energy of the neutron, may set up a number of oscillations in the spherical nucleus (a). Due to oscillations, the shape may change from spherical to ellipsoid (b). With large excitation energy the oscillations may be so violent as to change the shape of the nucleus to a dumb bell (c). Since both the balls of the dumb bell have a positive charge, the electrostatic repulsion may cause a fission (d).

The activation energy, equal to the sum of the kinetic energy and the binding energy of the captured neutron, required for the fission in a $_{92}U^{235}$ nucleus is 6.4 ~~energy~~ Mev, while for the $_{92}U^{238}$ nucleus it is equal to 6.6 Mev. For $_{92}U^{235}$, the binding energy released by captured neutron in forming a $_{92}U^{236}$ nucleus is 6.6 Mev. This is greater than the required activation energy (6.4 Mev) for fission. Hence fission will follow even if the kinetic energy of incident neutron is nearly zero. That is why a $_{92}U^{235}$ nucleus can be fissioned even by the thermal neutrons (energy equal to 0.03 eV)

On the contrary, the binding energy released by the captured neutron in $_{92}U^{239}$ nucleus is only 5.5 Mev, while the activation energy required for fission is 6.6 Mev. Hence a balance of 1.1 Mev energy must come from the kinetic energy of the neutron. This explains the splitting of a $_{92}U^{238}$ nucleus by neutrons having energy about 1.0 Mev.

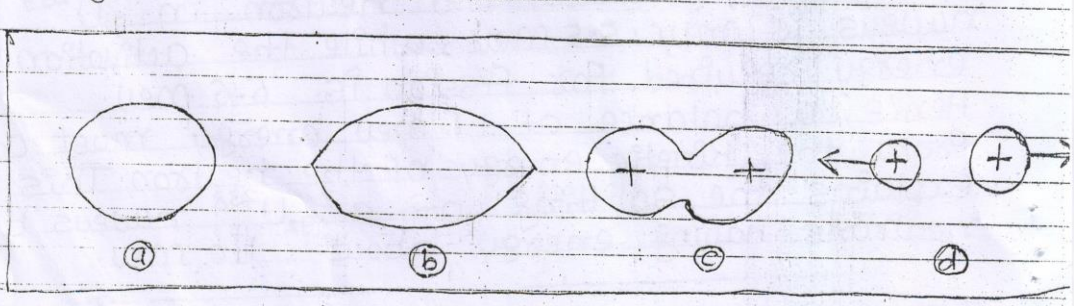
② as heavier group with mass numbers lying between 130 and 149.

* Bohr and wheeler's theory of nuclear fission:-

we have seen that a U^{235} nucleus can be disintegrated by fast neutrons having energy 1.0 Mev or more, while a U^{235} nucleus is fissionable by even slow neutrons.

An explanation of this behaviour was given by Bohr and wheeler on the basis of liquid drop model of the nucleus. A liquid drop has a spherical shape due to internal molecular forces that give rise to surface tension. A drop in an excited state may oscillate in a number of ways. The sphere may change into an ellipsoid shape by large external forces. If the external force is sufficiently large, the ellipsoid may change into a dumb bell shape and may finally break into two portions.

Coming to a nucleus, by the absorption of a neutron a nucleus forms a compound nucleus having a high energy as shown in fig.



* Energy Released in Nuclear Fission :-

During nuclear fission, enormous amount of energy is released. This energy is produced because the original mass of the nucleus is greater than the sum of the masses of the end products. The excess mass appears as energy in accordance with the Einstein's mass energy relation $E = mc^2$

In order to calculate the energy released. Let us consider the following fission reaction

$$_{92}U^{235} + {}_0n^1 \rightarrow \left({}_{92}U^{236}\right) \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q$$

Mass of $_{92}U^{235}$ nucleus $= 235.045733$ a.m.u

Mass of neutron $({}_0n^1) = 1.008665$ a.m.u

Total mass before fission $= 236.044398$ a.m.u

Mass of barium $= 140.9177$ a.m.u

Mass of Krypton $= 91.8854$ a-m.u

mass of 3 neutrons $= 3 \times 1.008665$

$= 3.025995$ a.m.u

Total mass after fission $= 235.829095$ a.m.u

Difference in mass $= 0.2153$ a.m.u

we Know that 1 a.m.u $= 931$ MeV

So the energy released per fission = 0.2153 × 931 MeV

$\cong$ 200.5 MeV

Thus, when one nucleus of Uranium undergoes fission by neutrons, an energy as high as 200 MeV is released.

If we calculate the energy released by the fission of one gram of Uranium, it comes out to be as high as 5.128×10^{23} MeV which is equivalent to 2.26×10^{4} Kilowatt hour. That is why the nuclear energy is being used for the generation of electricity.

* Nuclear Transmutations :-

(a) Transmutation by α-particles :-

① (α, p) reaction :- Rutherford and Chadwick studied the disintegration of a number of elements by bombarding them with α-particles. In all these reactions, protons are liberated. A ~~new~~ few examples are given below:

$${}_{5}B^{10} + {}_{2}He^{4} \longrightarrow {}_{6}C^{13} + {}_{1}H^{1}$$

$${}_{7}N^{14} + {}_{2}He^{4} \longrightarrow {}_{8}O^{17} + {}_{1}H^{1}$$

$${}_{19}K^{39} + {}_{2}He^{4} \longrightarrow {}_{20}Ca^{42} + {}_{1}H^{1}$$

In these reactions, the atomic number of product nucleus increases by 1 and mass number by 3.

nov ② (α, n) reactions :- These reactions were studied during the discovery of neutrons by chadwick. when certain nuclei are bombarded by α-particles, neutrons are ejected. A few examples are given below

$$_4Be^9 + _2He^4 \longrightarrow _6C^{13} \longrightarrow _6C^{12} + _0n^1$$

$$_4B^{11} + _2He^4 \longrightarrow _7N^{15} \longrightarrow _7N^{14} + _0n^1$$

In these reactions, the atomic number increases by 2 and mass number by 3.

With the advent of artificial accelerating machines, it could be possible to impart any desired energy to all the ~~particles~~ projectile particles and hence the nuclear reactions could be studied in all the elements of the periodic table. The scope of nuclear reactions become so large to form a full volume. However, a few important nuclear reactions can be summarised as follows:

ⓑ Transmutation by protons:

(i) (P, α) reactions :- The reactions were studied by cock croft and walton. Accelerated protons having energies from 100 Kev to 700 Kev were bombarded to disintegrate the lithium nucleus. As a result of the reaction, the lithium nucleus was found to break into two α-particles.

$$_3Li^7 + _1H^1 \longrightarrow 2\,_2He^4$$

A few other proton induced reactions can be described as follows:

$$_{5}B^{11} + {}_{1}H^{1} \longrightarrow {}_{6}C^{12} \longrightarrow {}_{4}Be^{8} + {}_{2}He^{4}$$

(ii) (p, d) reactions :- In these reactions protons are used as projectiles and the deuterons are emitted.

Ex.

$$_{4}Be^{9} + {}_{1}H^{1} \longrightarrow {}_{5}B^{10} \longrightarrow {}_{4}Be^{8} + {}_{1}H^{2}$$

(iii) (p, n) reactions :- The reactions in which the neutrons are emitted.

$$_{5}B^{11} + {}_{1}H^{1} \longrightarrow {}_{6}C^{12} \longrightarrow {}_{6}C^{11} + {}_{0}n^{1}$$

(iv) (p, γ) reactions :- The proton induced reactions in which we have some energy liberated in the form of γ ray photon can be represented as

$$_{3}Li^{7} + {}_{1}H^{1} \longrightarrow {}_{4}Be^{8} \longrightarrow {}_{4}Be^{8} + \gamma$$

(c) Transmutation by deuterons :-

Accelerated high energy deuterons when used as bombarding particles may yield many types of reactions summarised below:

(i) (d, α) reactions:

$$_{3}Li^{6} + {}_{1}H^{2} \longrightarrow {}_{4}Be^{8} \longrightarrow {}_{2}He^{4} + {}_{2}He^{4}$$

$$_{8}O^{16} + {}_{1}H^{2} \longrightarrow {}_{9}F^{18} \longrightarrow {}_{7}N^{14} + {}_{2}He^{4}$$

(ii) (d, p) reactions :-

$$_6C^{12} + {}_1H^2 \longrightarrow {}_7N^{14} \longrightarrow {}_6C^{13} + {}_1H^1$$

$$_{11}Na^{23} + {}_1H^2 \longrightarrow {}_{12}Mg^{25} \longrightarrow {}_{11}Na^{24} + {}_1H^1$$

(iii) (d, n) reactions

$$_6C^{12} + {}_1H^2 \longrightarrow {}_7N^{14} \longrightarrow {}_7N^{13} + {}_0n^1$$

$$_4Be^9 + {}_1H^2 \longrightarrow {}_5B^{11} \longrightarrow {}_5B^{10} + {}_0n^1$$

(d) Transmutation by Neutrons :

Neutrons, on account of being electrically neutral, are not affected by electrostatic fields. Hence they are very effective in producing transmutations. They have high penetrating power into the nuclei than the α-particles, protons and deuterons. Some of the neutron induced reactions are given below

(i) (n, α) reactions :-

$$_3Li^6 + {}_0n^1 \longrightarrow {}_3Li^7 \longrightarrow {}_1H^3 + {}_2He^4$$

$$_5B^{10} + {}_0n^1 \longrightarrow {}_5B^{11} \longrightarrow {}_3Li^7 + {}_2He^4$$

(ii) (n, p) reactions :

$$_7N^{14} + {}_0n^1 \longrightarrow {}_7N^{15} \longrightarrow {}_6C^{14} + {}_1H^1$$

$$_{13}Al^{27} + {}_0n^1 \longrightarrow {}_{13}Al^{28} \longrightarrow {}_{12}Mg^{27} + {}_1H^1$$

(iii) (n, 2n) reactions :

$$_{13}Al^{27} + {_0}n^1 \longrightarrow {_{13}}Al^{28} \longrightarrow {_{13}}Al^{26} + 2\,{_0}n^1$$

(iv) (n, γ) reactions :

$$_{13}Al^{27} + {_0}n^1 \longrightarrow {_{13}}Al^{28} \longrightarrow {_{13}}Al^{28} + \gamma$$

$$_{92}U^{238} + {_0}n^1 \longrightarrow {_{92}}U^{239} \longrightarrow {_{92}}U^{239} + \gamma$$

These reactions are known as radioactive captures and are used to produce radioactive isotopes.

(e) Transmutation by Electrons :

The electrons accelerated to high energy in betatrons cause the disintegration of a number of elements e.g. when beryllium target is bombarded with high energy electrons, a neutron is emitted.

$$_4Be^9 + {_{-1}}e^0 \longrightarrow {_3}Li^9 \longrightarrow {_3}Li^8 + {_0}n^1$$

(f) Photodisintegration :

When some target nuclei are bombarded by γ-ray photons having high energy, the resulting combined nucleus disintegrates with the emission of a neutron.
For ex., A 2.62 MeV photon breaks the deutron into a proton and neutron
i.e.

$$_1H^2 + \gamma \longrightarrow {_1}H^2 \longrightarrow {_1}H^1 + {_0}n^1$$

Photons have no mass but can give their kinetic energy to a nuclear reaction.

If the energy of the photon is greater than the binding energy of the nucleus, then other reactions which take place are

$${}_4Be^9 + \gamma \longrightarrow {}_4Be^9 \longrightarrow {}_4Be^8 + {}_0n^1$$

$${}_{15}P^{31} + \gamma \longrightarrow {}_{15}P^{31} \longrightarrow {}_{15}P^{30} + {}_0n^1$$

* Conservation laws in nuclear reactions :-

In all the artificial transmutation processes the following quantities remain conserved:

① The total energy (kinetic energy + rest mass energy) of the product particles is equal to the total energy of the initial particles.

② The total linear momentum of the product particles is equal to the total linear momentum of the incident particles.

③ The total electric charge remains conserved.

④ Total number of nucleons (mass number) remains constant, i.e. total number of protons and neutrons in a nuclear reaction remains constant.

⑤ Parity is also conserved in case of strong interactions. Thus parity before the reaction is equal to the parity after the reaction.

⑥ Angular momentum, spin and isotropic spin are the other physical quantities which are also conserved in a nuclear reaction.

However, magnetic dipole moment and the electric quadrapole moment of the reacting nuclei, are not conserved in nuclear reactions.

* Nuclear Reaction Kinematics :-

At any nuclear reaction, the conservation of energy and momentum imposes certain restrictions. mathematical of nuclear reaction with the restrictions of conservation laws is known as nuclear reaction kinematics.

Consider a nuclear reaction,

$$X + x \longrightarrow Y + y \text{ ——— } (1)$$

where X, x, Y and y are the target nucleus bombarding particle, product nucleus and product particle, respectively. It will be assumed that target nucleus is at rest so it has no kinetic energy. since total energy is conserved in a nuclear reaction, we get

$$M_x c^2 + (E_x + m_x c^2) = (E_y + m_y c^2) + (E_Y + M_Y c^2) \text{ ---- } (2)$$

where m_x, M_x, m_y, M_Y all represent respective masses of incident particle, target nucleus, product particle and product nucleus.

We now introduce a quantity Q which represents the difference between the kinetic energy of the products of reaction and that of the incident particle.

~~$Q = E_y - E_x$ ——— (3)~~

$$Q = E_Y - E_x \text{ ——— } (3)$$

from equations ② and ③ we have

$$m_X + m_x = (M_Y - m_y) c^2 = Q \quad \text{--- ④}$$

The quantity Q is called the energy balance of the reaction or more commonly Q-value of the reaction.

If Q is +ve the reaction is said to be exoenergic. this occurs if sum of the masses of incident particle and target nucleus is greater than that of masses of the product nuclei the kinetic energy of the product nuclei being greater than that of the incident particle.

If Q is -ve the reaction is said to be endoenergic that is energy must be supplied usually as a kinetic energy of the incident particle.

* Q Value:-

"Nuclear reactions resemble ordinary chemical reactions and are accompanied by energy changes. The energy liberated or absorbed during a nuclear reaction is called nuclear reaction energy."

This energy is denoted by Q in reaction equation and is called the energy balance of the reaction or more commonly its Q ~~-ve~~ Value. The Q value of a reaction can be positive or negative depending on the nature of the reaction.

According to Einsteins mass-energy equivalence, Q value must be balanced by the changes in mass associated with the nuclear reaction

Let us consider a general nuclear reaction

$$X + a \longrightarrow Y + b$$

Further, let M_0 be the mass of the stationary target nucleus X.
$M_1 E_1$ - be the mass and energy of the nucleus projectile (a) and
$M_3 E_3$ - be the mass and energy of the emitted particle (b)
The nuclear reaction may be written as,

$$M_0 + (M_1 + E_1) = (M_2 + E_2) + (M_3 + E_3)$$

or

$$(M_0 + M_1) - (M_2 + M_3) = E_2 + E_3 - E_1$$

As Q value is the energy balance, it may be expressed as,

$$Q = (E_2 + E_3) - E_1$$

$$Q = (M_0 + M_1) - (M_2 + M_3) \quad \text{--- (1)}$$

above eqn (1) shows that the Q value of a nuclear reaction may be determined either (i) from the known kinetic energies of the particles involved or (ii) from the known masses of the reactants and the product nuclei.

From eqn (1) Q value may be defined as the difference in mass of the reactants and the products

Case-I : $(M_0 + M_1) > (M_2 + M_3)$ and Q is positive
in this case the total mass of products is less than that of the interacting nuclei.
The difference in masses (mass defect) is converted into energy and hence the reaction

is accompanied by liberation of energy. Such a reaction is called as exothermic

$^{7}_{3}Li\ (P,\alpha)\ ^{4}_{2}He$ reaction is an example of exoergic reaction.

$M_0\ (^{7}_{3}Li) = 7.01822$ amu, $M_2\ (^{4}_{2}He) = 4.0038$ amu

$M_1\ (^{1}_{1}H) = 1.00814$ amu; $M_3\ (^{4}_{2}He) = 4.00387$ amu.

$M_0 + M_1 = 8.02636$ amu; $M_2 + M_3 = 8.00774$ amu

$\Delta M = (M_0 + M_1) - (M_2 + M_3) = 0.01862$ amu

$Q = c^2 \cdot \Delta M = 931.4\ (\Delta M)$ MeV.

$= 931.4\ (0.01862)$ MeV

$Q = 17.34$ MeV

case - II

$(M_0 + M_1) < (M_2 + M_3)$ & and Q is negative. In that case the total mass of the products is greater than that of reactants. It implies that there is a gain of mass in the reaction which happen when there is an absorption of energy. such reactions are called endothermic.

Let us consider the nuclear reaction

$^{14}_{7}N\ (\alpha, P)\ ^{17}_{8}O$

$M_0 \left({}^{14}_{7}N \right) = 14 \cdot 00753$ amu;

$M_2 \left({}^{17}_{8}O \right) = 17 \cdot 00450$ amu

$M_1 \left({}^{4}_{2}He \right) = 4 \cdot 00387$ amu;

$M_3 \left({}^{1}_{1}H \right) = 1 \cdot 00814$ amu.

$M_0 + M_1 = (M_0 + M_1) - (M_2 + M_3) = 0 \cdot 00124$ amu

$\therefore Q = -931 \cdot 4 \, (0 \cdot 00124)$ mev.

$Q = -1 \cdot 15$ Mev

· An endothermic reaction will not unless the bombarding particle has a kinetic energy greater than Q.

The minimum threshold energy necessary for an endothermic nuclear reaction to occur.