Nuclear and Particle Physics

Chapter-1: Nuclear Structure and General Properties of Nuclei

● Constituents of nuclei:

- Atomic nuclei are composed of two different types of elementary particles-protons and neutrons. Collectively, the neutrons and protons are referred to as nucleons.
- The proton is identified as the nucleus of the lightest and the commonest isotope of hydrogen. It carries one electronic charge, +e and has mass about 1836 times the electronic mass, m_e.
- The neutron, on the other hand, possesses no charge–electrically neutral. Its mass is almost equal to, but slightly more than, the mass of the proton.
- The electronic mass is negligibly small.
- According to Coulomb's law, the positively charged protons, closely spaced within the nucleus, should repel each other strongly and they should fly apart. Nucleons (protons and neutrons) are held together under the influence of very strong short range attractive force. This force is different from commonly known forces like gravitational or electrical and is classified as strong interactions.
- The number of protons in the nucleus determines the atomic number of the nuclide.
- The sum of the numbers of protons (Z) and neutrons (N) inside the nucleus is mass number A i.e. $A = N + Z$
- The neutron number N is therefore given by $N = A - Z$
- A nucleus of an atom X of atomic number Z and a mass number A, that is, a nuclide is symbolically represented by ${}^{A}_{Z}X$.

● Isotopes, Isobars, Isotones and Mirror nuclei:

- Nuclei with same atomic number Z, but different mass number A are called isotopes. Example: ${}^{6}_{3}Li$ and ${}^{7}_{3}Li$, ${}^{20}_{10}Ne$ and ${}^{22}_{10}Ne$.
- Nuclei with the same mass number A but different atomic number Z are called isobars. Example: ${}^{16}_{8}O$ and ${}^{16}_{7}N$. • Nuclei with the same number of neutrons, that is having the same N, are known as isotone. Example: ${}^{23}_{11}Na$ and ${}^{24}_{12}Mg$
- The pairs of isobaric nuclei such as $\left({}^{11}_{6}C \text{ and } {}^{11}_{5}B\right), \left({}^{13}_{7}N \text{ and } {}^{13}_{6}C\right)$ are known as mirror nuclei in which the proton number Z and the neutron number N are interchanged and differ by one unit.

● Nuclear mass and binding energy:

- The nuclear mass M_{nuc} is obtained from the atomic mass M(A, Z) by subtracting the masses of Z orbital electrons i.e. $M_{nuc} = M(A,Z) - Zm_e$
- The above expression however is not exact in that the binding energies of the electrons have not been taken into consideration. In forming a nucleus out of the constituent particles, a fraction of the total mass of the constituents disappears and the evaluation of energy equal to E_B takes place. If ΔM be the amount of mass disappeared, then binding energy, $E_B = \Delta Mc^2$
- If M_H, M_n be the masses of hydrogen atom and the neutron respectively, then

$$\Delta M = ZM_H + N\,M_n - M(A,Z)$$

where M(A, Z) is the mass of the atom of mass number A and atomic number Z.

Therefore,

$$E_B = \left[ZM_H + NM_n - M(A,Z)\right]c^2$$

$$= \left[ZM_p + NM_n + Zm_e - M_{nuc} - Zm_e\right]c^2$$

$$= \left[ZM_p + NM_n - M_{nuc}\right]c^2$$

● **Unit of atomic mass:**

- The unit of atomic mass is presently defined to be one-twelfth of the mass of the atom of carbon isotope ^{12}C taken to be exactly 12 units, and is symbolised by u, being the abbreviation for 'unified atomic mass unit'.
- 1 a.m.u. = 1.660566×10^{-27} kg
- Energy equivalent of 1 a.m.u. = 931.501 MeV
- The energy equivalent of the rest mass of electron, proton and neutron are as

$$\text{Electron}(m_e) = 9.10953\times10^{-31}\text{kg} = 5.48580\times10^{-4}\text{a.m.u}$$

$$\text{Proton}(m_p) = 1.677265\times10^{-27}\text{kg} = 1.0072765\text{ a.m.u}$$

$$\text{Neutron}(m_n) = 1.67495\times10^{-27}\text{kg} = 1.0086650\text{ a.m.u.}$$

● **Binding energy and stability of nucleus:**

If $E_B > 0$, i.e. positive, the nucleus is stable and energy from outside is to be supplied to disrupt the nucleus into its constituents separately. If, however, $E_B < 0$, i.e. negative, the nucleus is unstable and will disintegrate of itself. The E_B-value is a measure of the stability of the nucleus. More the E_B, more is the stability.

● **Mass defect and packing fraction:**

- **Mass defect:** The difference between the measured atomic mass M(A, Z), expressed in u, and the mass number A of a nuclide is called the mass defect, $\Delta M'$ i.e. $\Delta M' = M(A,Z) - A$

  The mass defect of ^{4}He = (4.002603 –4)a.m.u. = +0.002603 a.m.u. and that of ^{16}O = (15.994915–16) a.m.u. = –0.005085 a.m.u. The mass defect can therefore be both positive and negative. It is found that the mass defect is positive for very light and very heavy atoms, and it is negative in the region between the two.
- **Packing fraction:** It is defined as the mass defect per nucleon in the nucleus.

$$f = \frac{\Delta M'}{A} = \frac{M(A,Z)-A}{A} = \frac{M(A,Z)}{A} - 1 \quad \Rightarrow M(A,Z) = A(1+f)$$

The binding energy per nucleon E_B / A or f_B (binding fraction) is given by

$$f_B = \frac{E_B}{A} = \frac{ZM_H + NM_n - M(A,Z)}{A}$$

● **Binding fraction vs mass number curve:**

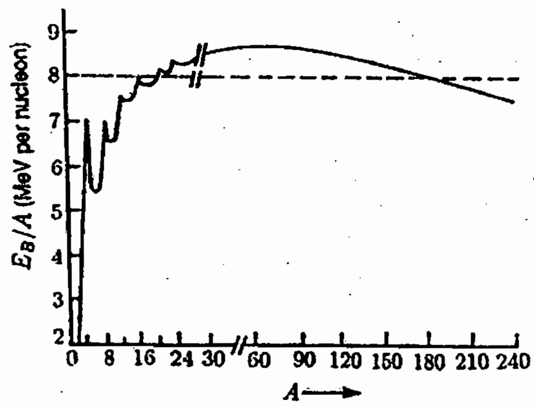


Figure: **Binding fraction curve**

- f_B is very small for very light nuclei and goes on increasing rapidly with increasing A and reaches a value ~ 8 MeV/nucleon for the mass number A ~ 20. Thereafter the rise of the curve is much slower, reaching a maximum value of 8.7 MeV per nucleon for A = 56. If A is increased still further, the curve decreases slowly.
- The variation in f_B is very slight in the mass number range $20 < A < 180$ and in this region f_B may be considered to be virtually constant with a mean value ~ 8.5 MeV/nucleon.
- For A > 180, that is, for heavy nuclei, the f_B-value decreases monotonically with increasing A and it is ~ 7.5 MeV/nucleon for the heaviest nuclei.
- A rapid fluctuation in f_B is noted for very light nuclei with peaks in the curve of this region, corresponding to even-even nuclei, such as ^4He, ^8Be, ^{12}C, ^{16}O i.e., with mass number A = 4n, where n = 1, 2, 3, ... Peaks in the curve are also seen at Z or N equal to 20, 28, 50, 82, 126. These are called magic numbers.

Nuclear size:

- Experiments indicate that the majority of atomic nuclei are spherical or nearly so in shape. The radius R of various nuclei is approximately given by $R = r_0 A^{1/3}$, where A is the mass number and r_0, a constant called nuclear radius parameter. The value of r_0 ranges from $(1.1\text{–}1.5)\times 10^{-15}$ m, i.e. (1.1–1.5)F.
- Since nuclear charge parameter $Z \propto A$ and the nuclear charge density ρ_c is nearly the same throughout the nuclear volume, the distribution of nuclear charge +Ze follows the pattern of nuclear mass distribution. Hence charge radius $\simeq$ mass radius, of the nucleus.
- The size of the nucleus was first estimated from Rutherford's α – ray scattering by various atoms. The larger the angle of scattering of α – particle closer is its approach to the nucleus. If the kinetic energy of α – particles be equal to the repulsive coulomb energy between α – particle (of charge 2e, mass 'm' and velocity 'v') and the nucleus (of charge Ze), it would momentarily come to rest such that the distance of closest approach R is given by

$$\frac{1}{2}mv^2 = \frac{2e \times Ze}{4\pi \epsilon_0 R} \quad \Rightarrow R = \frac{Ze^2}{\pi \epsilon_0 mv^2}$$

- The mean squared radius $\langle r^2 \rangle$ of nuclear charge distribution is given by

$$\langle r^2 \rangle = \frac{\int_0^R r^2 . 4\pi r^2 \rho(r) dr}{\int_0^R 4\pi r^2 \rho(r) dr} = \frac{\int_0^R r^4 dr}{\int_0^R r^2 dr} = \frac{3}{5}R^2$$

for a uniformly charged sphere of radius R, ρ = constant and $\rho = 0$ for r > R.

Nuclear density:

The nuclear density, ρ_N can be estimated from the relation, $\rho_N = \frac{\text{nuclear mass}}{\text{nuclear volume}}$

Nuclear magnetic moment: Nuclear magneton:

- An electron possesses a magnetic moment, associated with it due to its angular momentum in discrete orbits and is given by 1 Bohr magneton, μ_B i.e. $\mu_B = \frac{eh}{4\pi m_e} = 9.2 \times 10^{-24}$ J / tesla

In analogy, there is associated with a nucleon, a nuclear magneton given by

$$\mu_N = \frac{eh}{4\pi m_p} = 5.05 \times 10^{-27} \text{ J / tesla}$$

- The nuclear magneton is thus 1836 times smaller than the Bohr magneton and is also called Rabi magneton.
- The magnetic moment of a proton is not 1 nuclear magneton; it has instead a moment $+2.79\mu_N$. The positive sign indicates that the direction of the magnetic moment vector, $\vec{\mu}$, coincides with that of the angular momentum vector $\vec{I}$.
- The neutron has no net electric charge. But it has also a magnetic moment equal to $-1.91\mu_N$. The negative sign points out that the direction of angular momentum vector $\vec{I}$ is opposite to that of the magnetic moment vector $\vec{\mu}$.
- The net magnetic moment $\vec{\mu}$ of a nucleus depends on the resultant total angular momentum of the nucleus, $\vec{I}$ and is given by $\vec{\mu} = \gamma\vec{I} = g\mu_N\vec{I}$, where γ is the product of the gyromagnetic ratio 'g' and the nuclear magneton μ_N.

- **Electric quadrupole moment:**

If a nucleus is not spherically symmetric in shape, its deviation from spherically can be expressed by what is called electric quadrupole moment Q. Let a nucleus of charge number Z be deformed from a spheroid to an ellipsoid shape with the major axis 2a perpendicular to the symmetry axis and minor axis 2b along the axis of symmetry. The quadrupole moment Q of such deformed nucleus is

$$Q = \frac{2}{5}Z\left(a^2 - b^2\right)$$

If R be the average nuclear radius, ΔR the deviation of R from the direction of symmetry axis then

$$Q = \frac{6}{5}ZR^2\left(\frac{\Delta R}{R}\right)$$

If $\rho(x, y, z)$ be the volume density of charge of the nucleus with its centre of mass at the origin, so that $r^2 = x^2 + y^2 + z^2$, the quadrupole moment Q is defined from the electrostatic energy from interaction of electric potential and charge density as

$$Q = \frac{1}{e}\int\left(3z^2 - r^2\right)\rho d\tau$$

where the integration is carried out over the entire volume of the nucleus, and 'e' is the charge of the proton; the nucleus is assumed to have a symmetry axis along z.

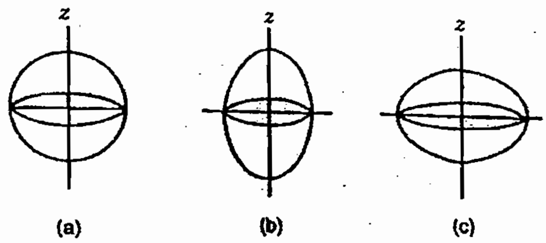


Figure: Electric quadrupole moment
(a) spherical charge distribution (Q = 0)
(b) Prolate spheroid (Q > 0)
(c) Oblate spheroid (Q<0)

- Obviously, Q = 0 for a spherically symmetric nucleus.
- For a nucleus in the shape of a prolate spheroid (the charge distribution stretched in the z-direction) a > b and the quadrupole moment Q > 0(positive).
- For a nucleus in the shape of an oblate spheroid, however, the charge distribution is stretched perpendicular to z-direction and b > a. This makes Q < 0 (negative) for such nuclei.
- Q is usually measured in barns i.e. 1 barn = 10^{-28} m^2.

- The magnitude of Q mostly lies in the range 10^{-28} to 10^{-30} m^2 and depends on the radius and charge of the nucleus and its deviation from spherical symmetry.

Spin angular momentum: Both the proton and the neutron, like the electron, have an intrinsic spin. The spin angular momentum is computed by $L_s = \sqrt{l(l+1)}.\frac{h}{2\pi}$ where the quantum number 1 is called the spin which is equal to 1/2. Hence spin angular momentum has a value $L_s = \frac{\sqrt{3}}{2}\frac{h}{2\pi}$.

- **Nuclear spin: (Total angular momentum)**

The spin of a nucleus is the resultant of the spins of its constituent nucleus–protons and neutrons. It turns out that the spins of protons and neutrons can be represented, like that of an electron, by the same quantum number 1/2.

The total angular momentum $\bar{I}$ of a nucleus is also loosely called the 'spin' of the nucleus, but it is different from the spin angular quantum number. The total angular momentum of a nucleus can be computed from the multiplicity and relative spacings of spectral lines in an applied magnetic field. If a nucleus with total angular momentum $\bar{I}$ be placed in an externally applied magnetic field, the magnetic quantum number m_I have values ranging from +I to –I and thus split the energy levels into 2I+I sub-levels. The transitions between these sub-states may be used to estimate I from the multiplicity of the spectral lines.

Total angular momentum $L_N = \sqrt{l_N(l_N+1)}.\frac{h}{2\pi}$. This is called nuclear spin.

- **Parity of nuclei:**

A particle moving with a large velocity can be quantum mechanically associated with a wave and the wave motion can be described by a wave function $\psi(x, y, z)$ which depends on the space coordinates (x, y, z).

$$\psi(-x,-y,-z) = +\psi(x,y,z) \text{....even } (+) \text{ parity}$$
$$\psi(-x,-y,-z) = -\psi(x,y,z) \text{....odd } (-) \text{ parity}$$

The parity π is determined by the orbital quantum number $\ell : \pi = (-1)^\ell$, so that for $\ell = 0$ or even, the parity is even and for l = odd, the parity is odd.

- **Rutherford's $\alpha-$ray scattering:**

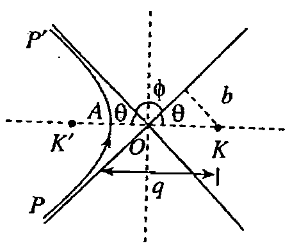


Figure: Path of a-particles during scattering by a nuclues

- 'b' is called the collision (or impact) parameter which is defined as the minimum distance to which the $\alpha-$particle would approach the nucleus if there were no force between them.

$$q = b\frac{1+\cos\theta}{\sin\theta} = b\cot\frac{\theta}{2}, \quad b = k\tan\theta = k\cot\phi/2$$

where $k = ZeE/4\pi\varepsilon_0 Mv^2$, E = charge of α – particles and the angle of scattering ϕ

- Probability of deviation of α – particles per unit solid angle, $W(\phi) = \frac{k^2}{4}\frac{1}{\sin^4(\phi/2)}$

Solved Examples

Example-1: Calculate the binding energy in MeV of ^4He from the following data: Mass of ^4He = 4.003875 a.m.u; Mass of ^1H = 1.008145 a.m.u and mass of a neutron = 1.008986 a.m.u.

Soln. ^4He nucleus contains 2 protons and 2 neutrons. So, mass of the constituents

= 2(1.008145 + 1.008986)a.m.u. = 4.034262 a.m.u. But mass of ^4He nucleus = 4.003875 a.m.u.

Mass difference (loss) = $(4.034262 - 4.003875) = 0.030387$ a.m.u.

Binding energy, $E_B = 0.030387 \times 931\ \text{MeV} = 28.29\ \text{MeV}$

Example-2: The mass of the hydrogen atom and of neutron are 1.008142 and 1.008982 a.m.u. respectively. Calculate the packing fraction and the binding energy per nucleon of ^{16}O nucleus.

Soln. ^{16}O nucleus consists of 8 protons and 8 neutrons.

Mass of constituents = $8(1.008142 + 1.008882)$a.m.u. = 16.136992 a.m.u.

Mass of ^{16}O nucleus = 15.994915 a.m.u.

Mass difference (loss) = 0.142077 a.m.u.

Binding energy, E_B = (0.142077×931) a.m.u. = 132.27 MeV

Mass defect, $\Delta M = M(A,Z) - A = (15.994915 \sim 16)$a.m.u. = 0.005085 a.m.u.

Packing fraction = $\Delta M / A = 0.005085/16 = 3.178 \times 10^{-4}$

Example-3: The radius of a $^{64}_{29}Cu$ nucleus is measured to be 4.8 × 10^{-13}cm. Find the radius of a $^{27}_{12}\mu g$ nucleus.

Soln. We know that, $R = R_0 A^{1/3}$

Now, $R_1 = R_0 A_1^{1/3}$, 4.8 × 10^{-13} = $R_0(64)^{1/3}$

So, $R_0 = \frac{4.8\times10^{-13}}{4} = 1.2\times10^{-13}$

Again, $R_2 = R_0 A_2^{1/3}$ = 1.2 × 10^{-13} Y $(27)^{1/3}$

= 3.6 × 10^{-13} cm

Example-4: Since $^{27}_{14}Si$ and $^{27}_{13}Al$ are mirror nuclei, their ground states are identical except for charge. If there mass difference is 6 MeV, estimate their radius (neglect the proton-neutron mass difference)

Soln. The mass-difference between mirror nuclei can be attributed to the difference in electrostatic energy. Now, the electrostatic energy of a charge Q distributed uniformly throughout a sphere of radius R is $W = 3Q^2/5R$

So, $\Delta W = \frac{3e^2}{5R}\left(Z_1^2 - Z_2^2\right) \Rightarrow R = \frac{3e^2}{5\Delta W}\left(14^2 - 13^2\right) = \frac{3hc}{5\Delta W}\left(\frac{e^2}{hc}\right)\times 27$

where e^2/hc = the fine structure constant = 1/137

$$\Rightarrow \quad R = \frac{3\times1.97\times10^{-11}}{5\times6}\times\frac{1}{137}\times27 = 3.88\times10^{-11}\text{cm} = 388\ \text{fm}$$

Example-5: An α – particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. Calculate the distance of closest approach.

Soln. The distance of closest approach is:

$$d = \frac{Ze^2}{\pi\varepsilon_0 Mv^2} = \frac{92\times\left(1.6\times10^{-19}\right)^2}{3.14\times8.85\times10^{-12}\times2\times5\times1.6\times10^{-13}} m = 5.3\times10^{-14} \text{ m}$$

Example-6: Calculate the binding energies of the following Isobars and their B.E/nucleon: Given $_{28}Ni^{64}$ = 63.9280 a.m.u, $_{29}Cu^{64}$ = 63.9298 a.m.u., M_N = 1.008665 a.m.u. and M_H = 1.007825 a.m.u.

Soln. Hence for $_{28}Ni^{64}$: $NM_N + ZM_H$ = [36 × 1.008665 + 28 × 1.007825] a.m.u.= 64.531 a.m.u

ΔM = 64.531 – 63.98 = 0.603 a.m.u.

Hence, B.E = 561.4 MeV and B.E / nucleon = 8.77 MeV.

For $_{29}Cu^{64}$: $NM_N + ZM_H = [35\times1.008665 + 29\times1.007825]$a.m.u.

ΔM = 64.5272 – 63.9298 = 0.5974 a.m.u $\Rightarrow$ B.E. = 556.18 MeV and B.E/nucleon = 8.7 MeV

Example-7: Find the energy release if two, $_1H^2$ nuclei can fuse together to form $_2He^4$ nucleus. The B.E/nucleon of $_1H^2$ and $_2He^4$ is 1.1 MeV and 7.0 MeV respectively.

Soln. Number of nucleon in $_2He^4$ is 4, hence B.E for $_2He^4$ = 28.0 MeV
Similarly B.E for $_1H^2$ = 2.2 MeV

Energy equivalent of mass of $_2He^4$ nucleus = $\left[\left\{2M_p + 2M_n\right\}c^2 - 28.0\right]\text{MeV} = E\left({}_2He^4\right)$

Energy equivalent of mass of $_1H^2$ nucleus = $\left[\left\{M_p + M_n\right\}c^2 - 2.2\right]\text{MeV} = E\left({}_1H^2\right)$

$$\Delta E = 2E\left({}_1H^2\right) - E\left({}_2He^4\right) = 23.6 \text{ MeV}$$

Example-8: Find the binding energy of a nucleus consisting of equal no's of protons and neutrons and having the radius one and a half times smaller than that of Al^{27} nucleus.

Soln. Mass number of given nucleus $\frac{27}{(3/2)^3} = 8$; Nucleus is Be^8

$$\text{B.E.} = \left\{(4\times1.00867) + (4\times1.00783) - 8\right\}\times931.5 \text{ MeV} = 61.48 \text{ MeV (approx.)}$$

Example-9: The atomic mass of $_6C^{12}$ is 12.00 amu and that of $_6C^{13}$ 13.00354 amu, find the energy required to remove a neutron from $_6C^{13}$ in MeV. The mass of neutron is 1.008665 a.m.u.

Soln. The nuclear equation is $_6C^{13} \rightarrow {}_6C^{12} + {}_0n^1$. The mass of $_6C^{12} + {}_0n^1$ is 12.00 + 1.008665 = 13.008665 amu

Mass defect = 13.008665 – 13.003354 = 0.005311 amu

Its energy equivalent = ΔE = 0.005311 × 931.5 = 4.95 MeV

Example-10: Calculate the binding energy of the last neutron in $^{15}N^7$ and of the last proton in $^{15}O^8$, and contrast with the last neutron in $^{16}N^7$ and in $^{16}O^8$.

Soln. From the CRC handbook, we know that

$M(^1H^1) = 1.0078$amu, $m_n = 1.0087$amu,

$M(^{14}N^7) = 14.0031$amu, $M(^{15}N^7) = 15.0001$amu,

$M(^{16}N^7) = 16.0061$amu, $M(^{15}O^8) = 15.0030$amu,

$M(^{16}O^8) = 15.9949$amu.

Using these values and the conversion between "amu" and "MeV" units, we can calculate the binding energy of the last neutron in $^{15}N^7$.

$$\text{B.E.} = -(M(^{14}N^7) + m_n - M(^{15}N^7))c^2$$

$$= -(14.0031 + 1.0087 - 15.0001)\text{amu} \times c^2$$

$$\approx -0.0117 \times 931.5\text{MeV}/c^2 \times c^2 = -10.8985\text{MeV}$$

Similarly, the binding energy of the last proton in $^{15}O^8$ is

$$\text{B.E.} = -(M(^{14}N^7) + M(^1H^1) - M(^{15}O^8))c^2$$

$$= -(14.0031 + 1.0078 - 15.0030)\text{amu} \times c^2$$

$$\approx -0.0079 \times 931.5\text{MeV}/c^2 \times c^2 = -7.3588\text{MeV}$$

Furthermore, the binding energy for the last neutron in $^{16}N^7$ is given by

$$\text{B.E.} = -(M(^{15}N^7) + m_n - M(^{16}N^7))c^2$$

$$= -(15.0001 + 1.0087 - 16.0061)\text{amu} \times c^2$$

$$\approx -0.0027 \times 931.5\text{MeV}/c^2 \times c^2 \approx -2.5150\text{MeV}$$

Finally, the binding energy of the last neutron in $^{16}O^8$ is

$$\text{B.E.} = -(M(^{15}O^8) + m_n - M(^{16}O^8))c^2$$

$$= -(15.0030 + 1.0087 - 15.9949)\text{amu} \times c^2$$

$$\approx -0.0168 \times 931.5\text{MeV}/c^2 \times c^2 = -15.6492\text{MeV}$$

Example-11: Chlorine-33 decays by positron emission with a maximum energy of 4.3 MeV. Calculate the radius of the nucleus from this.

Soln. The decay scheme is ${}_{17}Cl^{33} \rightarrow {}_{16}S^{33} + {}_{+1}e^0 + \nu + E_\beta$

When the positron emits with a maximum energy, the neutrino energy will be zero and the daughter nucleus S^{33} will be formed in the ground state.

$$\therefore \quad E_\beta = \frac{3}{5}\frac{e^2 A^{2/3}}{4\pi\epsilon_0 R_0} - 1.80MeV$$

or $\frac{3}{5}\frac{e^2 A^{2/3}}{4\pi\epsilon_0 R_0} = 6.1 \times 1.6 \times 10^{-13}$ Joule or $R_0 = \frac{3}{5}\frac{e^2 A^{2/3}}{4\pi\epsilon_0 \times 6.1 \times 1.6 \times 10^{-13}}$

$$= \frac{3}{5}\frac{(1.6 \times 10^{-19})^2 (33)^{2/3} \times 9 \times 10^9}{6.1 \times 1.6 \times 10^{-13}} = 1.41 \times 10^{-15}\text{meter}$$

$$\therefore R = R_0 A^{1/3} = 1.41 \times 10^{-15} \times 33^{1/3} = 4.54 \times 10^{-15}\text{m}$$

Chapter-2: Nuclear Models

- **Liquid drop model:**

The liquid drop model of the nucleus was first proposed by Niels Bohr and F.Kalcar in the year 1937. They observed that there exists many similarities between the drop of a liquid and a nucleus. For instance,
(i) both the liquid drop and the nucleus possess constant density,
(ii) The constant binding energy per nucleon of a nucleus is similar to the latent heat of vaporization of a liquid,
(iii) The evaporation of a drop corresponds to the radioactive properties of the nucleus, and
(iv) The condensation of drops bears resemblance with the formation of compound nucleus, etc.

- According to this model, the nucleus is supposed to be spherical in shape in the stable state with radius $R = r_0 A^{1/3}$, just as a liquid drop is spherical due to symmetrical surface tension forces. The surface tension effects are analogous to the potential barrier effects on the surface of the nucleus.
- The density of a liquid drop is independent of the volume, as is the case with the nucleus. But whereas the nuclear density is independent of the type of nucleus, the density of a liquid does depend on its nature.
- Like the nucleons inside the nucleus, the molecules in the liquid drop interact only with their immediate neighbours.
- The non-independence of the binding energy per nucleon on the number of nucleons in the nucleus is analogous to the non-independence of the heat of vaporization of a liquid drop on the size of the drop.
- Molecules in a liquid drop evaporate from the liquid surface in raising the temperature of the liquid due to their increased energy of thermal agitation. Analogously, if high energy nuclear projectiles bombard the nucleus, a compound nucleus is formed in which the nucleons quickly share the incident energy and the emission of nucleons occurs.
- The phenomenon of nuclear fission is easily explained as the splitting of the liquid drop into two more or less equals parts if set into vibration with sufficient energy.

- **Semi-empirical binding energy or mass formula:**

C.V. Weizsacker, a German physicist, proposed the following semi-empirical formula for the nuclear binding energy B.E. (in MeV) for the nucleus (Z, A)

$$\text{B.E.} = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

with the constants having the value, $a_v = 15.75, a_s = 17.80, a_c = 0.71, a_n = 22.7$ and $\delta = 33.6$ are in MeV.

(i) **Volume energy:**

The first term, $E_V = a_v A$, is the volume effect representing the volume energy of all nucleous. The more the total number of nucleons A, more difficult it becomes to remove an individual nucleon from the nucleus. Since the nuclear density is nearly constant, the nuclear mass is proportional to the nuclear volume, which again is proportional to R^3. But $R \propto A^{1/3} \Rightarrow R^3 \propto A$. So, the volume energy $E_v \propto A$. $\Rightarrow \quad E_v = a_v A$

This energy corresponds to the amount of heat energy (the heat of vaporisation) required to transform a liquid to its vapour state being proportional to the mass of the liquid.

(ii) Surface energy:

The second term, $E_s = a_s A^{2/3}$, is the surface effect being similar to the surface tension in liquids, like the molecules on the surface of a liquid, the nucleons at the surface of the nucleus are not completely surrounded by other nucleons. It results in reducing the total binding energy due to nucleons on the surface. This correction due to surface energy E_s, which is proportional to the surface area of the nucleus i.e. to $4\pi R^2$ i.e. $E_s \propto R^2$

$$\Rightarrow \quad E_s \propto A^{2/3} \quad \Rightarrow E_s = a_s A^{2/3}$$

(iii) Coulomb energy:

The third term, E_c, is the Coulomb electrostatic repulsion between the charged particles in the nucleus. Since each charged particle repulses all the other charged particles, this term would be directly proportional to the possible number of combinations for a given proton number Z, which is Z(Z–1)/2. The energy of interaction between protons is again inversely proportional to the distance of separation R, so the energy associated with Coulomb repulsion is:

$$E_c = k\frac{Z(Z-1)}{R} = k\frac{Z(Z-1)}{r_0 A^{1/3}} = a_c \frac{Z(Z-1)}{A^{1/3}} \quad (\text{-ve quantity})$$

(iv) Asymmetry energy:

The fourth term E_a originates from the asymmetry between the number of protons and the number of neutrons in the nucleus. Nuclear data for stable nuclei indicate that for lighter nuclei, the number of protons is almost equal to that of neutrons: N = Z. As A increases, the symmetry of proton and neutron number is lost and the number of neutrons exceeds that of protons to maintain the nuclear stability. This excess of neutrons over protons, i.e. N–Z, is the measure of the asymmetry and it decreases the stability or the B.E. of medium or heavy nuclei.

So, $E_a \propto (N-Z)$, and $E_a \propto (N-Z)A$

$$\Rightarrow \quad E_a = a_n \frac{(N-Z)^2}{A} = a_n \frac{(A-2Z)^2}{A}$$

(v) Pairing energy:

The last term, a pure corrective term, is called pairing energy E_p.

$$E_p = \pm\frac{\delta}{A^{3/4}}$$

Z	N	A	δ	E_p
even	even	even	34	$+\delta / A^{3/4}$
even	odd	odd	0	0
odd	even	odd	0	0
odd	odd	even	35	$-\delta / A^{3/4}$

- Binding fraction, $f_B = \frac{B.E.}{A} = a_v - \frac{a_s}{A^{1/3}} - \frac{a_c Z(Z-1)}{A^{4/3}} - a_n \frac{(A-2Z)^2}{A^2} \pm \frac{\delta}{A^{7/4}}$
- Mass of nucleus,

$${}^{A}_{Z}M = Z M_p + (A-Z)M_n - B.E./c^2$$

$$= ZM_p + (A-Z)M_n - \frac{1}{c^2}\left[a_0A - a_sA^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_n\frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}\right]$$

The above formula is known as the semi-empirical mass formula.

Assuming, $F_A = A(M_n - a_v + a_n) + a_sA^{2/3}$

$$p = -4a_n - (M_n - M_p);\ q = \frac{1}{A}\left(a_cA^{2/3} + 4a_n\right)$$

$\Rightarrow$ $M(A,Z) = F_A + pZ + qZ^2$

$\Rightarrow$ $\left(\frac{\partial M}{\partial Z}\right)_A = p + 2qZ = 0$ at $Z = Z_A$, whence, we get

$$\Rightarrow \quad Z_A = -\frac{p}{2q} = \frac{(M_n - M_p + 4a_n)A}{2\left(a_cA^{2/3} + 4a_n\right)} \quad \Rightarrow \quad Z_A = A/\left(1.98 + 0.015A^{2/3}\right)$$

In most cases, the value of Z nearest to Z_A gives the actual stablest nucleus for a given A.

- **Shell model:**

The large binding energy of He-nucleus (α – particle) suggests that 2 protons and 2 neutrons form a stable nuclear configuration. Taking the clue from the chemical stability of closed electron sub-shells and shells in atoms, the physicsts enquired if nucleons too form similar closed sub-shells and shells in nuclei, i.e., if protons and neutrons in a nucleus are also arranged in some type of a shell structure.

- **Points in favour of shell model:**

The main points in favour of the idea of the shell model are

1. Just as inert gases, with 2, 10, 18, 36, 54, ... electrons, having closed shells show high chemical stability, nuclei with 2, 8, 20, 50, 82 and 126 nucleous-the so called magic numbers of the same kind (either proton or neutron) are particularly stable. (Nuclei with Z = N = a magic number are said to be doubly magic.)
2. The number of stable isotope (Z = constant) and isotones (N = constant) is larger with respective number of protons and neutrons equal to either of the magic numbers, e.g. Sn(Z = 50) has 10 stable isotopes, Ca(Z = 20) has 6; the biggest group of isotone is at N=82, then at N = 50 and N = 20.
3. The three naturally occurring radioactive series decay to the stable end product $^{208}_{82}Pb$ with Z = 82 and N = 126 indicating extra stable configuration of magic nuclei.
4. The neutron absorption cross-section is low for nuclei with N = magic numbers like 50, 82 and 126, indicating reluctance of magic nuclei to accept extra neutrons in their completely filled shells.
5. Isotopes like $^{17}_{8}O$, $^{87}_{36}K$ and $^{137}_{54}Xe$ are spontaneous neutron emittes when excited by preceding β – decay. These isotopes have N = 9, 15 and 83 respectively. i.e. N = (8 +1), (50+1) and (82+1). One can interpret this loosely bound neutron as a valence neutrons which the isotopes emit to assume some magic N-value for stability.
6. Electric quadrupole moment Q of magic nuclei is zero indicating spherical symmetry of nucleus for closed shells. When Z-value or N-value is gradually increased from one magic number to the next, Q increases from zero to a maximum and then decreases to zero at the next magic number.
7. The energy of α or β-particles emitted by magic radioactive nuclei is larger.

- **Salient features (Assumptions) of shell model:**

- In the shell model, therefore, each nucleon in the nucleus is considered as a single particle that moves independently of others in the time-averaged field of the remaining (A–1) nucleons, and is confined to its own orbit unperturbed by others. In terms of Schrodinger's equation, each nucleon thus moves in the same potential V(r) which may be taken as an average harmonic oscillator potential i.e.

  $V(r) = \frac{1}{2}kr^2$. Schrodinger equation then becomes $\left(-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}kr^2\right)\psi = E\psi$

  having the solution, $E_n = \left(N + \frac{1}{2}\right)\hbar\omega$

  where N = oscillator quantum number = 0, 1, 2, 3, ... so that in the harmonic oscillator model, all the energy states are equally spaced. The wave function ψ has both angular (orbital) and the radial part.

- Each nucleon is supposed to have an orbital angular momentum $|\vec{\ell}| = \sqrt{\ell(\ell+1)}\hbar$ where l = 0, 1, 2, 3,........, the nuclear orbital quantum number.

- Each nucleon has also spin angular momentum $|\vec{s}| = \sqrt{s(s+1)}\hbar$ where s=1/2 and behaves as an independent particle subject to Pauli's principle that no two identical nucleons can be in the same quantum state.

- The quantum mechanical rules for angular momenta dictate that total angular momentum $j\hbar$ formed by vector addition of orbital angular momentum $\ell\hbar$ and spin $s\hbar$ must be such that 'j' (total angular momentum quantum number) is restricted to the following two values: $j = \ell + \frac{1}{2}$ and $j = \ell - \frac{1}{2}$.

  The level $j = \ell + \frac{1}{2}$ corresponds to $\vec{s}$ and $\vec{\ell}$ parallel $(\uparrow\uparrow)$ to each other and $j = \ell - \frac{1}{2}$ to $\vec{s}$ and

n	l	j	Designation and no. of p or n to fill sub-levels	Progessive Total
1	0	1/2	$(1s_{1/2})^2$ 2	2
1	1	3/2	$(1p_{3/2})^4$	
1	1	1/2	$(1p_{1/2})^2$ 6	8
1	2	5/2	$(1d_{5/2})^6$	
2	0	1/2	$(2s_{1/2})^2$	
1	2	3/2	$(1d_{3/2})^4$ 12	20
1	3	7/2	$(1f_{7/2})^8$	
2	1	3/2	$(2p_{3/2})^4$	
1	3	5/2	$(1f_{5/2})^6$	
2	1	1/2	$(2p_{1/2})^2$	
1	4	9/2	$(1g_{9/2})^{10}$ 30	50
1	4	7/2	$(1g_{7/2})^8$	
2	2	5/2	$(2d_{5/2})^6$	
2	2	3/2	$(2d_{3/2})^4$	
3	0	1/2	$(3s_{1/2})^2$	
1	5	11/2	$(1h_{11/2})^{12}$ 32	82
1	5	9/2	$(1h_{9/2})^{10}$	
2	3	7/2	$(2f_{7/2})^8$	
2	3	5/2	$(2f_{5/2})^6$	
3	1	3/2	$(3p_{3/2})^4$	
3	1	1/2	$(3p_{1/2})^2$	
1	6	13/2	$(1i_{13/2})^{14}$ 44	126

Fig. Nucleonic sub-shells and shells

$\vec{\ell}$ anti-parallel $(\uparrow\downarrow)$ to each other. Empirically, it is found that the nuclear energy level with higher 'j' always lies below that with smaller j. So, $j = \ell + 1/2$ sub-level has a lower energy than $j = \ell - 1/2$ sub-level, the former giving a more tightly bound nucleonic state.

• Each sub-level can have a maximum of (2j + 1) nucleons of the same kind, for a given j. So it can house $(2j+1)$ protons and $(2j+1)$ neutrons.

• For instance, for $\ell = 0, j = \ell + \frac{1}{2} = \frac{1}{2}$ and the number of nucleons in the level $= 2j+1 = 2\times\frac{1}{2}+1 = 2$

and the state is designated along with n-value as $(1s_{1/2})^2$. Similarly, for $\ell = 1, j = \ell \pm s = \ell \pm \frac{1}{2} = \frac{3}{2}$ and $\frac{1}{2}$.

The number of nucleons in the two sub-levels are thus $2\times 3/2+1 = 4$ and $(2\times 1/2+1)$ =2 . The number sub-states are designed as $(1p_{1/2})^2$ and $(1p_{3/2})^4$ respectively. So the total number of nucleons in this level = 4 + 2= 6.

• **Success of shell model:**

1. It very well explains the existence of magic numbers and the stability and high binding energy on the basis of closed shells.
2. The shell model provides explanation for the ground state spins and magnetic moments of the nuclei. The neutrons and protons with opposite spins pair off so that the mechanical and magnetic moment cancel and the odd or left out proton or neutron contributes to the spin and magnetic moment of the nuclei as a whole.
3. Nuclear isomerism, i.e., existence of isobaric, isotopic nuclei in different energy states of odd-A nuclei between 39-49, 69-81, 111 to 125 has been explained by shell model by the large difference in nuclear spins of isomeric states are their A-values are close to magic numbers.

• **Limitations of the shell model:**

1. The model does not predict the correct value of spin quantum number I in certain nuclei, e.g., $^{23}_{11}Na$ where the predicted value is $I = 5/2$ while the correct value is $\frac{1}{2}$.
2. The following four stable nuclei $^{2}_{1}H$, $^{6}_{3}Li$, $^{10}_{5}B$ and $^{14}_{7}N$ do not fit into this model.
3. The model cannot explain the observed first excited states in even-even nuclei at energies much lower than those expected from single particle excitation. It also fails to explain the observed large quadrupole moment of odd-A nuclei, in particular of those having A-value far away from magic numbers.

• **Quadrapole moment:**

$$Q = -\frac{3}{5}\left(\frac{2I-1}{2I+2}\right)R^2$$

• In the even Z-odd N nucleus, the protons couple to J = 0 and neutrons to $J = J_n$. Thus the resultant angular momentum of the whole nucleus is $J = J_n$.

In an odd Z-even N nucleus $J = J_p$

In an even-even nucleus J = 0

For odd-odd nuclides the angular momentum J is given by Nordheim's rules can be stated as:

(a) Strong rule: For $j_p = l + \frac{1}{2}$ and $j_n = l - \frac{1}{2}$ or the reverse, $I(=J) = |J_p - J_n|$

(b) Weak rule: For both $j = l + \frac{1}{2}$ or both $j = l - \frac{1}{2}$

$|j_p - j_n| \le I(=J) \le j_n + j_p$

Above rules can also be written as:

For $N(j_p + j_n - l_p - l_n) = 0 \quad I = |j_p - j_n|$

and for $N = \pm 1$, I is either $|j_p - j_n|$ or $j_n + j_p$

odd-odd nuclei case

$\boxed{{}^{16}_{7}N}$

$Z = 7$

$\Rightarrow (1s_{1/2})^2\ (1p_{3/2})^4\ (1p_{1/2})^1$ here $l_p = 1, j_p = 1/2$

$N = 9$

$\Rightarrow (1s_{1/2})^2\ (1p_{3/2})^4\ (1p_{1/2})^2\ (1d_{5/2})^1$

$l_n = 2,\quad f_n = 5/2$

$N(j_p + j_n - l_p - l_n) = \frac{1}{2} + \frac{5}{2} - 1 - 2 = 0$

$I = |j_p - j_n| = \left|\frac{1}{2} - \frac{5}{2}\right| = 2$

- **Exceptional cases:**

1. For odd A nuclei which are heavier like ${}^{75}_{33}As$ or ${}^{61}_{28}Ni$ sometimes high spin state filled initially in outermost shell.

   ${}^{75}_{33}As = (1s)^2 \left(1p_{3/2}\right)^4 \left(1p_{1/2}\right)^2 \left(1d_{5/2}\right)^6 \left(2s_{1/2}\right)^2 \left(1d_{3/2}\right)^4 \left(1f_{7/2}\right)^8 \left(2s_{3/2}\right)^3 \left(1f_{5/2}\right)^2$

   ⇓ Should have been

   $\left(2s_{3/2}\right)^4 \left(1f_{5/2}\right)^1$

2. In case of $1d_{5/2}, 1f_{7/2}, 1g_{9/2}$ filled to extent of 3 nucleous. The nucleus outside .... do not pair up first.

   $\boxed{{}^{23}_{11}Na}$ $1d_{5/2}$ $\Rightarrow$ | p | p | p | → Actually happens

   | pp | p | | → Should have been this

3. For ${}^{207}_{82}Pb$ $\Rightarrow$ spin $= \frac{1}{2}$ instead of $I = \frac{3}{2}$ as predicted by shell model. Energetically favourable to have $3p_{1/2}$ state instead of $1Li_{1/2}$ → paring energy is larger for larger value.

- **Magnetic moment of nuclei.**

$$\vec{\mu}_j = \vec{\mu}_\ell + \vec{\mu}_s; \quad j = \ell \pm s = \ell \pm \frac{1}{2}; \quad \vec{\mu}_j = \frac{\mu_N}{\hbar}\left(g_\ell \vec{\ell} + g_s \vec{s}\right); \quad \text{where, } \mu_N = \frac{\ell\hbar}{zm_p}$$

Proton: $g_\ell = 1, g_s = g_p = 5.58;$

Neutron: $g_\ell = 0, g_s = g_n = -3.82$

$$\mu_j = \mu_N j\left[g_\ell\left(\frac{j(j+1)+\ell(\ell+1)-s(s+1)}{2j(j+1)}\right)+g_s\left(\frac{j(j+1)+s(s+1)-\ell(\ell+1)}{2j(j+1)}\right)\right]$$

$j = \ell+\frac{1}{2}$ $\quad \mu_j = \left(j-\frac{1}{2}\right)g_\ell + \frac{g_s}{2}$; $j = \ell-\frac{1}{2}$ $\quad \mu_j = \frac{j}{j+1}\left(j+\frac{3}{2}\right)g_\ell - \frac{g_s}{2}$

$j = \ell+\frac{1}{2}$ $\quad \mu_j = \left(j-\frac{1}{2}\right)g_\ell + \frac{g_s}{2}$; $j = \ell-\frac{1}{2}$ $\quad \mu_j = \frac{j}{j+1}\left\{\left(j+\frac{3}{2}\right)g_\ell - \frac{g_s}{2}\right\}$

Odd protons: $g_\ell = 1 \quad g_s = 5.58$

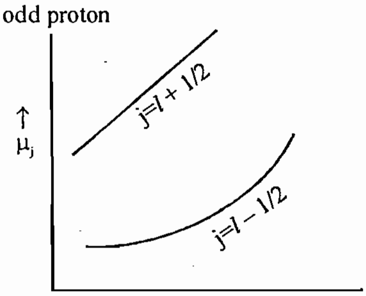
$j = \ell+\frac{1}{2}$ $\quad \mu_j = j+2.29$; $j = \ell-\frac{1}{2}$ $\quad \mu_j = j - \frac{2.29\,j}{j+1}$

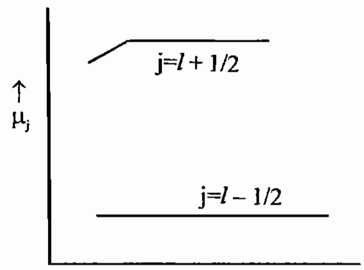
Odd neutron: $g_\ell = 0, g_s = -3.82$

$j = \ell+\frac{1}{2}$ $\quad \mu_j = -1.91;$

$j = \ell-\frac{1}{2}$ $\quad \mu_j = \frac{1.91\,j}{j+1}$

These relations taken in graph called Schmidt lines.





Problems:

$^{67}_{30}Zn$ $\quad n(p) = 30, \quad n(N) = 37$, one unpaired neutron.

$$\left(1s_{1/2}\right)^2\left(1p_{3/2}\right)^4\left(1p_{1/2}\right)^2\left(1d_{5/2}\right)^6\left(2s_{1/2}\right)^2\left(1d_{3/2}\right)^4\left(1f_{7/2}\right)^8\left(2p_{3/2}\right)^4\left(1f_{5/2}\right)^5$$

Ground state spin I = 5/2 and parity = $(-1)^3 = -1$.

$^{209}_{83}Bi$ $\quad n(p) = 83, \quad n(N) = 126$

unpaired $p \to \left(1h_{9/2}\right)^1$ $\quad I = \frac{9}{2}$ and parity=–1

$\Rightarrow \quad \mu_j = j+2.99$ for $j = \ell+\frac{1}{2} = j-2.29\frac{j}{j+1} \qquad j = \ell-\frac{1}{2}$

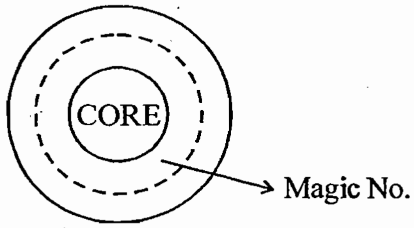
Here, $$\mu_j = \frac{9}{2} - \frac{2.29\left(\frac{9}{2}\right)}{\left(\frac{9}{2}+1\right)} = 2.63 \text{ units}$$

Quadrapole moment:

$$Q=-\frac{3}{5}\left(\frac{2I-1}{2I+2}\right)R^2=-\frac{3}{5}\left(\frac{9-1}{9+2}\right)\left(1.2\times(209)^{1/3}\times10^{-15}\right)^2=-0.176\text{ Barn}$$

Collective Model:

- Liquid drop model fails to explain spin, magnetic moment, magic number.
- Shell model can not explain deviation of magnetic moment from schmidt limit and large quadrupole moment of certain nuclei and fission and fusion.
- A nucleus consist of a central CORE and some extra nucleons around the CORE. CORE consists of number of nucleons equal to the magic number.



Nucleus

The loose nucleons outside the CORE have large centrifugal force on the CORE. If their number is large, they are able to deform the CORE. The total energy of deformed nucleus can be written as:

$$\boxed{E=E_N+E_V+E_R}$$

E_N – due the extra nucleons
E_V – Vibrational motion
E_R – Because of rotational motion.
The rotational energy is

$$\boxed{E_R\propto I(I+1)}$$ Where I = spin of the rotational state.

Solved Examples

Example-1: If the masses of ${}^{40}_{20}Ca$, ${}^{41}_{20}Ca$ and ${}^{39}_{20}Ca$ are 39.962589 a.m.u, 40962275 a.m.u and 38.970691 a.m.u. respectively, calculate the energy difference between $1d_{3/2}$ and $1f_{7/2}$ neutron shells.

Soln. Ca-39 has one neutron missing in the $1d_{3/2}$ shell; Ca-40 completes this shell and Ca-41 adds a neutron to $1f_{7/2}$ shell.

Therefore, B.E. of $1d_{3/2}$ neutron in Ca-40 is:

$$B_1=(M_{39}+M_n-M_{40})c^2=(38.97069+1.008665-39.962589)\times931\text{ MeV}$$
$$=0.016766\times931\text{ MeV}=15.61\text{ MeV}$$

B.E. of $1f_{7/2}$ neutron in Ca-40 is:

$$B_2=(M_{40}+M_n-M_{41})c^2$$
$$=(39.962589+1.008665-40.96225)\times931\text{ MeV}$$
$$=0.008979\times931\text{ MeV}=8.36\text{ MeV}$$

Therefore, the energy separation ΔE is the difference B_1-B_2

$\therefore\quad \Delta E=B_1-B_2=15.61-8.36=7.25\text{ MeV}$

Example-2: Predict (i) ground state spins, (ii) parities and (iii) the magnetic moments of $^{27}_{13}Al$, $^{33}_{16}S$ and $^{41}_{18}Ar$.

Soln.

$^{27}_{13}Al$: $13P-(1s_{1/2})^2(1p_{3/2})^2(1p_{1/2})^2(1d_{5/2})^5$

$14N-(1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^2(1d_{5/2})^6$

$\therefore \quad J=\frac{5}{2};\ \ell=2$

Parity = $(-1)^2 = +1$, even parity.

$\left[\ell=0,1,2,3,.....\text{for } s,p,d,f,.......\text{states respectively and parity}=(-1)^{\ell}\right]$

$^{33}_{16}S$: $16P-(1s_{1/2})^2(1p_{3/2})^2(1p_{1/2})^2(1d_{5/2})^6(2s_{1/2})^2$

$17N-(1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^2(1d_{5/2})^2(2s_{1/2})^2(1d_{3/2})^1$

$\therefore \quad J=3/2;\ \ell=2$

Parity $=(-1)^2=+1$, even parity

$^{41}_{18}Ar$: $18P-(1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^2\left(1d_{\frac{5}{2}}\right)^6\left(2d_{\frac{3}{2}}\right)^2$

$23N-\left(1s_{\frac{1}{2}}\right)^2\left(1p_{\frac{3}{2}}\right)^4\left(1p_{\frac{1}{2}}\right)^2\left(1d_{\frac{5}{2}}\right)^6\left(2s_{\frac{1}{2}}\right)^2\left(1d_{\frac{3}{2}}\right)^4\left(1f_{\frac{7}{2}}\right)^3$

$\therefore \ J=7/2;\ \ell=3$

Parity = $(-1)^3 = -1$, odd parity.

Example-3: Find the total angular momentum and parity for the ground state of $^{33}_{16}S$ nucleus, using the shell model, and also its electric quadrupole moment from the collective model.

Soln. $^{33}_{16}S$: $16P-\left(1s_{\frac{1}{2}}\right)^2\left(1p_{\frac{3}{2}}\right)^2\left(1p_{\frac{1}{2}}\right)^2\left(1d_{\frac{5}{2}}\right)^6\left(2s_{\frac{1}{2}}\right)^2$

$17N-\left(1s_{\frac{1}{2}}\right)^2\left(1p_{\frac{3}{2}}\right)^4\left(1p_{\frac{1}{2}}\right)^2\left(1d_{\frac{5}{2}}\right)^2\left(2s_{\frac{1}{2}}\right)^2\left(1d_{\frac{3}{2}}\right)^1$

The total angular momentum or spin of the nucleus $^{33}_{16}S$ is the total angular momentum of the last unpaired neutron.

$\therefore \quad J=3/2; \quad =2$ for d state

$\therefore \quad$ Parity $=(-1)^2=+1$, even parity

The electric quadrupole moment, Q of a nucleus with spin J is given, according to collective model, by

$$Q=-\frac{3}{5}\left(\frac{2J-1}{2J+2}\right)R_0^2$$

where $R_0=1.2\times A^{1/3}\ \text{fm}=1.2\times(33)^{1/3}\times10^{-15}\,\text{m}$ (since A=33 here)

$$\therefore \quad Q=-\frac{3}{5}\left\{\frac{\left(2\times\frac{3}{2}\right)-1}{2\times\frac{3}{2}+2}\right\}\times\left[1.2\times(33)^{1/3}\times10^{-15}\right]^2$$

$$=-0.0355\times10^{-28}\text{ m}^2=-0.0355\text{ barn} \qquad \left(\because 1\text{ barn}=10^{-28}\text{ m}^2\right)$$

Example-4: Compute the binding energy of the last proton in a nucleus of ^{12}C if the mass of ^{12}C-nucleus is 12.00052 was a.m.u. and the mass of the ^{11}B-nucleus is 11.01006 a.m.u. The mass of proton is 1.00759 a.m.u

Soln. On the addition of a proton, the ^{11}B-nucleus is converted into ^{12}C nucleus. The excess of mass of ^{12}C over ^{11}B is:

$$12.00052-11.01006=0.99046\text{ a.m.u.}$$

The mass of proton is 1.00759 a.m.u. Thus the proton, when added to the nucleus, suffers mass loss.

$$\text{Mass loss, } \Delta m=1.00759-0.99046=0.01713\text{ a.m.u.}$$

$$\therefore \quad \text{Equivalent energy, } \Delta E=0.01713\times931\simeq15.95\text{ MeV}$$

$$\therefore \quad \text{Binding energy of the last proton}=15.95\text{ MeV}$$

Example-5: Establish the relation $A\simeq 2Z$ for light nuclei using the semi-empirical mass formula, given $a_c=0.71\text{ MeV}, a_n=22.7\text{ MeV}, M\left({}^1_1H\right)=1.0078, M(n)=1.0086$ unit.

Soln. The mass M of a nucleus of mass number A and charge number Z according to the semi-empirical formula, is given by

$$M=Z M_H+(A-Z)M_n-\frac{1}{c^2}\left(a_v A-a_s A^{2/3}-a_c\frac{Z^2}{A^{1/3}}-\frac{a_n(A-2Z)^2}{A}\pm E_\delta\right)$$

For odd A nuclei, we have $E_\delta=0$

The mass of the most stable nucleus in a family of isobars is given by the condition $(\partial M/\partial Z)_A=0$.

$$\Rightarrow \quad (\partial M/\partial Z)_A=(M_H-M_n)c^2+2a_c\frac{Z}{A^{1/3}}-4a_n\frac{(A-2Z)}{A}=0$$

$$\Rightarrow \quad 2a_c\frac{Z}{A^{1/3}}-4a_n\frac{(A-2Z)}{A}=(M_n-M_H)c^2$$

$$\Rightarrow \quad 2Z\left(\frac{a_c}{A^{1/3}}+\frac{4a_n}{A}\right)=(M_n-M_H)c^2+4a_n$$

$$\Rightarrow \quad \frac{2Z}{A}\left(a_c A^{2/3}+4a_n\right)=(M_n-M_H)c^2+4a_n$$

$$\therefore \quad \frac{2Z}{A}\left\{\frac{a_c A^{2/3}}{4a_n}+1\right\}=\left\{\frac{(M_n-M_H)c^2}{4a_n}+1\right\}$$

$$\text{Or,} \quad Z=\frac{A}{2}\left\{\frac{1+(M_n-M_H)c^2/4a_n}{1+\frac{a_c}{4a_n}A^{2/3}}\right\}$$

Now, $a_c = 0.71\,\text{MeV},\ a_n = 22.7\,\text{MeV}$.

$\therefore\ a_c/4a_n = 0.0078$ and $(M_n - M_H)c^2/4a_n = 0.0082$

$$\therefore\quad Z = \frac{A}{2}\Big/\left\{\frac{1+0.0082}{1+0.0078\,A^{2/3}}\right\} \simeq \frac{A}{2},\ \text{for light nuclei.}$$

$\therefore\ A \simeq 2Z$, for light nuclei.

Example-6: Using the semi-empirical binding energy formula, calculate the binding energy of $^{40}_{20}Ca$.

Soln. The semi-empirical binding energy formula is:

$$B.E. = a_v A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} + \delta A^{-3/4}$$

where $a_v = 15.75\,\text{MeV}, a_s = 17.80\,\text{MeV}, a_c = 0.71\,\text{MeV};$

$a_n = 22.7\,\text{MeV}$ and δ=34 as A = even=40, Z = even=20

$$\therefore\quad a_v A = 15.75\times 40 = 630\,\text{MeV};\ a_s A^{2/3} = 17.80\times 40^{2/3} = 17.80\times 11.696 = 208.2\,\text{MeV}$$

$$a_c \frac{Z(Z-1)}{A^{1/3}} = \frac{0.71\times 20\times 19}{40^{1/3}} = \frac{0.71\times 20\times 19}{3.2} = 84.3\,\text{MeV}$$

$$a_n \frac{(A-2Z)^2}{A} = a_n \times 0 = 0$$

$$\delta A^{-3/4} = 34\times 40^{-3/4} = 34\times 0.063 = 2.14\,\text{MeV}$$

$$\therefore\quad B.E. = 630 - [208.2 + 84.3 - 2.14] = 339.64\,\text{MeV}$$

Example-7: Using the semi-empirical binding energy formula, find the atomic number of the most stable nucleus for a given mass number A. Hence explain which is the most stable among ^5_2He, ^6_2Be and ^6_3Li

Soln. Writing E_b for binding energy, $E_b = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$

where, $Z(Z-1) \simeq Z^2$ has been taken.

Now, for most stable nucleus, E_b must be maximum for a given mass number A, i.e.,

$$\left(\frac{\partial E_b}{\partial Z}\right)_{A=\text{const}} = -2a_c A^{-1/3} Z + 4a_n (A-2Z)A^{-1} = 0$$

$$\Rightarrow\quad 4a_n - 8a_n A^{-1} Z = 2a_c A^{-1/3} Z;\ Z\left(4a_n + a_c A^{2/3}\right) = 2a_n A$$

$$\therefore\quad Z = \frac{A}{2 + (a_c/2a_n)A^{2/3}} = \frac{A}{2 + 0.015\,A^{2/3}}$$

Substituting the values of a_c and a_n.

He, Be and Li are all light nuclei for which $0.015\,A^{2/3}$ is negligible and $Z = A/2$. This shows that of the three nuclei, ${}^{6}_{3}\text{Li}$ is most stable.

Example-8: Show, by way of computation, which nuclei you would expect to be more stable:

$${}^{7}_{3}\text{Li} \text{ or } {}^{8}_{3}\text{Li};\ {}^{9}_{4}\text{Be} \text{ or } {}^{10}_{4}\text{Be}$$

Soln. For a given mass number A, the atomic number Z of the most stable nucleus is:

$$Z = \frac{A}{2 + 0.015\,A^{2/3}}$$

Now, for $A = 7, Z = \dfrac{7}{2 + 0.015 \times 7^{2/3}} = \dfrac{7}{2 + 0.055} = \dfrac{7}{2.055} = 3.4$

for $A = 8, Z = \dfrac{8}{2 + 0.15 \times 8^{2/3}} = \dfrac{8}{2 + 0.060} = \dfrac{8}{2.060} = 3.88$

Since of the two Z-values, 3.4 is nearer to 3, the ${}^{7}_{3}\text{Li}$ nucleus is more stable.

Again, for $A = 9,\ Z = \dfrac{9}{2 + 0.015 \times 9^{2/3}} = \dfrac{9}{2 + 0.065} = \dfrac{9}{2.065} = 4.36$

for A = 10, $Z = \dfrac{10}{2 + 0.015 \times 10^{2/3}} = \dfrac{10}{2 + 0.067} = \dfrac{10}{2.067} = 4.80$

Since the two Z-values, 4.36 is nearer to 4, the ${}^{9}_{4}\text{Be}$ nucleus is more stable.

Example-9: Consider a nuclear level corresponding to a closed shell plus a single proton in a state with the angular momentum quantum numbers l and j. Of course $j = \ell \pm 1/2$. Let g_p be the empirical gyromagnetic ratio of the free proton. Compute the gyromagnetic ratio for the level in question, for each of the two cases $j = \ell + 1/2$ and $j = \ell - 1/2$

Soln. According to the shell model, the total angular momentum of the nucleons in a closed shell is zero, so is the magnetic moment. This means that the magnetic moment and angular momentum of the nucleus are determined by the only proton outside the closed shell.

As, $\mu_j = \mu_\ell + \mu_s,\ g_j j = g_\ell \ell + g_s S$

We have, $g_j j.j = g_\ell \ell.j + g_s s.j$

With $\ell.j = \frac{1}{2}\left(j^2 + \ell^2 - s^2\right) = \frac{1}{2}\left[j(j+1) + \ell(\ell+1) - s(s+1)\right]$

$$s.j = \frac{1}{2}\left(j^2 + s^2 - \ell^2\right) = \frac{1}{2}\left[j(j+1) + s(s+1) - \ell(\ell+1)\right]$$

$$g_j = g_\ell \frac{j(j+1) + \ell(\ell+1) - s(s+1)}{2j(j+1)} + g_s \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}$$

For proton, $g_\ell = 1, g_s = g_p$ the gyromagnetic ratio for free proton $(\ell = 0, j = s), s = \frac{1}{2}$. Hence we have

$$g_j = \begin{cases} \dfrac{2j-1}{2j} + \dfrac{g_p}{2j} & \text{for } j = \ell + 1/2 \\[2ex] \dfrac{1}{j+1}\left(j + \dfrac{3}{2} - \dfrac{g_p}{2}\right) & \text{for } j = \ell - 1/2 \end{cases}$$

The simplest model for low-lying states of nuclei with N and Z between 20 and 28 involves only f7/2 nucleons.

Example-10: Using this model in previous question, predict the magnetic dipole moments of $^{41}_{20}Ca$ and $^{41}_{21}Sc$.

Soln. ^{41}Ca has a neutron and ^{41}Sc has a proton outside closed shells in state $1f_{7/2}$. As closed shells do not contribute to the nuclear magnetic moment, the latter is determined by the extra-shell nucleons. The nuclear magnetic moment is given by $\mu = g_j \mu_n$

where 'j' is the total angular momentum, μ_n is the nuclear magneton. for a single nucleon in a central field, the g-factor is

$$g = \frac{(2j-1)g_\ell + g_s}{2j} \quad \text{for } j = \ell + \frac{1}{2}$$

$$g = \frac{(2j+1)g_\ell - g_s}{2(j+1)} \quad \text{for } j = \ell - \frac{1}{2}$$

For neutron, $g_\ell = 0, g_s = g_n = -\frac{1.91}{\frac{1}{2}} = -3.82$. As $\ell = 3$ and $j = \frac{7}{2} = 3 + \frac{1}{2}$, we have for ^{41}Ca.

$$\mu\left(^{41}_{20}Ca\right) = -\frac{3.82}{2j} \times j\mu_N = -1.91\,\mu_N$$

For proton, $g_\ell = 1, g_s = g_p = \frac{2.79}{1/2} = 5.58$. As $j = \frac{7}{2} = 3 + \frac{1}{2}$, we have for ^{41}Sc.

$$\mu\left(^{41}_{21}Sc\right) = \frac{(7-1)+5.58}{7} \times \frac{7}{2}\mu_N = 5.79\,\mu_N$$

Example-11: The band spectrum of U^{238} base on $(O)^+$ ground state. If the energy of $(2)^+$ state is 44.7 KeV. What is the spin and parity of state having energy 5.25 KeV.

Soln. Since $E_I \propto I(I+1)$

$$E_I = k\,I(I+1)$$

$$E_2 = k\,2(2+1) \quad \Rightarrow \quad E_2 = k\,2\times 3 \quad \text{... (2)}$$

equation (2) divided by (1)

$$\Rightarrow \quad \frac{E_2}{E} = \frac{6}{I(I+1)} \quad \Rightarrow \quad \frac{6}{I(I+1)} = \frac{44.7}{525}$$

$$\Rightarrow \quad I(I+1) = \frac{6\times 525}{44.7} \approx 72 \quad \Rightarrow I^2 + I - 72 = 0 \quad \Rightarrow I^2 + 9I - 8I - 72 = 0$$

$$\Rightarrow \quad I(I+9) - 8(I+9) = 0 \quad \Rightarrow (I-8)(I+9) = 0$$

$I = +8$ $\quad I \neq -\text{ive}$

Or, $\boxed{I = 8^{+}}$ $\quad I^P = 8^+$

Example-12: The rotation energy level of an even-even nucleus are (0^+) ground state, energy of $2^+ = 44$ KeV and higher state energies are 146, 304, 514 KeV respectively for three exited state. Assign

Soln. (i) $E_I \propto I(I+1) \Rightarrow E_I = KI(I+1)$

$E_2 = k\,2\times 3$

$E = k\,I(I+1)$

$$\frac{E_2}{E} = \frac{6}{I(I+1)} \Rightarrow \frac{6}{I(I+1)} = \frac{44}{146} \Rightarrow I(I+1) = \frac{6\times 146}{44} = 6\times 3.3 = 19.8 = 20$$

$\Rightarrow$ $I^2 + I = 20 \Rightarrow I^2 + I - 20 = 0 \Rightarrow I^2 + 5I - 4I - 20 = 0$

$\Rightarrow$ $I(I+5) - 4(I+5) = 0$

$\Rightarrow$ $(I-4) = 0 \Rightarrow \boxed{I=4}$

(ii) $$\frac{E_2}{E_3} = \frac{I(I+I)}{I(I+1)} \Rightarrow \frac{146}{304} = \frac{4(4+1)}{I(I+1)} \Rightarrow I(I+1) = \frac{20\times 304}{146} = \frac{3040}{73}$$

$\Rightarrow$ $I^2 + I = 41.64 \approx 42 \Rightarrow I^2 + I - 42 = 0 \Rightarrow I^2 + 7I - 6I - 42 = 0$

$\Rightarrow$ $I(I+7) - 6(I+7) = 0 \Rightarrow (I+7)(7-6) = 0 \Rightarrow \boxed{I=6}$

(iii) $$\frac{E_3}{E_4} = \frac{I(I+1)}{I(7+1)} \Rightarrow \frac{304}{514} = \frac{6\times 7}{I(I+1)}, \quad I^P = 8^+$$

Example-13: The difference in the coulomb energy between the mirror nuclei $^{49}_{24}Cr$ and $^{49}_{25}Mn$ is 6MeV. Assuming that the nuclei have a spherically symmetric charge distribution and that e^2 is approximately 1.0 MeV-fm. Find the radius of the $^{49}_{25}Mn$ nucleus.

Soln. Kinetic coulomb energy of uniformly charged sphere of radius R is

$$E_C = \frac{3}{5}\frac{z^2 e^2}{R}$$

According to the question, $E_{Cr} - E_{Mn} = 6MeV$

$$\frac{3}{5}\frac{(Z_{Cr}^2 - Z_{Mn}^2)e^2}{R} = 6MeV \Rightarrow R = \frac{3(Z_{Cr}^2 - Z_{Mn}^2)e^2}{6\times 5MeV}$$

$= 4.9$ fm $= 4.9 \times 10^{-15}$ fm

where $Z_{Cr} = 25$ and $Z_{Mn} = 24$

Example-14: For what elements should stable isobars exist for (a) A = 97 and (b) A = 80?

Soln. Semi-empirical mass formula for a $_zX^A$-atom is

$$_zM^A = M_N(A-Z) + M_H Z - a_v A + a_s A^{2/3} + a_c Z(Z-1)A^{-1/3} + a_a(A-2Z)^2 A^{-1} \pm a_p A^{-3/4}$$

Substituting values of constants in terms of MeV, we have

$$_zM^A = 939.6(A-Z) + 938.8Z - 14A + 13A^{2/3} + 0.60Z(Z-1)A^{-1/3}$$

$$+19(A-2Z)^2 A^{-1} \pm 34A^{-3/4}$$

For a most stable isobar

$$(\partial M / \partial N)_{z-zA} = -0.8 + 0.60(2Z_A - 1)A^{-1/3} - 76(A - 2Z_A)A^{-1} = 0$$

or $$Z_A = \frac{76.8 + 0.6A^{-1/3}}{152 + 1.2A^{2/3}} A$$

For odd value of A, there should be only one stable isobar, the neighbouring isobars are unstable and decay to stable isobar. for A = 97.

$$Z_A = \frac{76.8 + 0.6 \times (97)^{-1/3}}{152 + 1.2(97)^{2/3}}(97) = 42.1$$

Hence the most stable isobar is ${}_{42}Mo^{97}$.

By similar calculations for A = 80, we get $Z_A = 35.32$. Hence the most stable isobar will be ${}_{35}Br^{80}$. It is the case of Z-odd, N-odd. It has been discussed in theory that for even-A values, we get two parabolas due to the term $\pm a_V A^{-3/4}$. The odd Z values are associated with upper and even Z-values with the lower. The possible stable isobars with Z-even values are ${}_{34}Se^{80}$ and ${}_{36}Kr^{80}$.

Chapter-3: Radioactivity and α, β, γ Decays

- **Radioactivity:**

- **Laws of radioactive decay (Disintegration):**

(i) Atoms of every radioactive substance are constantly breaking into fresh radioactive products with the emission of α, β and γ rays.

(ii) The rate of breaking is not affected by external factors (temperature, pressure, chemical combination etc.) but is based upon probability concept and depends entirely on the law of chance i.e., the no. of atoms breaking per second at any instant is proportional to the number present.
If there are N atoms of any substance at time 't' and a number dN breaks in time dt

Then Rate of breaking $R = \frac{-dN}{dt} \propto N \quad \Rightarrow R = \frac{-dN}{dt} = \lambda N$

λ = radioactive constant = ratio of the amount of the substance which disintegrates in a unit time to the amount of substance present

$$\Rightarrow \quad \frac{dN}{N} = -\lambda dt \quad \Rightarrow \log_e N = -\lambda t + c$$

Now when $t = 0$, $\quad N = N_0 \quad \Rightarrow \log_e^{N_0} = C \quad \Rightarrow \quad \log_e \frac{N}{N_0} = -\lambda t$

Now, $\quad N = N_0 e^{-\lambda t} \quad \Rightarrow \quad \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$

N_0 = number of atoms at $t = 0$ and N = number of atoms left after time t.
i.e., $\quad N_0 - N \rightarrow$ Converted to daughter.

Growth of daughter $= N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0\left(1 - e^{-\lambda t}\right)$

At time $t = \frac{1}{\lambda}$, the no. of atoms of radioactive substance left behind is given by $N = N_0 e^{-\lambda \times \frac{1}{\lambda}} = \frac{N_0}{e}$

Hence the radioactive constant is also defined as the reciprocal of the time during which the number of atoms of a radioactive substance falls to 1/e of its original value.

- **Average life:** The atoms of a radioactive substance are constantly disintegrating and then the life of every atom is different. The atoms which disintegrate earlier have a very short life and others which disintegrates the end have a long life.

$$\text{Avgerage life} = \frac{\text{The sum of the lives of all atoms}}{\text{Total number of atom}}$$

We have $\frac{dN}{dt} = -\lambda N \quad \Rightarrow -dN = \lambda N dt = \lambda N_0 e^{-\lambda t} dt$

Total life of $-dN$ atom $= -tdN$
Since the possible life of any one of the total no. of atoms varies from 0 to ∞, the total life of all N_0 atoms is given by $\int_0^{N_0} -tdN$.

Average life, $\quad T_a = \frac{1}{N_0}\int_0^{N_0} -tdN = \frac{1}{N_0}\int_0^{\infty} \lambda N_0 e^{-\lambda t} t dt = \lambda \int_0^{\infty} e^{-\lambda t} t dt$

$$= \lambda\left[\frac{e^{-\lambda t}}{-\lambda}t - \int\frac{e^{-\lambda t}}{-\lambda}dt\right]_0^\infty = \lambda\left[\frac{e^{-\lambda t}}{-\lambda}t - \frac{e^{-\lambda t}}{\lambda^2}\right]_0^\infty = -\frac{1}{\lambda}\left[(\lambda t+1)e^{-\lambda t}\right]_0^\infty = \frac{1}{\lambda}$$

- **Half life:** We have $N = N_0 e^{-\lambda t}$

  At half life $\frac{N_0}{2} = N_0 e^{-\lambda T} \Rightarrow \frac{1}{2} = e^{-\lambda T} \Rightarrow 2 = e^{+\lambda T} \Rightarrow \lambda T = \log_e 2$

  $T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.6931}{\lambda}$

- **Relation between half life and average life:** $T_{1/2} = 0.693 T_a$
- **Units of Radioactivity:**

  SI unit Bacqurrel.
  1 Bq = 1 disintegration per second
  1 curie = 3.70×10^{10} disintegration per sec.
  1 Rutherford = 10^6 disintegration per second.

- **Activity or strength:**

  The activity or strength A_t of a radioactive sample at any instant 't' is thus defined as the number of disintegrations occurring in the sample in unit time at 't', that is,

  Activity, $A_t = \left|\frac{dN_t}{dt}\right| = \lambda N_t = \frac{0.693}{T}N_t$

  The activity per unit mass of a sample is called its specific activity.

  Differentiating the decay equation, $N_i = N_0 e^{-\lambda t}$, with respect to time 't'.

  $$\frac{dN_t}{dt} = -\lambda N_0 e^{-\lambda t}$$

  When t = 0, $\left(\frac{dN_t}{dt}\right)_0 = -\lambda N_0$. Hence from the relation (i) above

  $$\frac{dN_t}{dt} = \left(\frac{dN_t}{dt}\right)_0 e^{-\lambda t}$$

  Or, $A_t = A_0 e^{-\lambda t}$

  where, $A_t = dN_t/dt$ and $A_0 = (dN_t/dt)_0$

  A_t is called the activity or the strength of the sample and is proportional to the rate of disintegration.

- **Theory of Successive Disintegration:**

  Consider the equilibrium which is set up when radioactive body A disintegrate into a radioactive body B which disintegrated into a radioactive C. Let N_1 and N_2 be the no. of atom of A and B respectively present in the mixture after a time t and N_0 the no. of atoms of A at start $(t = 0)$. If λ_1 is the disintegration constant, then for A. $\Rightarrow N_1 = N_0 e^{-\lambda_1 t}$

  Every time an atom of A (called the parent) disappears, an atom B (called the daughter) is produced. Rate of formation of the daughter $B = \lambda_1 N_1$

Rate at which B decays $= \lambda_2 N_2$

Hence the Net rate at which B is produced $= \frac{dN_2}{dt} = (\lambda_1 N_1 - \lambda_2 N_2) = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_2 N_2$

$$\Rightarrow \quad \frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_0 e^{-\lambda_1 t}$$

Multiply both sides by $e^{\lambda_2 t}$

$$\Rightarrow \quad \frac{dN_2}{dt} e^{\lambda_2 t} + \lambda_2 N_2 e^{\lambda_2 t} = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} \Rightarrow \frac{d}{dt}\left[N_2 e^{\lambda_2 t}\right] = \lambda_1 N e^{(\lambda_2 - \lambda_1)t}$$

$$\Rightarrow \quad N_2 e^{\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 e^{(\lambda_2 - \lambda_1)t} + c$$

When $t = 0$, $N_2 = 0 \Rightarrow c = -\frac{\lambda_1}{\lambda_2 - \lambda_1} N_o$

$$\Rightarrow \quad N_2 e^{\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_o \left[e^{(\lambda_2 - \lambda_1)t} - 1\right] \Rightarrow N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right].$$

- **Secular (or permanent) Equilibrium:**

  If A, the parent is very long lived i.e., half life T_1 of A >> Half life T_2 of B, then $\lambda_1 << \lambda_2$. In such cases λ_1 can be neglected as compared to λ_2 and $e^{-\lambda_2 t}$ can be neglected as compared to $e^{-\lambda_1 t}$ provided t is very large. Thus above equation becomes $N_2 = N_o \frac{\lambda_1}{\lambda_2} e^{-\lambda_1 t}$

  If $T_{1/2}$ of A very large then $e^{-\lambda_1 t} \approx 1$ $\quad \Rightarrow N_0 = N_1$ or $N_2 = N_1 \frac{\lambda_1}{\lambda_2}$ $\quad \Rightarrow \lambda_1 N_1 = \lambda_2 N_2$.

- **Transient Equilibrium:**

  If $T_{1/2}$ of A is not very large as compared to the time during which we make observation, then $e^{-\lambda_1 t} \not\approx 1$ and λ_1 can't be neglected as compared to λ_2

  Thus, $\quad N_2 = N_o \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_1 t} = \frac{N_1 \lambda_1}{\lambda_2 - \lambda_1} \left(\because N_1 = N_o e^{-\lambda_1 t}\right) \Rightarrow \frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1}$

- **Radioactive displacement law:** During a radioactive disintegration the nucleus which undergoes disintegration is called parent-nucleus and that which remains after the disintegration is called a daughter nucleus

α **-decay:** When a radioactive nucleus disintegrates by emitting an α -particle, the atomic number decreases by two and mass number decreases by four. It can be represented as

$${}_Z X^A \rightarrow {}_{Z-2} Y^{A-4} + {}_2 He^4$$

Example: Radium $({}_{88}Ra^{226})$ is converted to radon $({}_{86}Rn^{222})$ due to α -decay as

$${}_{88}Ra^{226} \rightarrow {}_{86}Rn^{222} + {}_2 He^4$$

β **-decay:** When a radioactive nucleus disintegrates by emitting a β -particles, the atomic number increases by one and the mass number remains the same. It can be represented as

$${}_Z X^A \rightarrow {}_{Z+1} Y^A + {}_{-1} e^0$$

Example: Thorium ($_{90}Th^{234}$) is converted to protoactinium ($_{91}Pa^{234}$)

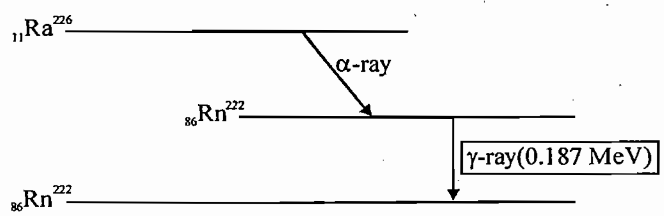
$$_{90}Th^{234} \rightarrow {}_{91}Pa^{234} + {}_{-1}e^{0}$$

At a time, either α or β-particles is emitted. Both α and β particles are not emitted during a single decay.

γ-ray: When a radioactive nucleus emits γ-ray, only the energy level of the nucleus changes and the atomic number and mass number remain the same.

During α or β decay, the daughter nucleus is mostly in the excited state. It comes to ground state with the emission of γ-rays.

Example: During the radioactive disintegration of radium ($_{88}Ra^{226}$) into Radon ($_{86}Rn^{222}$), gamma ray of energy 0.187 MeV is emitted, when radon returns from the excited state to the ground state as shown below:



Solved Examples

Example-1: A radioactive sample has its half-life equal to 60 days. Calculate its (i) disintegration constant, (ii) Its average life, (iii) the time required for 2/3 of the original number of atoms to disintegrate and (iv) the time taken for 1/4 of the original number of atoms to remain unchanged.

Soln. (i) Since $T_{1/2} = 60$ days, $\lambda = 0.693/T_{1/2} = 0.693/60 = 0.01155\ \text{day}^{-1}$

(ii) Since $\lambda = 0.01155\ \text{day}^{-1}$, $\bar{T} = 1/\lambda = 1/0.01155 = 86.58$ days

(iii) Number to be disintegrated $= \frac{2}{3}N_0$. So, the number to remain unchanged $= \frac{1}{3}N_0$

Thus, $N/N_0 = \frac{1}{3}$

Therefore, From the relation $N = N_0 e^{-\lambda t}$, we obtain $\frac{1}{3} = e^{-\lambda t} \Rightarrow \lambda t = \ell n\, 3$

$$\Rightarrow \quad t = \frac{\ell n\, 3}{\lambda} = \frac{2.3026 \times 0.4771}{0.01155} = 95.1 \text{ days}$$

(iv) Here $N/N_0 = \frac{1}{4}$. So, from the decay law, $\frac{1}{4} = e^{-\lambda t} \Rightarrow \lambda t = \ell n 4$

$$\Rightarrow \quad t = \frac{\ell n\, 4}{\lambda} = \frac{2.3026 \times 0.6021}{0.01155} = 120 \text{ days}$$

Example-2: (a) A radioactive substance disintegrates for a time equal to its average life. Calculate the fraction of the original substance disintegrated.

(b) The half-life of a radon is 3.82 days. What fraction of freshly prepared sample of radon will disintegrate in 10 days?

Soln. (a) Here $\bar{T} = t = \frac{1}{\lambda}$ Since, $\frac{N}{N_0} = e^{-\lambda t} = e^{-\lambda \bar{T}} = e^{-1} = 0.368$

Therefore, Fraction disintegrated = 1–0.368 = 0.632

(b) Here $T_{1/2} = 3.82$ days and t = 10 days. Therefore, we have $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.82} = 0.181\ \text{day}^{-1}$

Now, $\frac{N}{N_0} = e^{-\lambda t} = e^{-0.181 \times 10} = e^{-1.81} = \frac{3}{20}$

$\Rightarrow$ Fraction disintegrated $= 1 - \frac{3}{20} = \frac{17}{20}$

Example-3: The half-life of UX_1 is 24.1 days. How many days, after UX_1 has been isolated, will it take for 90% of it to change to UX_2?

Soln. Here $T_{1/2} = 24.1$ days $\Rightarrow$ $\lambda = 0.693/24.1 = 0.0287\ \text{day}^{-1}$

Amount disintegrated = 90/100 $\Rightarrow$ $N/N_0 = 10/100 = 1/10$

Since, $\frac{N}{N_0} = e^{-\lambda t}$, $\frac{1}{10} = e^{-0.0287t}$

$$\Rightarrow \quad t = \frac{\ell n\, 10}{0.0287} = \frac{2.3026 \times 1}{0.0287} = 80 \text{ days}$$

Example-4: The half-life of radioactive K-40 is 1.83×10^8 years. Find the number of β – particles emitted per sec per g. of K-40, assuming $\lambda = 1.2 \times 10^{-17}\ s^{-1}$, Avogrado number = 6.02×10^{23}.

Soln. Let N_t = Number of atoms of K-40 in 1g at t = $6.02 \times 100^{23}/40$

$$\Rightarrow \quad \left|\frac{dN_t}{dt}\right| = \lambda N_t = \frac{1.2 \times 10^{-17} \times 6.02 \times 10^{23}}{40} = 1.8 \times 10^5$$

So, the number of particles emitted per g of K-40 is 1.8×10^5

Example-5: It is observed that 3.67×10^{10} α – particles are emitted per g of Ra- 226. Calculate the half-life of Ra-226. Avogadro number = 6.023×10^{23}.

Soln. 1g of Ra-226 = $6.023 \times 10^{23}/226$ atoms of Ra-226. Of these 3.67×10^{10} disintegrate per sec. So, the decay constant.

$$\lambda = \frac{3.67 \times 10^{10}}{6.023 \times 10^{23}/226}\ s^{-1} \quad \Rightarrow \quad T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693 \times 6.023 \times 10^{23}}{3.67 \times 10^{10} \times 226} s = 1595 \text{ years}$$

Example-6: A large amount of radioactive material of half-life 20 days got spread in a room making the level of radiation 40 times the permissible level of normal occupancy. After how many days would the room be safe for occupation.

Soln. Let after 't' days the room would be safe for occupation. So, in 't' days, the activity would drop down to 1/40 of its initial value.

$\Rightarrow \quad \frac{N}{N_0} = \frac{1}{40}$. From the relation $\frac{N}{N_0} = e^{-\lambda t}, \frac{1}{40} = e^{-(0.693/20)t}$

$\left(\because T_{1/2} = 40 \text{ days}, \lambda = (0.693/20)\text{day}^{-1}\right) \Rightarrow \ell n\, 40 = \frac{0.693}{20} t$

$\Rightarrow \quad t = \frac{2.303 \times 20}{0.693} \log_{10} 40 = \frac{2.303 \times 20 \times 1.6021}{0.693} = 106.4$

Therefore, room would be safe for occupation after 107 days.

Example-7: Calculate the amount of Ra-226 in secular equilibrium with 1 kg of pure U-238, given the half-lives of Ra-226 and U-238 as 1620 years and 4.5×10^9 years respectively.

Soln. Let x in g be the required amount of Ra-226 in secular equilibrium with 1 kg of pure U-238. Now:

$$1 \text{ kg of pure U-238} = \frac{6.02\times10^{23}}{238}\times10^3 \text{ atoms of U}$$

$$x \text{ g of Ra-226} = \frac{6.02\times10^{23}}{226}\times x \text{ atoms of Ra}$$

The condition of secular equilibrium gives, $\frac{N_U}{(T_{1/2})_U} = \frac{N_{Ra}}{(T_{1/2})_{Ra}}$

$$\Rightarrow \quad \frac{6.02\times10^{23}\times10^3}{238\times4.5\times10^9} = \frac{6.02\times10^{23}\times x}{226\times1620}$$

$$\Rightarrow \quad x = \frac{226\times1620\times10^3}{238\times4.5\times10^9} = 34.18\times10^{-5} \text{ g} = 0.34 \text{ mg}$$

Example-8: The Half life of ${}_{92}U^{238}$ is 4.51×10^9 yrs. What %age of ${}_{92}U^{238}$ that existed 10^{10} years ago still survives.

Soln. $\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{4.5\times10^9}$

If N_0 = no. of atom of ${}_{92}U^{238}$ excited 10^{10} year ago $N = N_0$.

Now present, $N = N_0 e^{-\lambda t}$ where $t = 10^{10}$ yrs $\Rightarrow \frac{N}{N_0} = e^{\lambda t}$ or $\log_e \frac{N_0}{N} = \lambda t$

$\Rightarrow \quad 2.3026 \log_{10} \frac{N_0}{N} = \lambda t = \frac{0.693\times10^{10}}{4.51\times10^9} \quad \Rightarrow \log_{10}\frac{N_0}{N} = \frac{0.693\times10}{2.3026\times4.51} = 0.6673.$

$\Rightarrow \quad \frac{N_0}{N} = \text{anti}\log 0.6673 = 4.648 \quad \Rightarrow \frac{N}{N_0} = 0.215$

% of ${}_{92}U^{235}$ now present = $0.215 \times 100 = 21.5\%$

Example-9: The half life of a radioactive substance is 5 hr. What will be its one third life time?

Soln. $T_{1/2}$ = 5 hrs. $T_{1/2} = \frac{0.693}{\lambda} \quad \Rightarrow \lambda = \frac{0.693}{5} = 0.1386$ per hour also $\frac{N}{N_0} = e^{-\lambda t}$.

In this case $\frac{N}{N_0} = \frac{1}{3} \Rightarrow \frac{1}{3} = e^{-\lambda t} \Rightarrow 3 = e^{\lambda t}$

Hence, $\log_e 3 = \lambda t$ or $t = \frac{2.3026 \log_{10} 3}{0.1386} = 7.93$ hrs

Example-10: The activity of certain radio nuclide decreases to 15% of its original value in 10 days. Find its half life.

Soln. Let N_0 be the original no. of nuclei and N left behind after 10 days. If λ is the radioactive constant, then

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t} \Rightarrow \frac{15}{100} = e^{-\lambda 10}$$

$$\Rightarrow \log_e \frac{100}{15} = 10\lambda \Rightarrow \lambda = \frac{1}{10}\log_e \frac{100}{15} = \frac{1}{10} \times 2.3026 \log_{10} \frac{100}{15} = 0.1897$$

$$\Rightarrow T_{1/2} = \frac{0.6931}{\lambda} = \frac{0.6931}{0.1897} = 3.65 \text{ day}$$

- **α-decay:**

  - **Velocity of α-particles:**
    Let 'v' be the velocity of the α – particles, M be its mass, q the charge, 'r' the radius of the track and B the magnetic field, then $v = Bqr/M$.

  - **Range of α-particles:**
    The distance through which an α – particle travels in a specified material before stopping to ionise it is called the range of α-particles in that material. The range is thus a sharply defined ionisation path-length.

    The range depends on (i) The initial energy of the α – particle, (ii) the ionisation potential of the gas and (iii) the chances of collision between the α – particles and on the nature and the temperature and pressure of the gas. With increase of pressure, the range decreases; it increases if the temperature of the gas increased.

    The range R in standard air is proportional to v^3, i.e. $R \propto v^3 \Rightarrow R = av^3$
    The relation is known as the Geiger law, which is valid only in a limited velocity range.

    Since $R \propto v^3$ and the energy $E = \frac{1}{2}mv^2$, the range-energy relationship is

    $$R \propto E^{3/2} \Rightarrow R = bE^{3/2}$$

  - **Specific ionisation:**
    The number of ion-pairs formed per unit path-length at any point in the path of the α – particle is called specific ionisation and is symbolised by I.

    Since, $E \propto R^{2/3} \Rightarrow \frac{dE}{dR} \propto R^{-\frac{1}{3}} \propto \frac{1}{v}$

  - **Geiger-Nuttal law:**
    An important quantitative relation between the range R of the α – particles and the decay constant λ of the emitting nuclei was experimentally discovered by Geiger and Nuttall(1911) and is called the Geiger-Nutall law. The relation runs as:

    $$\ell n\, \lambda = A + B\, \ell n\, R$$

where A and B are constants having values different for different radioactive series.

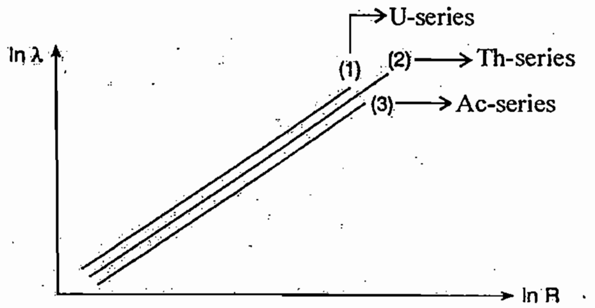


Figure: Variation of ln λ with *l*nR: Geiger-Nuttall law

Since $R \propto E^{3/2} \quad \Rightarrow \ell n\,\lambda = C + D\,\ell n\,E$
where C, D are two constants.

- **α-disintegration energy:**

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

The Q-value of the decay process is known as the α – disintegration energy which is the total energy released in the disintegration process and is given by $Q_\alpha = (M_X - M_\alpha - M_Y)c^2$ where M's are the masses of the particles and 'c' the velocity of light in vacuum.

For heavy nuclei, Q_α is positive, so the decay can occur spontaneously as it does. According to laws of conservation of momentum and energy.

$$0 = M_\alpha v_\alpha - M_Y v_Y$$

and $$Q_\alpha = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y v_Y^2$$

$\Rightarrow$ $$Q_\alpha = \frac{1}{2}M_\alpha v_\alpha^2\left(1 + \frac{M_\alpha}{M_Y}\right) = T_\alpha\left(1 + \frac{M_\alpha}{M_Y}\right)$$

Kinetic energy of ejected α – particles, $T_\alpha = \dfrac{Q_\alpha}{1 + M_\alpha / M_Y}$

$\Rightarrow$ $$Q_\alpha = T_\alpha \frac{[My + M_\alpha]}{M_y} = T_\alpha\left[\frac{A}{A-4}\right]$$

$\Rightarrow$ $$\boxed{T_\alpha = Q_\alpha\left[1 - \frac{4}{A}\right]} \quad \& \quad T_d = Q_\alpha - T_\alpha = \frac{4Q}{A}$$

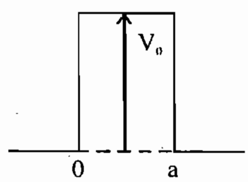
Where K.E. of α-particle = T_α

As $A \sim 200 \quad \Rightarrow \quad T_\alpha \sim Q_\alpha$

- Gamow explained the emission of α-particle from heavy nucleus using quantum mechanical tunnel effect and the transmission probability of α-particle is given as

$$\boxed{T \simeq e^{-2k_2 a}}$$

where $k_2 = \dfrac{\sqrt{2m(V_0 - E)}}{\hbar}$



- **β –decay:**

Let 'v' be the velocity of a given β – particle, B the magnetic flux density, 'm' the relativistic mass of the β – particle and 'r' the radius of the circular track.

Then $\frac{mv^2}{r} = Bev \Rightarrow r = \frac{mv}{Be} = \frac{m_0}{\sqrt{1-v^2/c^2}} \cdot \frac{v}{Be}$

- β – decay, **we write:**

$${}^{A}_{Z}X \to {}^{A}_{Z+1}Y + {}^{0}_{-1}e$$

The disintegration energy in β^- decay is:

$$Q_{\beta^-} = \left[M_n(A,Z) - M_n(A,Z+1) - m_e\right]c^2$$

$$= \left[M(A,Z) - Zm_e - M(A,Z+1) + (Z+1)m_e - m_e\right]c^2 \text{ (in terms of atomic mass)}$$

$$= \left[M(A,Z) - M(A,Z+1)\right]c^2$$

where M_n is the nuclear mass, M the atomic mass and m_e the mass of electron.

$Q_{\beta^-} > 0$, if M(A, Z) > M(A, Z + 1)

implying that β^- decay occurs only if the mass of the parent atom is greater than that of the daughter atom.

- β^+ - **decay:**

$${}^{A}_{Z}X \to {}^{A}_{Z-1}Y + {}^{0}_{+1}e$$

$$\Rightarrow \quad Q_{\beta^+} = \left[M_n(A,Z) - M_n(A,Z-1) - m_e\right]c^2$$

$$= \left[M(A,Z) - Zm_e - M(A,Z-1) + (Z-1)m_e - m_e\right]c^2$$

$$= \left[M(A,Z) - M(A,Z-1) - 2m_e\right]c^2$$

For β^+ - decay to occur: $Q_{\beta^+} > 0$, if $M(A,Z) > M(A,Z-1) + 2m_e$

i.e. the mass of the parent atom is greater than the daughter atom at least twice the electronic mass, i.e. 1.02 MeV.

- **Electron capture:**

$${}^{A}_{Z}Y + {}^{0}_{-1}e \to {}^{A}_{Z-1}Y$$

Therefore, disintegration energy, $Q_e = \left[M_n(A,Z) + m_e - M_n(A,Z-1)\right]c^2 - B_e$ where B_e is the binding energy of the electron to the orbit.

$$\Rightarrow \quad Q_e = \left[M(A,Z) - Zm_e + m_e - M(A,Z-1) + (Z-1)m_e\right]c^2 - B_e$$

$$= \left[M(A,Z) - M(A,Z-1)\right]c^2 - B_e$$

For electron capture to occur: $Q_e > 0$, if $M(A,Z) > M(A,Z-1) + B_e$

i.e. the mass of the parent atom is greater than that of the daughter atom by at least the binding energy of the electron.

- **Selection rule for β^+ - decay:**

If ℓ_β is odd, initial and final nuclei must have opposite parities (parity changes in these transitions); for even ℓ_β values the initial and final nuclei must have same parity (no change in parity). Furthermore, as in allowed transitions, the emission of leptons (electron and neutrino) in the singlet state (Fermi-selection rule) requires $\Delta I \le \ell_\beta$, whereas triplet-state (G-T selection rule) emission requires $\Delta I \le \ell_\beta + 1$. Thus selection rules for forbidden transitions are:

First forbidden – For these transitions $\ell_\beta = 1$ and parity changes.

Fermi-selection rules: $\Delta I = \pm 1, 0 \left(\text{except } 0 \to 0\right)$

Gamow Teller rules: $\Delta I = \pm 2, \pm 1, 0 \left(\text{except } 0 \to 0, \frac{1}{2} \to \frac{1}{2}, 0 \leftrightarrow 1\right)$

The examples are:

${}^{39}_{18}Ar \to {}^{39}_{19}K + \beta^-$	$(7/2^- \to 3/2^+)$	GT.
${}^{14}_{6}C \to {}^{14}_{7}N + \beta^-$	$0 \to 1$	F
${}^{38}_{17}Cl \to {}^{38}_{18}Ar + \beta^-$	$(2^- \to 2^+)$	F + GT mixed
${}^{139}_{56}Ba \to {}^{139}_{57}La + \beta^-$	$(7/2^- \to 7/2^+)$	F + GT mixed
${}^{143}_{59}Pr \to {}^{143}_{60}Nd + \beta^-$	$(7/2^+ \to 7/2^-)$	F + GT mixed
${}^{147}_{61}Pm \to {}^{147}_{62}Sm + \beta^-$	$(7/2^+ \to 7/2^-)$	F + GT mixed
${}^{85}_{36}Kr \to {}^{85}_{37}Rb + \beta^-$	$(9/2 \to 5/2)$	GT.
${}^{89}_{38}Sr \to {}^{89}_{39}Y + \beta^-$	$(5/2^+ \to 1/2^-)$	GT.
${}^{137}_{55}Cs \to {}^{136}_{56}Ba^* + \beta^-$	$(7/2^+ \to 11/2^-)$	GT.

Second forbidden. For these transition $\ell_\beta = 2$ and no change in parity.

Fermi-selection rules: $\Delta I = \pm 2, \pm 1$ $\left(\text{except } 0 \leftrightarrow 0\right)$

Gamow Teller rules: $\Delta I = \pm 3, \pm 2$ $\left(\text{except } 0 \leftrightarrow 2\right)$

The examples are:

${}^{137}_{55}Cs \to {}^{137}_{56}Ba + \beta^-$	$(7/2^+ \to 3/2^+)$	F + GT mixed
${}^{10}_{4}Be \to {}^{10}_{5}B + \beta^-$	$(0^+ \to 3^+)$	GT.
${}^{22}_{11}Na \to {}^{22}_{10}Ne + \beta^+$	$(3^+ \to 0^+)$	GT.

- **γ-decay:**

In it no change in Z and A. γ-ray photons are emitted when the nucleus jumps from an excited state to the lower state (after 10^{-14} sec). It is always not possible that when nucleus jumps from higher energy state to lower γ-photon will be emitted.

Sometimes the nucleus in higher energy state directly give energy to the atomic e^- and hence an e^- may come out rather than a γ-ray photon. This process is known as internal conversion and the e^- so emitted is called as conversion e^-.

- **Selection rule for γ-decay:**

Type	Symbol	Change in Angular momentum, L	Parity change
Electric dipole	E_1	1	Yes
Magnetic dipole	M_1	1	No
Electric quadrupole	E_2	2	No
Magnetic quadrupole	M_2	2	Yes
Electric octupole	E_3	3	Yes
Magnetic octupole	M_3	3	No
Electric 2^L – pole	E_L	L	[No for L even. Yes for odd]
Magnetic 2^L – pole	M_L	L	[Yes for L even. No for L odd]

$h_{11/2} \rightarrow d_{5/2}$

Example: $\left(\frac{11}{2}\right)^- \rightarrow \left(\frac{5}{2}\right)^+$, parity changes, so $\pi(p) = (-1)$

$$L = \left|\frac{11}{2} - \frac{5}{2}\right| \ldots\ldots \left|\frac{11}{2} + \frac{5}{2}\right| = 3, 4, 5, 6, 7, 8$$

So, allowed transitions are of following types: $\underset{E_3}{3}, \underset{M_4}{4}, \underset{E_5}{5}, \underset{M_6}{6}, \underset{E_7}{7}, \underset{M_8}{8}$

- **Pair production:**
  When a γ-ray photon passes near to the nucleus and if its energy is equal or more than 1.02 MeV, then due to the electromagnetic interaction with nucleus it breaks up and a pair of e^- and positron (Particle-antiparticle) is created. This process is known as pair creation (or production).

$$\underset{E_r}{\gamma} \rightarrow \underset{m_0c^2}{e^-} + \underset{m_0c^2}{e^+} \Rightarrow E_r \geq 2m_0c^2 \, (=1.02 \text{ eV})$$

- It is not possible in free space. (Because momentum is not conserve).
  When nucleus get energy from outside it goes to high energy state, stay there for short duration of 10^{-14} (**See Fig.1**) and jump to ground state by any of the two process.

(i) Nucleus jump to ground state by emitting of a γ-photon as shown in **Fig.2.** In this case the energy of emitted γ-photon is: $h\nu = E_2 - E_1$

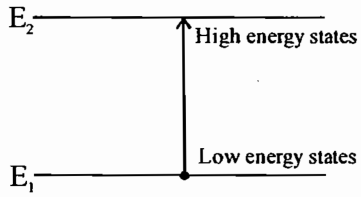
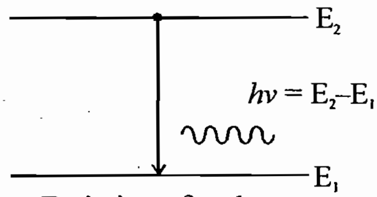
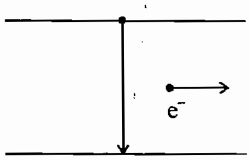


Fig.1.



Emission of γ-photon
Fig.2.



Internal conversion
Fig.3.

(ii) Nucleus in higher energy state has surplus energy $(E_2 - E_1)$, gives this energy directly to the one of the atomic e^- and jump to the ground state. The e^- which get the surplus energy, then comes out.

The energy lost by nucleus or gained by e^- is divided into two parts as given below.

$$E_2 - E_1 = E_e + B_e$$

B_e → B.E of e^- from nucleus.

E_e → K.E of emitted e^-

because the k-shell is near to nucleus, therefore more chances of emission of e– are from k-shell. Thus in this case equation (1) becomes.

$$E_2 - E_1 = E_{ek} + B_{ek}$$
$$E_2 - E_1 = E_{eL} + B_{eL}$$
$$E_2 - E_1 = E_{eM} + B_{eM}$$

→ For L shell e^-.
→ Also for M shell e^-.

Thus from (2). $E_{e_k} + B_{e_k} = E_{e_L} + B_{e_L} = E_{e_M} + B_{e_M} = \ldots\ldots\ldots$

As $\quad B_{e_k} > B_{e_L} > B_{e_M} \rightarrow E_{e_k} < E_{e_L} < E_{e_M}$

This is K.E. of emitted k-shell e^- is minimum because it is tightly bound from the nucleus. These emitted e^{-s} are called conversion e^{-s} and this process is called internal conversion.
The total decay probability per unit time (λ) from higher to lower energy state of a nucleus is thus given as

$$\lambda = \lambda_e + \lambda_r$$

λ_r → γ-photon emission probability

λ_e → Convenience e^- probability

And conversion e^- coefficient $(\alpha_e) = \dfrac{\lambda_e}{\lambda_r}$

Conversion e^- process or internal conversion takes place at the cost of γ-photon emission. More is the internal conversion less is the emission of γ-photon.

- **Internal pair creation:**
  The γ-ray photon are produced when nucleus jumps from an excited state to the ground state. As a result of it there is an electromagnetic interaction between nucleus and γ-ray photon and hence e^- - e^+ pair is created as a result of e.m. interaction of γ-ray photon with parent nuclei. This is known as internal pair creation. The difference in internal pair creation and pair creation is that pair creation is due to some other (external) nucleus and it is more probable for high z nucleus. While internal pair creation also taken place in low z medium.
- **Annihilation:**
  In this process particle and antiparticle destroy the existence of each other and energy is produced.

  Example: $e^- + e^+ \longrightarrow \gamma + \gamma$

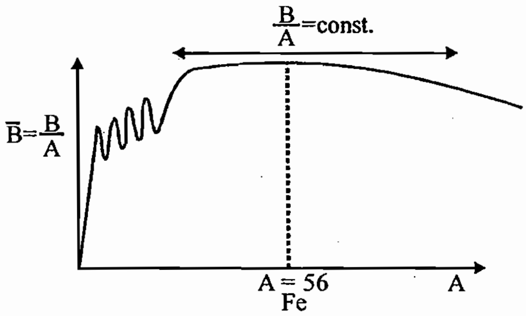
Here two γ-photons are emitted just to conserve momentum.

- **Interaction of γ-rays with matter:**
  When γ-rays falls on a matter, they get absorbed in matter and their intensity decreases, it has been found that $\dfrac{dI}{dx} \propto -I$ [x is the penetration depth] $\quad \Rightarrow \dfrac{dI}{dx} = -\mu I$

Solving it, we get $\boxed{I = I_0 e^{-\mu x}}$ where, μ = absorption coeff. (per meter), I_0 = Initial intensity (at x = 0)

$$\Rightarrow \quad \boxed{\mu = \frac{2.3026}{x}\log_{10}\frac{I_0}{I}}$$

Fission:



In the process of fission a heavy nucleus such as uranium splitts into two lighter nuclei. Since the lighter nuclei are about 1MeV/nucleon more tightly bound than the heavy nuclei, therefore there will be a energy conversion of 1MeV/Nucleon × 200 ≃ 200MeV.

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U^* \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$$

Mass of uranium $\Rightarrow$ $m(^{235}U) \equiv 235.04278u$

$m(^{1}_{0}n) \equiv 1.00866u$

$m(kr) \equiv 91.89719u$

$m(Ba) \equiv 140.9129u$

Calculate energy released

$$Q = [M(U) + m(^{1}_{0}n) - m(Ba) - m(Kr) - 3m(^{1}_{0}n)]c^2$$

$$Q = [235.04278 + 1.00866 - 140.9129 - 91.89719 - 3\times 1.0086]$$

$$Q = 0.21537u \times 931.5$$

$$Q = 200.6MeV$$

Example-1: In a nuclear reactor Plutonium $^{239}_{94}Pu$ is used as a fuel releasing energy by its fission into the isotopes of Ba and Sr through the reaction.

$$^{239}_{94}Pu + ^{1}_{0}n \rightarrow ^{146}_{56}Ba + ^{91}_{38}Sr + 3^{1}_{0}n$$

B.E. / Nucleon for each of these nuclei is given as

7.6 8.2 8.6 MeV/nucleon

Using this information find the number of such fission reactions / sec in a 100 MW reactor.

Soln. Power = energy released per sec ≡ 100 M watt

Energy released in one sec. from the reactor = 100 × 10^6 Joule/sec

$$= \frac{100MeV/\sec}{1.6\times 10^{-19}}$$

200 MeV energy released = 1 reaction

$$1 \text{ MeV energy released} = \frac{1}{200}$$

$$\frac{100}{1.6\times10^{-19}} \text{ MeV energy released} = \frac{1}{200}\times\frac{100}{1.6\times10^{-19}}$$

$$= \frac{100}{32}\times10^{18} = 3.9 \times 10^{18}$$

Example-2: An atomic bomb consisting of ^{235}U explodes and releases as energy of 10^{14} J. It is known that each Uranium-235 which undergoes fission releases 3 neutrons and about 200 MeV energy. Further only 20% of ^{235}U atoms in the bomb undergo fission. Find total number of neutrons released and mass of ^{235}U in the bomb.

Soln. Energy released = 10^{14}J

200 MeV energy is released with → 3 neutrons

1 MeV energy is released with = $\frac{3}{200}$ neutrons

10^{14} Joule energy is released = $\frac{3}{200}\times\frac{10^{14}}{1.6\times10^{-19}} = 9.375 \times 10^{24}$ neutrons

$\simeq 9.7 \times 10^{24}$ neutrons

1 atom is ^{235}U undergoes fission = 200MeV

6.02×10^{23} undergoes fission = $6.02 \times 10^{23} \times 200$MeV

235 gm of undergoes fission = $6.02 \times 10^{23} \times 200$ MeV

Let the mass of ^{235}U in the bomb is x kg

$x\times\frac{20}{100}$ kg of $^{235}U \rightarrow 10^{14}$J

$6.02 \times 10^{23} \times 200$ MeV energy released when amount of U consumed = 235gm

1 MeV energy released when amount of U consumed = $\frac{235\times10^{-3}\text{Kg}}{6.02\times10^{23}\times200\times10^{6}}$

10^{14}J MeV energy released when amount of U consumed = $\frac{235\times10^{14}\times10^{-3}}{6.02\times200\times1.6\times10^{6}\times10^{23}\times10^{-19}}$

$$x\times\frac{20}{100} = \frac{235\times10^{-3}\times10^{14}}{6.02\times200\times1.6\times10^{-19}\times10^{23}\times10^{6}}$$

$$x = \frac{100}{20}\times\frac{235\times10^{7}}{6.02\times200\times1.6\times10^{6}} \text{ kg}$$

$$\boxed{x = 6.1kg}$$

Explanation of nuclear fission on the basis of liquid drop model: In the nucleus, there is a competition between nuclear forces which are attractive and repulsive forces (electrostatic) they try to tear apart, the nucleus. In heavy nucleus there is a delicate balance between these two.

If we imagine the nucleus to be the drop of liquid then by absorbing a neutron or high energy proton the drop begins to vibrate, the shape of the nucleus changes rapidly back and forth from more elongates to spherical. When nucleus is streached to highly elongated shape, coulomb repulsion being long range will not be change appreciably while the nuclear force being short range is reduced significantly with sufficient streaching the centre becomes pinched off and nucleus will be splitted into two pieces.

Unequal distribution of A ≡ Asymmetric fission

Equal distribution of A ≡ Symmetric fission

The probability of symmetric fission is least, if the excitation energy of compound nucleus is increased, the symmetric fission becomes more probable.

Spontaneous fission: ${}_Z^AX \to {}_{Z_1}^{A_1}X_1 + {}_{Z_2}^{A_2}X_2$

$M(Z,A)-M(Z_1,A_1)-M(Z_2,A_2)>0 \Rightarrow Q>0$

The process will be spontaneous

Symmetric fission, $Z_1 = Z_2 = \frac{Z}{2},\ A_1 = A_2 = \frac{A}{2}$

Condition $M(Z,A)-2M\left(\frac{Z}{2},\frac{A}{2}\right)>0$

From semi-impirical mass formula

$$M(Z,A) = ZM_H + (A-Z)M_n - a_vA - a_sA^{2/3} + a_c\frac{z^2}{A^{1/3}} + a_s\frac{(A-2z)^2}{A}$$

$$M\left(\frac{Z}{2},\frac{A}{2}\right) = \frac{Z}{2}M_H + \frac{(A-Z)}{2}M_n - a_v\frac{A}{2} + \frac{a_s(A)^{2/3}}{2^{2/3}} + a_c\frac{z^2}{A^{1/3}}\frac{2^{1/3}}{4} + 2a_{as}\frac{(A-2z)^2}{4A}$$

$$\left\{\frac{(A-2z)^2}{A} = \frac{(A/2-2z/2)^2}{A/2} = \frac{2(A-2z)^2}{4A}\right\}$$

Condition

$$M(Z,A)-2M\left(\frac{Z}{2},\frac{A}{2}\right) = \left(1-\frac{2}{2^{2/3}}\right)a_sA^{2/3} + \left(1-\frac{2^{1/3}}{2}\right)\frac{z^2}{A^{1/3}}a_c + 0$$

$$= -0.26a_sA^{2/3} + 0.37a_c\frac{z^2}{A^{1/3}} > 0,\ 0.37a_c\frac{z^2}{A^{1/3}} > 0.26a_sA^{2/3}$$

$$\frac{Z^2}{A} > \frac{0.26}{0.37}\frac{a_s}{a_c},\ \frac{Z^2}{A} > 17.6$$

$\boxed{\frac{Z^2}{A} \simeq 18}$ Symmetric spontaneous fission

$\boxed{\frac{Z^2}{A} > 45 \simeq 50}$ Asymmetric spontaneous fission

Fussion: Two or more nuclei fuse together to produce a heavy nucleus.

e.g. ${}_1^2H + {}_1^2H \to {}_2^4He$ (deuteron is the nucleus of deuterium)

$Q = B(\alpha) - 2B({}_1^2H)$ $\quad f \to$ Binding fraction

$= 4\times f_\alpha - 2\times 2\times f_d$

$= 28.3MeV - 2\times 2.225MeV$

$\simeq 24MeV$

Energy released per nucleon (in fusion) $= \frac{24}{4} = 6$ MeV

While B.E. / nucleon = 1MeV (in fission)

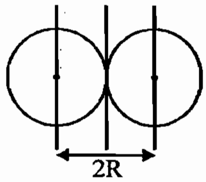
So B.E. / nucleon is more in fusion so fussion is more harmful.

Condition: Temperature should be high for fusion, B.E./N (Fussion) > B.E./N (Fission)

Mass of mass, fusion reaction gives more energy than fission reaction.

Example-1: Find the threshold temperature above which the nuclear reaction?

$^3_2He + ^3_2He \rightarrow ^4_2He + 2\,^1_1H + 12.86MeV$ can occur



(Use $\frac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-15}$ MeV met)

$E = \frac{3}{2}KT$ [$K = 1.38 \times 10^{-23}$ J/K]

$q_1 = +2e,\ q_2 = +2e$

$r = 2R \ = 2R_0 A^{1/3}$

$r = 2 \times 1.2 \times 10^{-15} \times (3)^{1/3}$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{3}{2}KT$$

$$\Rightarrow T = \frac{2}{3K}\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{2\times 4e^2}{3\times 1.38\times 10^{-23} J/K \times 4\pi\varepsilon_0 \times 2\times 1.2\times 10^{-15}\times (3)^{+1/3}}$$

$$= \frac{2\times 4\times 1.44\times 10^{-15}\times 1.6\times 10^{-19}\times 10^{6}}{3\times 1.38\times 10^{-23}\times 2\times 1.2\times 10^{-15}\times 1.44} = \frac{6.4\times 10^{10}}{4.968} = 1.288244\times 10^{10}$$

$$\boxed{T = 1.28\times 10^{10} K}$$

Solved Examples

Example-1: A radio-nuclide emits α – particles of energy 4.8 MeV and has a half-life 1620 years. Compute the velocity of α – particles and the probability of α – emission (Mass of α = 4.0026 a.m.u., radius of residual nucleus = 7.9×10^{-15} m)

Soln. The energy E_α of the α – particles is, non-relativistically, given by

$$E_\alpha = \frac{1}{2}mv^2 \quad \Rightarrow \ v = \sqrt{2E_\alpha / m}$$

Therefore, velocity, $v = \left(\frac{2\times 4.8\times 1.6\times 10^{-13}}{4.0026\times 1.66\times 10^{-27}}\right)^{1/2} = 1.516\times 10^{7}\ ms^{-1}$

Probability of α – emission, $P = \lambda/\omega$, where λ is the decay constant and ω, the frequency of hitting the barrier of the nucleus.

Now, $\lambda = \ell n\, 2/T_{1/2} = \frac{0.693}{1620\times 365\times 24\times 3600} = 1.356\times 10^{-11} s^{-1}$

And, $\omega = v/2R$, where R is the radius of the nucleus.

Therefore, $\omega = \frac{1.516\times 10^{7}\ ms^{-1}}{2\times 7.9\times 10^{-15} m} = 9.6\times 10^{20} s^{-1}$

Therefore, Probability of emission, $P = \frac{1.356\times 10^{-11}}{9.6\times 10^{20}} = 1.4\times 10^{-32}$

Example-2: Find the maximum height of the potentenail barrier for $\alpha-$penetration through ^{238}U nucleus. The radius of the residual nucleus is 9.3×10^{-13} cm

Soln. The Coulomb repulsion energy, $U(r)=Z\,ze^2/4\pi\varepsilon_0 r$. The maximum value corresponds to a distance r = R, the nuclear radius.

Therefore, $$B=\frac{1}{4\pi\varepsilon_0}\frac{Z\,ze^2}{R}=\frac{92\times2\times\left(1.6\times10^{-19}\right)^2}{4\times3.14\times8.85\times10^{-12}\times9.3\times10^{-15}}J$$

$$=\frac{92\times2\times2.56\times10^{-38}}{4\times3.14\times8.85\times10^{-12}\times9.3\times10^{-15}\times1.6\times10^{-13}}MeV\simeq28MeV$$

Example-3: Five groups of $\alpha-$particles are emitted $^{212}_{83}$Bi decays. Their energies are 6.08, 6.04, 5.76, 5.62 and 5.60, all being expressed in MeV. Calculate the $\alpha-$disintegration energies.

Soln. Using the relation, $Q_\alpha=\frac{A}{A-4}K_\alpha$, we have

$$(Q_\alpha)_1=\frac{212\times6.08}{208}=6.20\text{ MeV};\qquad (Q_\alpha)_2=\frac{212\times6.04}{208}=6.16\text{ MeV}$$

$$(Q_\alpha)_3=\frac{212\times5.76}{208}=5.87\text{ MeV};\qquad (Q_\alpha)_4=\frac{212\times5.62}{208}=5.73\text{ MeV}$$

$$(Q_\alpha)_5=\frac{212\times5.60}{208}=5.71\text{ MeV}$$

Example-4: A beam of monoenergetic $\gamma-$rays is incident on an Al-sheet of the thickness 10 cm. The sheet reduces the intensity of the beam to 21% of the original. Calculate the linear and mass absorption coefficients, given decay of Al = 27000 kg.m^{-3}.

Soln. Let I_0 be the original intensity and I the intensity on absorption by a thickness 't'.

Therefore, $I/I_0=e^{-\mu t}$, where μ = linear absorption coefficient. Here, $I/I_0=21/100$; $t=0.1$ m

Therefore, $0.21=e^{-0.1\mu}\;\Rightarrow\;-0.1\mu=n(0.21)=-1.56\Rightarrow\;\mu=15.6\text{ m}^{-1}$

Therefore, mass absorption coefficient, $\mu_m=\mu/p$

$$\mu_m=15.6/2700=5.78\times10^{-3}\text{ m}^2.\text{kg}^{-1}$$

Example-5: Show that $^{236}_{94}$Pu will spontaneously decay by $\alpha-$emission. Given: $M_{Pu}=236.046$ a.m.u, $M_v=232.037$ a.m.u. and $M_{He}=4.0020$ a.m.u.

Soln. The reaction is: $^{236}_{94}Pu\rightarrow{}^{232}_{92}U+{}^{4}_{2}He+Q$

Therefore, $$Q=\left(M_{Pu}-M_U-M_{He}\right)a.m.u.=236.046-232.037-4.002$$

$$=0.007a.m.u.=0.007\times931.5\text{ MeV}=6.52\text{ MeV}$$

Since Q is positive, Pu-236 will decay spontaneously by $\alpha-$emission.

Example-6: Calculate the energy of $\gamma-$rays emitted in the $\beta-$decays of $^{28}_{13}Al$. Given: The end-point energy $=2.81\,MeV$, $M\left(^{28}_{13}Al\right)=27.9819$ a.m.u., $M\left(^{28}_{14}Si\right)=27.9769$ a.m.u.

Soln. During the $\beta-$decay $^{28}_{13}Al\rightarrow{}^{28}_{14}Si$, $\gamma-$rays are being emitted. This implies that daughter nucleus $^{28}_{14}Si$ is in an excited state.

Substituting the atomic masses of parent and daughter

$$Q=(27.9819-27.9769)\text{a.m.u.}=0.005\text{ a.m.u.}=0.005\times931.5\text{ MeV}=4.65\text{ MeV}$$

But the end-point energy = 2.81 MeV. So, $\gamma-$ray energy = (4.65 –2.81) = 1.84 MeV.

Example-7: Which nuclide of each pair is unstable in the following cases? Indicate its mode of decay and energy released in each case.

(a) $^{7}_{3}Li(7.0182\text{ a.m.u.}),\ ^{7}_{4}Be(7.0192\text{ a.m.u.})$

(b) $^{13}_{6}C(13.0076\text{ a.m.u.}),\ ^{13}_{7}N(13.0100\text{ a.m.u.})$

(c) $^{19}_{9}F(19.0045\text{ a.m.u.}),\ ^{19}_{10}Ne(19.0080\text{ a.m.u.})$

(d) $^{34}_{15}P(33.9983\text{ a.m.u.}),\ ^{34}_{16}S(33.9978\text{ a.m.u.})$

As for each pair of isobars the atomic numbers differ by one, thus only $\beta-$decay or orbital electron capture is possible between them.

Neglecting the variation of the binding energy of the electron in different atoms and shells, the energy released in the $\beta^{-}-$decay is given by

$$E(\beta^{-})=\left[M_x(Z,A)-Zm_e-M_y(Z+1,A)+(Z+1)m_e-m_e\right]c^2$$

$$=\left[M_x(Z,A)-M_y(Z+1,A)\right]c^2$$

where M is the mass in atomic masses. Thus β^{-}-decay can take place if $M_x>M_y$. Similarly for $\beta^{+}-$decay, we have, $E(\beta^{+})=\left[M_x(Z,A)-M_y(Z-1,A)-2m_e\right]c^2$

Thus, $\beta^{+}-$decay can take place only if $M_x-M_y>2m_e=0.0011$ a.m.u.

For orbital electron capture, we have

$$E(EC)=\left[M_x(Z,A)-M_y(Z-1,A)\right]c^2-E_{B,\ell}$$

where $E_{B,\ell}$ represents the binding energy of an electron in the lth atomic shell, which is about 10 eV or 1.1×10^{-8} a.m.u. for K-shell

For pair (a), $\Delta m=M(Z-1,A)-M(Z,A)=0.001$ a.m.u. which is less than 0.0011 a.m.u. thus $^{7}_{4}Be$ is unstable against K-electron capture.

For pair (b), $\Delta m=0.0024\text{ a.m.u.}>0.0011\text{ a.m.u.}$ thus ^{12}N is unstable against $\beta^{-}-$decay and K – electron capture

For pair (c), $\Delta m=0.0035\text{ a.m.u.}>0.0011\text{ a.m.u.}$, thus $^{19}_{10}Ne$ is unstable against $\beta^{+}-$decay and K-electron capture.

For pair (d), $\Delta m=-0.0005$ a.m.u. thus $^{34}_{15}P$ is unstable against $\beta-$decay

Example-8: What are the expected types of gamma ray transitions between the following states of odd A – nuclei : $g_{9/2} \to p_{1/2}, f_{5/2}, h_{1/2} \to d_{5/2}, h_{11/2} \to d_{3/2}$?

Soln. We know that for a transition $I_i \to I_f$, the L of the emitted γ – rays is given by

$$|I_i - I_f| \le L \le (I_i + I_f)$$

Therefore, for transition $g_{9/2} \to p_{1/2} = \left(\frac{9}{2}\right)^+$ and $I_f = \left(\frac{1}{2}\right)^-$, hence possible L-values are 4 and 5.

Since the parity changes, the possible transitions are M4 and E5.

For transition $f_{5/2} \to p_{3/2}; I_i \left(\frac{5}{2}\right)^+$ and $I_f = \left(\frac{1}{2}\right)^-$, hence possible L-values are 1, 2, 3, 4. Since parity changes, the possible transitions are M1, E2, M3 and M4.

For transition $h_{11/2} \to d_{5/2}; I_i = \left(\frac{11}{2}\right)^-$ and $I_f = \left(\frac{5}{2}\right)^+$, hence possible L-values are 3, 4, 5, 6 7, 8..

Since parity changes, the possible transitions are E3, M4, E5, M6 , E7 and M8

For transition $h_{11/2} \to d_{3/2}; I_i = \left(\frac{11}{2}\right)^-$ and $I_f = \left(\frac{3}{2}\right)^+$, hence possible L-values are 4, 5, 6, 7. Since parity changes, the possible transitions are M4, E5, M6,E7.

Since the transition probability decreases rapidly with the increase of multiple order L. Hence the predominant decay modes are M4, M1, E3 and M4 in the above transitions.

Example-9: Examine the possibilities of isomeric transitions between nuclei $_4Be^7$ and $_3Li^7$.

Soln. The isotopic masses of these nuclei are $_3Li^7 = 7.016004mu$ and $_4Be^7 = 7.016929mu$

We know that for a β^- decay to be possible

$_ZM^A > _{Z+1}M^A + _{-1}e^0$ (Nuclear masses).

By adding z electron masses to both sides, the nuclear masses are changed into isotopic masses and the same condition appears now as

$_ZM^A > _{Z+1}M^A$ (Atomic masses).

Since mass of $_3Li^7$<mass of $_4Be^7$, hence β-decay is not possible.

We know that for a β^+-decay to be possible

$_ZM^A > _{Z-1}M^A + _{+1}e^0$ (Nuclear masses)

By adding z-electron masses to both sides, e-nuclear masses are changed into isotopic masses and the same condition appears as

$_ZM^A > _{Z-1}M^A + 2\,_{+1}e^0$ (Atomic masses).

Since the mass difference between given isotopes is 0.000925 mu = 0.860 MeV, which is less than the energy equivalent of two electron masses, i.e., 1.022 MeV, hence no β^+ – decay is possible.

The atomic binding energy of the K-electron in the Be^7-atom, however, is considerably less than the available energy 0.874 MeV, hence K-capture is energetically possible.

Example-10: Calculate the binding energies of the following isobars and their binding energies per nucleon.

$_{28}Ni^{64} = 63.9280mu$, $_{29}Cu^{64} = 63.9298mu$. Which of these would you expect to be β^- – active and how would it decay? Why?

Soln. (a) Since mass of neutron M_N = 1.008665mu and M_H = 1.007825 mu, hence for $_{38}Ni^{64}$,

$NM_N + ZM_H = 36 \times 1.008665 + 28 \times 1.007825 = 64.531mu$

$\therefore \quad \Delta M = 64.531 - 63.98 = 0.603mu$

Hence binding energy = 561.4 MeV and binding energy per nucleon is 8.77 MeV.

Similarly for $_{29}Cu^{64}$,

$\Delta M = NM_N + ZM_H - {_Z}M^A = 35 \times 1.008665 + 29 \times 1.007825 - 63.9298$

= 64.5272 – 63.9298 = 0.5974 mu.

Hence binding energy = 556.18 MeV and binding energy per nucleon is 8.7 MeV.

(b) These isobars are β^- emitters, i.e.,

$_{28}Ni^{64} \rightarrow {_{29}}Cu^{64} + \beta^-$ and $_{29}Cu^{64} \rightarrow {_{90}}Zn^{64} + \beta^-$

if the mass of the final isotope is less than of the corresponding initial isotope, i.e., if $_ZM^A > z\ {_{+1}}M^A$.

Since mass of Cu^{64} is greater than that of Ni^{64} and Zn^{64}, hence Ni^{64} is not a β^--emitter but Cu^{64} is a β^--emitter.

These isobars are β^+-emitter, if $_zM^A > z\ {_{-1}}M^A + 2\ {_{+1}}e^0$.

Since $M(Cu^{64}) - M(Ni^{64}) = 0.0018mu = 1.6758MeV$ is greater than the mass of two electrons, hence Cu^{64} is a β^+-emitter.

Example-11: Find the energy release, if two $_1H^2$ nuclei can fuse together to form $_2He^4$ nucleus. The binding energy per nucleon of H^2 and He^4 is 1.1 MeV and 7.0 MeV respectively.

Soln. Since no. of nucleons in $_2He^4$ nucleus is 4, hence B.E. for $_2He^4$ = 28.0 MeV, similarly B.E. for H^2 nucleus (combination of one proton and one neutron) = 2.2 MeV.

Mass of $_2He^4$ nucleus = 2 [Mass of proton] + 2 [Mass of neutron] – 28.0 MeV.

Mass of $_1He^2$ nucleus = [Mass of proton] + [Mass of neutron] – 2.2 MeV. In fusion reaction energy released.

$\Delta E = 2$ [Mass of $_1H^2$] – [Mass of $_2He^4$]

= $2[M_p + M_n - 2.2] - [2M_p + 2M_n - 28.0]$ = 23.6 MeV.

Example-12: Show from semi-empirical mass formula that for a given isotope the slope of the α-decay energy versus neutron number should be negative and the slope versus atomic number should be positive. Compute the slopes for N = 120, and Z = 82.

Soln. Assuming that the parent nucleus (Z, A) and the daughter nucleus (Z–2, A–4) are in their ground states before and after emission, we get

$E_\alpha = B(He^4) + B(Z-2, A-4) - B(Z, A)$,

where $B(He^4) = 28.3MeV$ and $B(Z,A) = a_vA - a_sA^{2/3} - a_cZ^2A^{-1/3} - a_a(A-2Z)^2A^{-1} \pm \delta$.

In α-decay, $(A–2Z)$ is constant and the slight variation of δ between parent and product nuclei is negligible. The finite difference $B(Z-2, A-4) - B(Z, A)$ can be well approximated by the corresponding derivatives with $dZ = -2$, $dA = -4$. Hence we get

$E_\alpha = 28.3 - 2\partial B/\partial Z - 4\partial B/\partial A$

$$= 28.3 - 4a_v + \frac{8}{3}\frac{a_s}{A^{1/2}} + 4a_c\frac{Z}{A^{1/3}}\left(1-\frac{Z}{3A}\right) - 4a_a\left(1-\frac{2Z}{A}\right)^2$$

Substitution of numerical values indicates that E_α is positive for all nuclides which are heavier than A~150.

The dependence of E_α on the neutron number N of the parent nucleus when Z is kept constant is same as that of $(\partial E_\alpha / \partial A)_{Z=const,}$ hence

$$\left(\frac{\partial E_\alpha}{\partial N}\right)_{Z=const} = -\frac{8}{9}\frac{a_3}{A^{4/3}} - \frac{4}{3}a_c\frac{Z}{A^{4/3}}\left(1-\frac{4Z}{3A}\right) - 16a_a\frac{Z}{A^2}\left(1-\frac{2Z}{A}\right)$$

This is a negative number for all values of Z and A, as every term is negative. For $Z = 82$, $N = 120$ (or $A = 202$), we get for a set of coefficients ($a_v = 14$, $a_s = 13$, $a_c = 0.60$ and $a_a = 19$ MeV),

$$\frac{\partial E_\alpha}{\partial N} = -\frac{8}{9}\times\frac{13}{202^{4/3}} - \frac{4}{3}\times 0.6\times\frac{82}{202^{4/3}}\left(1-\frac{4\times 82}{3\times 202}\right) - 16\times 19\times\frac{82}{202^2}\left(1-\frac{2\times 82}{202}\right)$$

$= -0.009747 - 0.02539 - 0.1149 = -0.15$ MeV per nucleon.

Similarly the second part of the question can be solved.

Example-13: What are the expected types of gamma ray transitions between the following states of odd A nuclei, $g_{9/2} \to p_{1/2}$, $f_{5/2} \to p_{3/2}$, $h_{11/2} \to d_{5/2}$, $h_{11/2} \to d_{3/2}$.

Soln. For transition $g_{9/2} \to p_{1/2}$, $J_a = \frac{9+}{2}$ and $J_b = \frac{1-}{2}$ hence possible L values are 4 and 5. Since the parity changes, the possible transitions are M4 and E5.

For transition $f_{5/2} \to p_{3/2}$, $J_a = \frac{5-}{2}$ and $J_b = \frac{3-}{2}$, hence possible L-values are 1, 2, 3, 4. Since parity does not change, the possible transition are M1, E2, M3 and E4.

For the transition $h_{11/2} \to d_{5/2}$, $J_a = \frac{11-}{2}$ and $J_b = \frac{5+}{2}$, hence possible L values are 3, 4, 5, 6, 7, 8. Since parity changes, the possible transitions are E3, M4, E5, M6, E7 and M8.

For the transition $h_{11/2} \to d_{3/2}$, $J_a = \frac{11-}{2}$ and $J_b = \frac{3+}{2}$, hence possible L-values are 4, 5, 6, 7. Since parity changes, the possible transitions are M4, E5, M6, E7.

Since the transition probability decreases rapidly with the increase of multiple order L. Hence the predominant decay modes are M4, M1, E3 and M4 in the above transitions.

Example-14: Calculate the fission rate for U^{235} required to produce 2 watt and the amount of energy that is released in the complete fissioning of 1/2 kg of U^{235}.

Soln. As we know that 200 MeV energy is released per fission of U^{235}

$\therefore$ Fission rate = 2 watt / 200 MeV per fission

$= 6.25 \times 10^{10}$ fission/sec

No. of U^{235} nuclei in 1/2 kg of U^{235} = $(0.5/235) \times 6.0247 \times 10^{26}$

On fissioning this number of U^{235} nuclei, the energy release will be

$= (0.5/235) \times 6.0247 \times 10^{26} \times 200$ MeV

$= 2.57 \times 10^{26}$ MeV $= 10^{10}$ kilocalories

Example-15: Calculate the excitation energy for U^{236*} and for U^{239*}. Estimate the rate of spontaneous fissioning of 1gm of U^{238}, given its half life $\sim 3\times10^{17}$ years.

Soln. The excitation energy $E_e = B(Z, A+1) - B(Z, A)$

For U^{236}

$$E_e = a_v(236-235) - a_s(236^{2/3} - 235^{2/3}) - a_c(92^2/236^{1/2} - 92^2/235^{1/2})$$

$$-a_a(52^2/236 - 51^2/235) + a_v(236^{-3/4} - 0)$$

$$= a_v - 0.1, a_s - 2.2a_c - 0.4a_a + 0.0167a_p = 6.8MeV$$

Similarly for U^{239}, E_c = 5.9 MeV.

Rate of spontaneous fissioning $dN/dt = \lambda N$, where

$N = 6.023 \times 10^{23}/235$ and $\lambda = \log_e 2/T_{1/2} = 0.693/3\times10^{17}\times3.15\times10^{7}$

$dN/dt = 2\times10^{-4}\ \text{sec}^{-1} = 0.7\ \text{hr}^{-1}$.

Example-16: If U 236 nucleus is fissioned by a neutron, two fission fragments of mass numbers 96 and 138 and two neutron are 235.12, 95.94, 137.95 and 1.009 amu. Calculate the amount of energy released.

Soln. Masses before fission = 235.12, 1.0089 = 236.126

Masses after fission = 95.94 + 137.95 + 2 X 1.0089

= 95.938 + 137.95 + 2.018 = 235.905 amu

Difference in mass = 236.127 – 235.905 = 0.221 amu

Energy released = 0.221 × 931 MeV = 206 MeV

Chapter-4: Nuclear Reactions

- **Compound Nucleus:**
The compound nucleus theory states that there are two steps involved in a nuclear disintegration and they are (i) the capture of the incident particle A by the target nucleus B forming a compound nucleus C^* in an excited state
(ii) the de-excitation of the compound nucleus C^* into a product nucleus D and the emission of a particle E or a $\gamma-$photon .

So, the nuclear reaction may be represented as $A+B \to C^* \to D+E$.

- Rutherford's first artificial nuclear transmutation may therefore be represented as

$${}^{14}_{7}N + {}^{4}_{2}He \to {}^{18}_{9}F^* \to {}^{17}_{8}O + {}^{1}_{1}H$$

- **Classification of nuclear reactions:**

(i) **Elastic scattering:** In elastic scattering, the same particles are scattered in different directions and there is no loss of energy. The residual nucleus is the same as the target nucleus and is left in the same state (ground state) as the latter so that it can be represented as X(x, y)X. An example is the scattering of neutrons by graphite: ${}^{12}C(n,n)\,{}^{12}C$. **Example:** ${}_{2}He^4 +_{79} Au^{197} \to {}_{79}Au^{197} + {}_{2}He^4$

(ii) **Inelastic scattering:** In inelastic scattering, the same particles are scattered in different directions with different energy, as there is loss of energy due to collision. The residual nucleus which is the same as target nucleus is left in an excited state so that the process can be represented as $X(x,y)X^*$. An example is the collision of fast neutrons with U-238.

(iii) **Radiative capture:** In radiative capture, the incident particle is absorbed or captured by the target nucleus to form the excited compound nucleus which disintegrates to produce one or more $\gamma-$photons and goes down to the ground state. The process may be represented as $X(x,\gamma)Y^*$.

Example: ${}_{92}U^{235} + {}_{0}n^1 \longrightarrow {}_{92}U^{236}$.

(iv) **Reaction of transformation:** Here the oncoming particle is retained in the nucleus and the compound nucleus emits a different particle so that the product nucleus is different from target nucleus, e.g. ${}^{9}Be(\alpha,n)^{12}C$

(v) **Photo-disintegration:** In photo-disintegration, a very energetic photon is absorbed by the target nucleus so that it is raised to an excited state and subsequently disintegrates. It can be represented as $X(\gamma,y)Y$. **Example:** ${}_{1}H^2 + \gamma \longrightarrow {}_{1}H^1 + {}_{0}n^1$.

(vi) **Stripping reactions:** In stripping reactions, one or more nucleons from the projectile are captured by the target nucleus, the remaining stripped nucleus is emitted in a different direction.

Example: ${}_{29}Cu^{63} + {}_{1}d^2 \longrightarrow {}_{29}Cu^{64} +_{1} H^1$

(vii) **Direct reactions:** A collision of an incident particle with the nucleus may immediately pull one of the nucleons out of the target nucleus and is called 'pick up reaction'.

Example: ${}_{1}H^1 + {}_{6}C^{13} \longrightarrow {}_{1}d^2 + {}_{6}C^{12}$

- **Conservation in Nuclear reactions:**
(i) Conservation of mass number
(ii) Conservation of atomic number
(iii) Conservation of energy (including mass-energy)
(iv) Conservation of linear momentum

(v) Conservation of angular momentum. (vi) Conservation of parity
(vii) Conservation of isotopic spin

- **Q-value and threshold energy of nuclear reaction:**

The law of conservation of energy and momentum imposes certain restrictions on the reactions. These restrictions are called the kinematic restrictions and this mathematical methods is known as kinematics.

Consider the nuclear reaction

$$x + X \longrightarrow y + Y$$

Where x, X, y and Y are the bombarding particle, target nucleus, outgoing particle and product nucleus respectively. It is assumed that the target nucleus is in rest. Since total energy is conserved in the nuclear reaction, therefore we get,

$$\left(m_x c^2 + E_x\right) + M_X c^2 = \left(E_y + m_y c^2\right) + \left(E_Y + M_Y c^2\right)$$

E_x, E_y and E_Y are the kinetic energies of respective particles.

Now the quantity $Q = E_y + E_Y - E_x \Rightarrow Q = \left(m_x + M_X - m_y - M_Y\right)c^2$

Where Q is called the Q-value of nuclear reaction.

(i) If Q is positive, the reaction is said to be exoergic (exothermic) and
(ii) If Q is negative, the reaction is called endoergic (endothermic).

The minimum K.E. required for incident particle (x) to start the nuclear reaction is called the threshold energy $(E_x)^{th}$. The relation between Q-values and threshold energy is:

$$E_x^{th} = -Q\frac{\left(m_y + M_Y\right)}{\left(M_Y + m_y - m_x\right)}$$

If $E_x^{th} = 0$ for exoergic or exothermic reactions i.e. these reaction are spontaneous process.

- **Mechanism of nuclear reactions:**
- **Transmutation by protons:**

(a) (p,α) reactions:

(i) $_3Li^7 + _1H^1 \rightarrow \left(_4Be^8\right)^* \rightarrow _2He^4 + _2He^4$

(ii) $_3Li^6 + _1H^1 \rightarrow \left(_4Be^7\right)^* \rightarrow _2He^3 + _2He^4$

(iii) $_4Be^9 + _1H^1 \rightarrow \left(_5B^{10}\right)^* \rightarrow _3Li^6 + _2He^4$

(iv) $_5B^{11} + _1H^1 \rightarrow \left(_6C^{12}\right)^* \rightarrow _4Be^8 + _2He^4$

(v) $_9F^{19} + _1H^1 \rightarrow \left(_{10}Ne^{20}\right)^* \rightarrow _8O^{16} + _2He^4$

(b) (p–n) Reaction:

(i) $_5B^{11} + _1H^1 \rightarrow \left(_6C^{12}\right)^* \rightarrow _6C^{11} + _0n^1$

(ii) $_8O^{18} + _1H^1 \rightarrow \left(_9F^{19}\right)^* \rightarrow _9F^{18} + _0n^1$

(iii) $_{11}Na^{23} + _1H^1 \rightarrow \left(_{12}Mg^{24}\right)^* \rightarrow _{12}Mg^{23} + _0n^1$

(iv) $_{28}Ni^{58} + _1H^1 \rightarrow \left(_{29}Cu^{59}\right)^* \rightarrow _{29}Cu^{58} + _0n^1$

(v) $_{29}Cu^{65} + _1H^1 \rightarrow \left(_{30}Zn^{66}\right)^* \rightarrow _{30}Zn^{65} + _0n^1$ (usually endoergic)

(c) (p, d) reaction:

(i) $_3Li^7 +_1H^1 \rightarrow _3Li^6 + _1H^2$ (ii) $_4Be^9 +_1H^1 \rightarrow _4Be^8 + _1H^2$

(d) Proton capture:

Compound nucleus in excited states come to ground state with γ – ray photon.

(i) $_3Li^7 +_1H^1 \rightarrow (_4Be^8)^* \rightarrow _4Be^8 + \gamma$ (ii) $_6C^{12} +_1H^1 \rightarrow (_7N^{13})^* \rightarrow _7N^{13} + \gamma$

(iii) $_7N^{14} +_1H^1 \rightarrow (_8O^{15})^* \rightarrow _8O^{15} + \gamma$ (iv) $_9F^{19} +_1H^1 \rightarrow (_{10}Ne^{20})^* \rightarrow _{10}Ne^{20} + \gamma$

(v) $_{13}Al^{27} +_1H^1 \rightarrow (_{14}Si^{28})^* \rightarrow _{14}Si^{28} + \gamma$ (vi) $_{24}Cr^{50} +_1H^1 \rightarrow (_{25}Mn^{51})^* \rightarrow _{25}Mn^{51} + \gamma$

If incident proton has an energy (> 20 MeV) the compound nucleus has sufficient excitation energy to permit the expulsion of two or more nucleons.

(2) Transmutation by Neutrons:-

Neutrons have no electric charge and can penetrate +vely charged nuclei without any experience of repulsive electrostatic force.

(a) (n–α) reaction:

With slow Neutrons

(i) $_3Li^6 + _0n^1 \rightarrow (_3Li^7)^* \rightarrow _1H^3 +_2He^4$ (ii) $_5B^{10} + _0n^1 \rightarrow (_5B^{11})^* \rightarrow _3Li^7 +_2He^4$

(iii) $_{13}Al^{27} + _0n^1 \rightarrow (_{13}Al^{28})^* \rightarrow _{11}Na^{24} +_2He^4$ (iv) $_7N^{14} + _0n^1 \rightarrow (_7N^{15})$

(v) $_5B^{11} +_2He^4$ (vi) $_3Li^7 +_2He^4 + _2He^4$

Capture of fast Neutrons - emission of α - particle are usually radioactive

(i) $_{11}Na^{23} +_0n^1 \rightarrow (_{11}Na^{24})^* \rightarrow _9F^{20} + _2He^4$ followed by $_9F^{20} \rightarrow _{10}Ne^{20} + \beta^-$

(ii) $_{13}Al^{27} +_0n^1 \rightarrow (_{13}Al^{28})^* \rightarrow _{11}Na^{24} + _2He^4$ followed by $_{11}Na^{24} \rightarrow _{12}Mg^{24} + \beta^-$

(b) (n, p) reaction: Proton in the nucleus is replaced by neutron mass no, does not change but change decreases by one unit.

(i) $_7N^{14} + _0n^1 \rightarrow (_7N^{15})^* \rightarrow _6C^{14} +_1H^1 + Q$ followed by $_6C^{14} \rightarrow _7N^{14} + \beta^- + E_{max}$

(ii) $_2He^3 + _0n^1 \rightarrow (_2He^4)^* \rightarrow _1H^3 +_1H^1 + Q$ followed by $_1H^3 \rightarrow _2He^3 + \beta^- + E_{max}$

(iii) $_{17}Cl^{35} + _0n^1 \rightarrow (_{17}Cl^{36})^* \rightarrow _{16}S^{35} +_1H^1 + Q$ followed by $_{16}S^{35} \rightarrow _{17}Cl^{35} + \beta^- + E_{max}$

With fast neutrons:

(i) $_{13}Al^{27} +_0n^1 \rightarrow (_{13}Al^{28})^* \rightarrow _{12}Mg^{27} + _1H^1$ (ii) $_{16}S^{32} +_0n^1 \rightarrow (_{16}S^{33})^* \rightarrow _{15}P^{32} + _1H^1$

(iii) $_{29}Cu^{65} +_0n^1 \rightarrow (_{29}Cu^{66})^* \rightarrow _{28}Ni^{65} + _1H^1$ (iv) $_{30}Zn^{64} +_0n^1 \rightarrow (_{30}Zn^{65})^* \rightarrow _{29}Cu^{64} + _1H^1$

(c) $(n-\gamma)$ Reaction:

(i) $_1H^1 + _0n^1 \rightarrow (_1H^2)^* \rightarrow _1H^2 + \gamma$ (ii) $_1H^2 + _0n^1 \rightarrow (_1H^3)^* \rightarrow _1H^3 + \gamma$

Product Nucleus is Radioactive:

(i) $_{45}Rh^{103} + {_0n^1} \rightarrow ({_{45}Rh^{104}})^* \rightarrow {_{45}Rh^{104}} + \gamma$ (ii) $_{49}In^{115} + {_0n^1} \rightarrow ({_{49}In^{116}})^* \rightarrow {_{49}In^{116}} + \gamma$

(iii) $_{92}U^{238} + {_0n^1} \rightarrow ({_{92}U^{239}})^* \rightarrow {_{92}U^{239}} + \gamma$ (iv) $_{29}Cu^{65} + {_0n^1} \rightarrow ({_{29}Cu^{66}})^* \rightarrow {_{29}Cu^{66}} + \gamma$

(v) $_{79}Au^{65} + {_0n^1} \rightarrow ({_{79}Au^{198}})^* \rightarrow {_{79}Au^{198}} + \gamma$

(d) **(n – d) and (n – t) reaction:** Bombardment of 90 MeV neutrons: $_7N^{14} + {_0n^1} \rightarrow {_6C^{12}} + {_1H^3}$

(e) **(n – 2n) reaction:** one neutron captured by nucleus and 2 neutrons are emitted. Q < 0 fast neutron are needed . Most cases residue nucleus unstable -followed by positron emission.

(i) $_6C^{11} + {_0n^1} \rightarrow ({_6C^{13}})^* \rightarrow {_6C^{11}} + 2\,{_0n^1}$ (ii) $_{19}K^{39} + {_0n^1} \rightarrow ({_{19}K^{40}})^* \rightarrow {_{19}K^{38}} + 2\,{_0n^1}$

(iii) $_{51}Sb^{121} + {_0n^1} \rightarrow ({_{51}Sb^{122}})^* \rightarrow {_{51}Sb^{120}} + 2\,{_0n^1}$

(f) **Neutron - Three or more particles:** Incident Neutron ~ 30 MeV sufficient energy to overcome coulomb Barrier, 3 neutrons or even 2 neutron and a proton are ejected from compound nucleus. Neutron (~ 100 MeV) Nuclei with moderate mass no. undergo spallation and those of high mass no. eg. Bi and Pb suffer fission probably accompanied by spallative.

- **(3) Transmutation by Deuteron:** High energy Deuteron.

(a) **$(d-\alpha)$ reaction**

(i) $_3Li^6 + {_1H^2} \rightarrow ({_4Be^8})^* \rightarrow {_2He^4} + {_2He^4}$ (ii) $_8O^{16} + {_1H^2} \rightarrow ({_9F^{18}})^* \rightarrow {_7N^{14}} + {_2He^4}$,

(iii) $_{10}Ne^{20} + {_1H^2} \rightarrow ({_{11}Na^{22}})^* \rightarrow {_9F^{18}} + {_2He^4}$

(iv) $_{12}Mg^{26} + {_1H^2} \rightarrow ({_{13}Al^{28}})^* \rightarrow {_{11}Na^{24}} + {_2He^4}$,

(v) $_{13}Al^{27} + {_1H^2} \rightarrow ({_{14}Si^{29}})^* \rightarrow {_{12}Mg^{25}} + {_2He^4}$

(b) **(d – p) Reaction: Isotope creation**

(i) $_6C^{12} + {_1H^2} \rightarrow ({_7N^{14}})^* \rightarrow {_6C^{13}} + {_1H^1}$ (ii) $_3Li^7 + {_1H^2} \rightarrow ({_4Be^9})^* \rightarrow {_3Li^8} + {_1H^1}$

(iii) $_{11}Na^{23} + {_1H^2} \rightarrow ({_{12}Mg^{25}})^* \rightarrow {_{11}Na^{24}} + {_1H^1}$ (iv) $_{15}P^{31} + {_1H^2} \rightarrow ({_{16}S^{33}})^* \rightarrow {_{15}P^{32}} + {_1H^1}$

(v) $_{48}Cd^{114} + {_1H^2} \rightarrow ({_{49}In^{116}})^* \rightarrow {_{48}Cd^{115}} + {_1H^1}$

(vi) $_{83}Bi^{209} + {_1H^2} \rightarrow ({_{84}Po^{211}})^* \rightarrow {_{83}Bi^{210}} + {_1H^1}$

(c) **(d – n) Reaction:**

(i) $_3Li^7 + {_1H^2} \rightarrow ({_4Be^9})^* \rightarrow {_4Be^8} + {_0n^1}$ (ii) $_4Be^9 + {_1H^2} \rightarrow ({_5B^{11}})^* \rightarrow {_5B^{10}} + {_0n^1}$

(iii) ${}_6C^{12} + {}_1H^2 \rightarrow \left({}_7N^{14}\right)^* \rightarrow {}_7N^{13} + {}_0n^1$

When two Deuterons Interact both the (d, n) and (d, p) reactions have been obtained.

$${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 \rightarrow {}_1H^3 + {}_1H^1 + 4.02\,\text{MeV}$$

$$\rightarrow {}_2He^3 + {}_0n^1 + 3.25\,\text{MeV}$$

- **Cross-section of nuclear reactions:**

One of the most important parameters in nuclear reactions is the reaction cross-section, symbolised by σ. It is quantitative measure of the probability of occurrence of a nuclear reaction.

Let a parallel beam of N monoenergetic particles be incident per unit time normally on a target foil of surface area A and thickness Δx, having 'n' nuclei per unit volume. Now, the number ΔN of nuclei in the foil undergoing nuclear reaction will be proportional to (i) the intensity of the beam and (ii) the number of target nuclei contained in the the foil.

But the intensity I of the beam = N/A and the number of nuclei in the foil $= nA.\Delta x$

Therefore, number of nuclei transmuted is $\Delta N \propto \frac{N}{A} nA.\Delta x$

$\therefore \quad \Delta N = \sigma N n.\Delta x = \sigma N n_1$ where σ = constant.

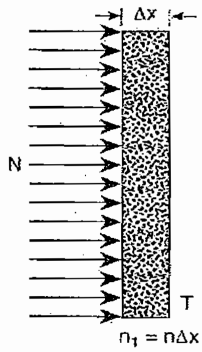


Figure: Nuclear reaction cross-section: bombardment of target foil.

where $n_1 = n.\Delta x$, the number of nuclei per unit area of the target foil used.

The constant σ is called the nuclear reaction cross-section.

Therefore, nuclear reaction cross-section, $\boxed{\sigma = \frac{\Delta N}{N n_1}}$

- The probability of the incident particle to interact with the target atom will be equal to

$$P = \frac{\text{number of interact particle}}{\text{number of incident particle}}$$

If dN is number of interacting particle and N is number of incident particles, then

$$P = \boxed{\frac{dN}{N} = \sigma\, n\, dx} \Rightarrow \int_{N_0}^{N} \frac{dN}{N} = \int_{z=z}^{z=0} \sigma\, n\, dx \Rightarrow \ell n\left[\frac{N}{N_0}\right] = -\sigma\, n\,(+x) \Rightarrow \frac{N}{N_0} = e^{-\sigma\, nx}$$

$$\boxed{N = N_0\, e^{-\sigma\, n\, x}}$$

Solved Examples

Example-1: Calculate the energy generated in MeV when 0.1 kg of ^{7}Li is converted to ^{4}He by proton bombardment. Given: masses of ^{7}Li, ^{4}He and ^{1}H in a.m.u. are 7.0183, 4.0040 and 1.0081 respectively.

Soln. $^7_3\text{Li} + ^1_1\text{H} = 2\,^4_2\text{He} + Q$

Masses of the reactants = (7.0183 + 1.0081) a.m.u. = 8.0264 a.m.u.
Masses of the products = (2×4.0040) a.m.u. = 8.0080 a.m.u.

Therefore, Difference = 0.0184 a.m.u.

Therefore, energy liberated when 7.0183 kg of Li are converted to He is 0.0184 kg. For 0.1 kg of Li, therefore, the amount of energy liberated is

$$E = \frac{0.1\times 0.184}{7.0183}\text{kg} = \frac{0.1\times 0.0184}{7.0183}\times\left(3\times 10^{8}\right)^{2}\text{J} = \frac{0.1\times 0.0184\times 9\times 10^{16}}{7.0183\times 1.6\times 10^{-13}}\text{MeV} = 14.74\times 10^{25}\text{ MeV}$$

Example-2: Calculate the binding energy in MeV of ^{4}He from the following data: Mass of ^{4}He = 4.003875 a.m.u; mass of ^{1}H = 1.008145 a.m.u. and mass of a neutron = 1.008986 a.m.u.

Soln. A ^{4}He-nucleus consists of 2 protons and 2 neutrons.

Mass of (2 protons + 2 neutrons) = 2(1.008145 + 1.008986) a.m.u.
= 4.034262 a.m.u.
Mass of ^{4}He-nucleus = 4.003875 a.m.u.

Therefore, Mass difference = 0.030387 a.m.u.

Binding energy = 0.030387 a.m.u. × c^2 = (931×0.030387) a.m.u. = 28.29 MeV

Example-3: Calculate the threshold energy for the nuclear reaction $^{14}\text{N}(n,\alpha)\,^{11}\text{B}$ in MeV

Soln. Masses of reactants = (14. 007550 + 1.008987) a.m.u. = 15.016537 a.m.u.
Masses of products = (4.003879 + 11.012811) a.m.u. = 15.016690 a.m.u.
Therefore,

$$Q = (15.016537 - 15.016690)\text{a.m.u.} = -0.000153\text{ a.m.u.} = -0.000153\times 931\text{ MeV} = -0.14\text{MeV}$$

$$\Rightarrow E_{th} = -Q\left[1+\frac{M(n)}{M(N)}\right] = 0.14\left(1+\frac{1.008987}{14.007550}\right)\text{MeV} = 0.14\left(1+\frac{1}{14}\right)\text{MeV} = 0.15\text{ MeV}$$

Example-4: Find the amount of energy in joule released during the process in which 0.001 kg of radium is converted into lead (masses: ^{226}Ra = 226.0955 a.m.u., ^{206}Pb = 206.0386 a.m.u. and α – particle = 4.003 a.m.u.)

Soln. In the conversion of 1 atom of ^{226}Ra into 1 atom of ^{206}Pb, 5α – particles are emitted in all.

Initial mass of ^{226}Ra = 226.0955 a.m.u.
Final mass of ^{206}Pb = 206.0386 a.m.u.

Therefore, Difference in masses = 20.0569 a.m.u.
Mass of 5α – particles = 20.0150 a.m.u.

Therefore, Mass converted into energy = 0.0419 a.m.u.

This is equivalent to an energy (0.0419 ×931) a.m.u. or 39 MeV (for 1 atom)

Now, 0.001 kg (= 1 g) of radium contains $\frac{6.023\times10^{23}}{226}$ Ra – atoms

Therefore, Total energy released, $E=\frac{6.023\times10^{23}\times39}{226}\text{MeV}=\frac{6.023\times10^{23}\times39\times1.6\times10^{-13}}{226}\text{J}$

$=16.63\times10^{9}\text{ J}$

Example-5: The Q-values in MeV of the following three reactions

$${}_1^2H+{}_1^2H\rightarrow{}_1^3H+{}_1^1H;\ {}_1^2H+{}_1^2H\rightarrow{}_2^3H+{}_0^1n\ \text{ and }\ {}_1^3H\rightarrow{}_3^2He+\beta^-$$

are 4.031, 3.265 and 0.0185 a.m.u. respectively. Calculate the mass difference between the neutron and the hydrogen atom from these data.

Soln. We have: $Q_1=4.031\text{ MeV}=\left(\text{mass of }2{}_1^2H-\text{mass of }{}_1^3H-\text{mass of }{}_1^1H\right)\times c^2$

$$Q_2=3.265\text{ MeV}=\left(\text{mass of }2{}_1^2H-\text{mass of }{}_2^3H-\text{mass of }{}_0^1n\right)\times c^2$$

$$Q_3=0.0185\text{ MeV}=\left(\text{mass of }{}_1^3H-\text{mass of }{}_2^3He\right)\times c^2$$

The mass of β^- is too small be taken into account and has been neglected

$$\Rightarrow\quad Q_1-Q_2+Q_3=\left(\text{mass of }{}_0^1n-\text{mass of }{}_1^1H\right)\times c^2$$

Therefore, Mass difference between neutron and H-atom is thus

$$Q_1-Q_2+Q_3=(4.031-3.265+0.0185)\times931.5\text{ MeV}=0.7845\text{ MeV}$$

$$\text{Mass difference}=0.7845/931\text{ a.m.u.}=0.000842\text{ a.m.u.}$$

Example-6: Calculate the Q-values of the following two reactions

(i) ${}_1^3H+{}_1^2H\rightarrow{}_2^4He+{}_0^1n$ and (ii) ${}_2^4He+{}_7^{14}N\rightarrow{}_8^{17}O+{}_1^1H$.

Given $M\left({}_1^3H\right)=3.0169982$, $M\left({}_1^2H\right)=2.0147361$, $M\left({}_2^4He\right)=4.0038727$, $M\left({}_0^1n\right)=1.0089832$, $M\left({}_7^{14}N\right)=14.003074$, $M\left({}_8^{17}O\right)=16.999133$ and $M\left({}_1^1H\right)=1.007825$ all in a.m.u. Indicate also if the reaction is exoergic or endoergic.

Soln. We have $Q=(\text{mass of reactants}-\text{mass of products})\times c^2=\Delta m.c^2$

(i) Here, $\Delta m=\left[\left\{M\left({}^3H\right)+M\left({}^2H\right)\right\}-\left\{M\left({}^4He\right)+M\left({}^1n\right)\right\}\right]$

$=\left[(3.0169982+2.0147361)-(4.0038727+1.0089832)\right]\text{a.m.u.}=0.0188784\text{ a.m.u.}$

$\Rightarrow Q=0.0188784\times931.48\text{ MeV}=+17.57\text{ MeV}$. Reaction is exoergic.

(ii) Here, $\Delta m=\left[\left\{M\left({}^4He\right)+M\left({}^{14}N\right)\right\}-\left\{M\left({}^{17}O\right)+M\left({}^1H\right)\right\}\right]$

$=\left[(4.003872+14.003074)-(16.999133+1.007825)\right]\text{a.m.u.}$

$=(18.0006946-18.006958)\text{a.m.u.}=-0.000012\text{ a.m.u.}$

$=-0.000012\times931.48\text{ MeV}=-0.11178\text{ MeV}$

So, the reaction is endoergic.

Example-7: Assume that 4 hydrogen nuclei are used to form a helium nucleus in the sun to provide the total energy to it. Calculate (i) the energy released when 1 gm-atom of hydrogen is fused to helium, (ii) how much hydrogen is to be converted to helium in the sun per second, given: mass of hydrogen nucleus = 1.00813 a.m.u, mass of He-nucleus = 4.00386 a.m.u, solar constant = 1.35 $kW.m^{-2}$. earth-sun distance = 1.5×10^{8} km and there is no other loss in the energy radiated by the sun.

Soln. (i) Mass of H-nucleus = 1.00813 a.m.u; Mass of He-nucleus = 4.00386 a.m.u.
Therefore, Mass difference on fusion = (4×1.00813 – 4.00386) a.m.u. = (4.03252 – 4.00386) a.m.u. = 0.02866 a.m.u.
Therefore, Energy released = 0.02866×931 MeV = 26.68 MeV. This amount is released due to fusion of 4 H-atom into helium.

Therefore, Energy released due to fusion of 1 gm-atom of H
$= (26.68 \times 6.02 \times 10^{23}/4)$ MeV $= 40.15 \times 10^{23}$ MeV

(ii) Energy released on fusion of 4 H-atom, $E = 0.02866$ a.m.u.$\times c^2 = 0.02866 \times 1.66 \times 10^{-27}$ kg
$= 0.02866 \times 1.66 \times 10^{-27} \times (3\times10^{8})^2$ J $= 4.28\times10^{-12}$ J

Therefore, Energy released per H-atom = $(4.28\times10^{-12}/4)$J = 1.07×10^{-12} J

Total energy radiated per sec. from the sun $= 1.35\times10^{3}\times4\pi\times\left(1.5\times10^{11}\right)^2 = 3.82\times10^{26}$ J

Therefore, number of H-atom required $= \dfrac{3.82\times10^{26}}{1.07\times10^{-12}} = 3.57\times10^{38}$

Therefore, required mass of hydrogen $= 3.57\times10^{38}\times1.00813\times1.66\times10^{-27}\text{ kg} = 5.97\times10^{11}\text{kg}$

Example-8: Consider the nuclear reaction $N^{14}(\alpha, p)O^{17}$. Mass of neutral atoms are given to be

$_1H^1 = 1.007825$ a.m.u. $\quad _2He^4 = 4.002603$ a.m.u.

$N^{14} = 14.003074$ a.m.u., $\quad O^{17} = 16.994131$ a.m.u.

determine the Q.value of the reaction in MeV. Calculate the threshold K.E. of α-particle for the reaction.

Soln. Reaction
$$_7N^{14} + {}_2He^4 \longrightarrow {}_1H^1 + {}_8O^{17}$$
X x y Y

$$Q = \left(M_X + m_x - M_Y - m_y\right)c^2 = -(0.001279 \text{ a.m.u.})\,c^2 = -0.001279\times931.5\text{ MeV}$$

$$E_x^{th} = -Q\left[\frac{m_y + M_Y}{m_y + M_Y - m_x}\right] = 1.535\,494\text{ MeV}$$

Example-9: Complete the following Nuclear Reactions

(i) $_{17}Cl^{35} + ? \rightarrow {}_{16}S^{32} + {}_2He^4$ (Ans. $_1H^1$)

(ii) $_5B^{10} + ? \rightarrow {}_3Li^7 + {}_2He^4$ (Ans. $_0n^1$),

(iii) $_3Li^6 + ? \rightarrow {}_4Be^7 + {}_0n^1$ (Ans. $_1H^2$)

(iv) $_{13}Al^{27} + {}_0n^1 \rightarrow {}_{12}Mg^{27} + ?$ (Ans. $_1H^1$),

(v) $_4Be^9 + {}_2He^4 \rightarrow ? + {}_0n^1$ (Ans. $_6C^{12}$)

(vi) $_3Li^7$ + proton → α – particle + ? (Ans. α – particle $_2He^4$)

(vii) Deuteron + Deuteron → proton + ? (Ans – tritium, $_1H^3$)

Example-10: Complete the following disintegration reaction by substituting the proper nuclide or particle for the question mark in each case.

(i) ${}_{14}Si^{28}(n,p)?$ **(ii)** ${}_{3}Li^{7}(p,\gamma)?$ **(iii)** ${}_{13}Al^{27}(\alpha,p)?$ **(iv)** ${}_{11}Na^{23}(p,?)\,{}_{10}Ne^{20}$

(v) ${}_{13}Al^{27}(?,\alpha)\,{}_{11}Na^{24}$ **(vi)** ${}_{5}B^{10}(?,\alpha)\,{}_{3}Li^{7}$ **(vii)** ${}_{11}Na^{23}(d,?)_{13}Mg^{24}$

Soln. (i) ${}_{13}Al^{28}$ (ii) ${}_{4}Be^{8}$ (iii) ${}_{14}Si^{30}$ (iv) ${}_{2}He^{4}$ (v) ${}_{0}n^{1}$ (vi) ${}_{0}n^{1}$ (vii) ${}_{0}n^{1}$

Example-11: Complete the following Reactions

(i) H^1 (n,γ) ?(ii) Li^7 (p, ?) Be^7 (iii) N^{14} (?, p) O^{17} (iv) ?(n, p) Na^{24}

Soln. (i) ${}_{1}H^{2}$, (ii) ${}_{0}n^{1}$, (iii) ${}_{2}He^{4}$ and (iv) ${}_{12}Mg^{24}$

Example-12: An α-particle with K.E. $T_\alpha = 7.0$ MeV is scattered elastically by an initially stationary Li^6 nucleus. Find the K.E. of the recoil nuclers if the angle of divergence of the two particles is $\theta = 60°$

Soln. Initial momentum of α – particle is $\sqrt{2mT_\alpha}\,\hat{i}$

Final momenta are respectively $\vec{p}_\alpha$ and $\vec{p}_{Li}$.

Conservation of momentum reads

$$\vec{p}_\alpha + \vec{p}_{Li} = \sqrt{2mT_\alpha}\,\hat{i} \quad \Rightarrow \quad p_\alpha^2 + p_{Li}^2 + 2p_\alpha p_{Li}\cos\theta = 2mT_\alpha \qquad ...(1)$$

Where θ is the angle between $\vec{p}_\alpha$ and $\vec{p}_{Li}$

Energy conservation: $$\frac{p_\alpha^2}{2m} + \frac{p_{Li}^2}{2M} = T_\alpha \quad \Rightarrow \quad p_\alpha^2 + \frac{m}{M}p_{Li}^2 = 2mT_\alpha \qquad ...(2)$$

Where, m = mass of α, M = mass of Li^6

Subtract (2) from (1) we get. $p_{Li}\left[\left(1-\frac{m}{M}\right)p_{Li} + 2p_\alpha\cos\theta\right] = 0 \qquad p_{Li} \neq 0$

$$\Rightarrow \; p_\alpha = -\frac{1}{2}\left(1-\frac{m}{M}\right)p_{Li}\sec\theta$$

Now p_α, p_{Li} are both +ve number (bring magnitudes of vectors) we must have $-1 \le \cos\theta < 0$ if $m < M$

Thus we write, $$\frac{p_{Li}^2}{2M}\left[1+\frac{M}{4m}\left(1-\frac{m}{M}\right)^2\sec^2\theta\right] = T_\alpha$$

Hence Recoil energy of Li nucleus is $$\frac{p_{Li}^2}{2M} = \frac{T_\alpha}{1+\frac{(M-m)^2}{4mM}\sec^2\theta}$$

Putting, $\theta = 120°$, we get recoil energy of Li = 6 MeV.

Example-13: Write missing symbols, denoted by x in the following nuclear reaction:

(a) $B^{10}(x,\alpha)Be^8 \to B^{10}(d,\alpha)Be^8$ (b) $O^{17}(d,n)x \to O^{17}(d,n)F^{18}$

(c) $Na^{23}(p,x)Ne^{20} \to Na^{23}(p,\alpha)Ne^{20}$ (d) $x(p,n)Ar^{37} \to Cl^{37}(p,n)Ar^{37}$

Example-14: What amount of heat is liberated during the formation of one gram of He^4 from Deuterium H^2? What mass of the coal with calorific value of 30 kJ/g is thermally equivalent to the magnitude obtained?

Soln. Reaction is $H^2 + H^2 \to He^4 + Q$

$Q = 2\Delta_{H^2} - \Delta_{He^4} = (0.02820 - 0.00260)c^2 = (0.02560\ \text{amu})c^2 = 23.8$ MeV.

Hence the energy released in the 1gm of He^4 is $\frac{6.023\times10^{23}}{4}\times23.8\times16.02\times10^{-13}\,\text{Joule} = 5.75\times10^{8}\,\text{kJ}$

This energy can be derived from $\frac{5.75\times10^{8}}{30000}\,\text{kg} = 1.9\times10^{4}\,\text{kg of coal}$

Example-15: Making use of the tables of atomic masses, determine the energies of the following reaction.

(a) $Li^7(p, n)\,Be^7$ (b) $Be^9\,(n, r)\,Be^{10}$ (c) $Li^7\,(\alpha, n)B^{10}$ (d) $O^{16}\,(d, \alpha)N^{14}$

Soln. (a) $Li^7\,(p, n)\,Be^7$ Energy of reaction is $Q = \left(M_{Li^7} + M_{Be^7}\right)C^2 + (M_P - M_n)C^2 = \left(\Delta_{Li_7} - \Delta_{Be^7}\right)C^2 + \Delta_p - \Delta_n$

$= [0.01601 + 0.00783 - 0.01693 - 0.00867]\ \text{amu} \times c^2 = -1.64$ MeV.

(b) $Be^9\,(n, \gamma)Be^{10}$: Mass of γ is taken as zero.

$Q = \left(M_{Be^9} + M_n - M_{Be^{10}}\right)c^2 = \left(\Delta_{Be^9} + \Delta_n - \Delta_{Be^{10}}\right)c^2 = (\ 0.01219 + 0.00867 - 0.01354\ )\ \text{amu} \times c^2$

$= 6.81$ MeV.

(c) $Li^7\,(\alpha, n)B^{10} : Q = \left(\Delta_{Li^7} + \Delta_\alpha - \Delta_n - \Delta_{B^{10}}\right) = (\ 0.01601 + 0.00260 - 0.00867 - 0.01294\ )\ \text{amu} \times c^2$

$= -\ 2.79$ MeV

(d) $O^{16}\,(d, \alpha)N^{14} : Q = \left(\Delta_{O^{16}} + \Delta_\phi - \Delta_\alpha - \Delta_N^{14}\right)c^2$

$= (\ -0.00509 + 0.01410 - 0.00260 - 0.00307\ )\ \text{amu} \times c^2 = 3.11$ MeV

Example-16: Find the velocity with which the products of the reaction $B^{10}\,(n, \alpha)Li^7$ come apart; the reaction proceeds via interaction of very slow neutrons with stationary Boron nuclei.

Soln. We have : $B^{10}\,(n, \alpha)Li^7$ $\quad Q = \left(\Delta_{B^{10}} + \Delta_n - \Delta_\alpha - \Delta_{Li^7}\right)c^2$

$= (0.01294 + 0.00867 - 0.00260 - 6.01601\ )\ \text{amu} \times c^2 = 2.79$ MeV.

Since the incident neutron is very slow and B^{10} is stationary, the final total momentum must also be Zero. So the reaction products must emerge in opposite direction. If their speeds are respectively, v_α and V_{Li}

then $4V_\alpha = 7V_{Li}$ and $\frac{1}{2}\left(4V_\alpha^2 + 7V_{Li}^2\right)\times1.672\times10^{-24} = 2.79\times1.602\times10^{-6}$

So, $\frac{1}{2}\times4V_\alpha^2\left(1+\frac{4}{7}\right) = 2.70\times10^{18}\,\text{cm}^2/\text{s}^2 \quad \Rightarrow V_\alpha = 9.27\times10^{6}\,\text{m/sec}$

$\Rightarrow \quad V_{Li} = 5.3 \times 10^6$ m/sec.

Example-17: Protons striking a stationary Lithium target activate a reaction $Li^7\,(p, n)Be^7$. At what value of the proton KE can the resulting neutron be stationary?

Soln. $Q = -1.64$ MeV.

Conservation of Momentum: $p_p = p_{Be}$ (since Initial Li and final neutron are both at rest)

$$\Rightarrow \frac{p_p^2}{2m_p} = \frac{p_{Be}^2}{2m_{Li}} + 1.64 \qquad \Rightarrow \frac{p_p^2}{2m_p}\left(1-\frac{m_p}{m_{Be}}\right) = 1.64$$

Hence, $T_p = \frac{p_p^2}{2m_p} = \frac{7}{6}\times1.64\ \text{MeV} = 1.91\ \text{MeV}$

Example-18: A beam of proton (of a certain energy) equivalent to a current of 1.6 mA is incident uniformly on a ^{56}F target containing 10^{22} atom per m². S0 that the following reaction take places, $p + ^{56}Fe \longrightarrow n + ^{53}Co$

If cross section for the reaction is 1 barn. Calculate the number of neutrons produced per sec.

Soln. $^{56}Fe + p \longrightarrow ^{53}Co + n$ Given : $I = 1.6\text{ mA} = 1.6\times10^{-3}\text{ A}$

$\Rightarrow q = it \quad \Rightarrow \frac{q}{t} = i$ Since, $q = ne \Rightarrow \frac{ne}{t} = i$

Number of incident particle per unit time is $N_0 = \frac{n}{t} = \frac{i}{e} = \frac{1.6\times10^{-3}}{1.6\times10^{-19}} = 10^{16}/\text{sec}$

No. of particle which interact with target is $N_0 - N = N_0 - N_0 e^{-\sigma nx} = N_0\{1 - e^{-\sigma nx}\}$

where $nx = 10^{22}$ atom per m².

$\Rightarrow N_0 - N = 10^{16}\left\{1 - e^{-\{10^{-6}\}}\right\} = 10^{10}$ = Number of neutron produced per second.

Example-19: A 16 μA beam of a particle having cross-sectional area of 10^{-4} m² is incident on a Rh target of thickness

1 μm. This produces neutrons through the reaction $\alpha + ^{100}Rh \longrightarrow Pd + 3n$

(1) Number of α particle hitting the target per second is:

(a) $0.5\times10^{+14}$ (b) 1×10^{14} (c) 2×10^{20} (d) 4×10^{20}

Soln. $\frac{n}{t} = \frac{i}{2e} = \frac{16\times10^{-6}}{2\times1.6\times10^{-19}} = 0.5\times10^{14}$

Example-20: Consider the decay process $\tau^- \rightarrow \pi^- + \nu_\tau$ in the rest frame of the τ^-. The masses of τ^-, π^- and ν_τ are μ_τ, μ_π and zero respectively. Find the energy and velocity of π^-.

Soln. Kinetic energy $= -Q\left[\frac{\text{Sum of masses of all particles involved in the reaction}}{2\times\text{mass of target}}\right]$

Where $Q = [m_i - m_f]c^2 = [m_\tau - m_\pi - 0]c^2$

Kinetic energy $= -(m_\tau - m_\pi)c^2\left[\frac{m_\tau + m_\pi}{2m_\tau}\right] = \left[\frac{-m_\tau^2 - m_\tau m_\pi + m_\pi m_\tau + m_\pi^2}{2m_\tau}\right]c^2$

$= \frac{(-m_\tau^2 + m_\pi^2)}{2m_\tau}c^2$

The energy of π^- is

E_{th} = Kinetic energy + rest mass energy

$= \frac{[-m_\tau^2 + m_\pi^2]c^2}{2m_\tau} + m_\tau c^2 = \frac{(m_\tau^2 + m_\pi^2)c^2}{2m_\tau}$

Chapter-5: Nuclear Force and Nuclear Scattering

● Nuclear forces:

According to Coulomb's law, the positively charged protons closely spaced within the nucleus should repel each other strongly and they should fly apart. It is therefore difficult to explain the stability of nucleus unless one assumes that nucleons are under the influence of some very strong attractive forces. The forces inside the nucleus binding neutron to neutrons, protons to protons and neutrons to protons are classified as strong interactions and are represented as n–n, p–p and n–p forces respectively.

Characteristics of nuclear forces:

(i) They are short range forces i.e. the forces between nucleons are attractive in nature when they are 0.5-25F apart and these forces are of short range having maximum value at about 2×10^{-15} m and falls off sharply with distance, becoming negligible beyond this range,

(ii) They are charge-independent i.e. They are charge-independent so that the nuclear force between a proton and neutron and are almost the same;

(iii) They are the strongest known forces in nature;

(iv) They get readily saturated by the surrounding nucleons i.e. a particular nucleon interacts with a limited number of nucleons around it and other surrounding ones remain unaffected. So, they become saturated over short distance.

(v) They are spin-dependent ie. the nuclear forces depend on the mutual orientation of spins of various nucleons and are different in parallel and antiparallel spins.

● Neutron - Proton Scattering at low energies

- Nucleus is a bound system means attractive force exists b/w neutron and proton.
  In scattering of free neutrons by protons a parallel beam of Neutron is allowed to impinge upon a target containing hydrogen atoms and no. of Neutrons dejected through various angles is determined as a function of Neutron energy.

- Since neutron have no charge, they are unaffected by the electrostatic field and their scattering will directly reflect the operation of nuclear force.

Two kinds of reactions can be involved in Neutron-proton interaction
(i) Scattering
(ii) Radiative Capture: It has low probability for high energy neutron as the cross-section for the competing radiative capture reaction decreases with $\frac{1}{2}\rightarrow$ neutron velocity

In practice protons are bound in nucleus the chemical binding energy of the proton in a molecule is about 0.1eV.
Thus for neutron energy > 1 eV proton can be assumed free. This sets lower limit the neutron energy.
If the neutron energy is less than 10 MeV, only the S-wave overlaps with the Nuclear potential and is scattered.
In the centre of mass system, the Schrodinger equation for the two body (n –p) system is

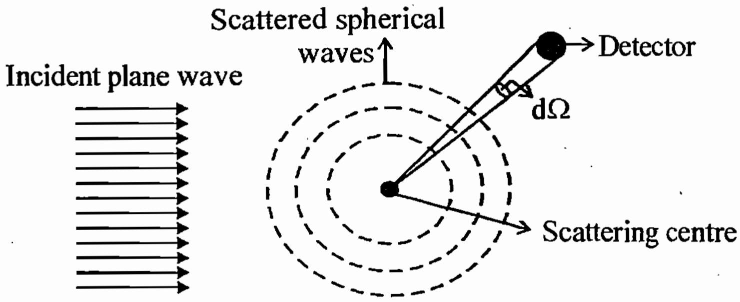
$$\nabla^2\Psi+\frac{M}{\hbar^2}\left[E-v(r)\right]\Psi=0$$

Where M = proton or Neutron mass = 2 × reduced mass of the system.
E = Incident K.E. in cm system = $\frac{1}{2}$(Incident K.E in L-coordinate) and V(r) = Inter Nucleon potential energy.
At large distances from the centre of scattering the soln of this equation is expected to be of the form

$$\psi=e^{ikz}+\frac{e^{ikr}}{r}f(\theta)$$

Where, $f(\theta)$ = scattering amplitude in the direction 'θ'

e^{ikz} = plane wave describing a beam of particles moving in z-direction



The differential cross-section

$\sigma = \int |f(\theta)|^2 d\Omega = 2\pi \int |f(\theta)|^2 \sin\theta d\theta$

Let us consider the wave equation in the absence of scattering centre [V(r) = 0 for all values of 'r'] i.e.

$$\nabla^2\Psi + \left[\frac{ME}{\hbar^2}\right]\Psi = 0$$

Soln. $\Psi = e^{ikz}$, where $k = \frac{1}{\lambda} = \frac{\sqrt{ME}}{\hbar}$

can be expanded in terms of spherical harmonics, $\Psi = e^{ikz} = e^{ikr\cos\theta} = \sum_{\ell=0}^{\infty} R_l(r) y_{l,0}(\theta)$

(where l is the integer representing the number of partial waves)

- Scattering amplitudes $f(\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)$

  where δ_ℓ = phase shift due to scattering from the potential.

- Differential cross-section $d\sigma = |f(\theta)|^2 d\Omega$
- Total cross-section

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell$$

- **Scattering length:**
- For neutrons of very low energy scattered by free protons λ is very large and hence k is very small we have $f(\theta) = \frac{e^{i\delta_0}}{k}\sin\delta_0$ and scattering cross-section $\sigma_{sc} = 4\pi\lambda^2 \sin^2\delta_0$ as $k \to 0$ δ_0 must approach zero or $f(\theta)$ would become infinite.

- For low energy neutrons $f(\theta)$ can be written as $f(\theta) \lim_{\delta_0 \to 0} \frac{e^{i\delta_0}\sin\delta_0}{k} = \frac{\delta_0}{k} = -a$

  where quantity +a is called scattering length.
- The zero energy scattering cross-section is $\sigma_0 = 4\pi a^2$ (Identical to impenetrable sphere of radius a).

● Deuteron:

Deuteron is the only two-nucleon bound system made up of a proton and a neutron. The two other possible two nucleon system the Diproton and Dineutron do not exist as bound system.

- **Experimentally Determined Properties of Deuteron**

(i) The BE of Deuteron is very small . Its value is 2.225 $\pm$ 0.003 MeV. This is very less as compared to the stable nuclei. i.e, 8MeV, hence loosely bound.

(ii) The Angular momentum quantum no . often called the nuclear spin of ground state of the Deuteron is 1. It suggest that spins are parallel (triplet state) and the orbital angular momentum of the Deuteron about their common center of mass is zero. This the ground state is ^3S state.

(iii) The sum of the magnetic dipole moments of the proton (2.79275 μ_N) and Neutron (–1.91315 μ_N), do not exactly equal to magnetic moment of Deuteron (0.85735 μ_N)

(iv) Deuteron has small but +ve Quadrupole moment suggesting that it deviates from spherical shape and has the probability of finding in the next higher state i.e., ^3D state also. Hence and state is a mixture of ^3S and ^3D state.

(v) n-p Combination, neutron is uncharged hence the force is not electrostatic, mass is less hence no gravitational, must be of nuclear origin and assumed to be central and attractive. This assumption somewhat disagrees with the experiment (central).

- Consider the example $\mu_p = 2.79281 \pm 0.00004$ nm and $\mu_n = -1.913148 \pm 0.000066$ nm

  The fact that $\mu_d + \mu_n - \mu_d \neq 0$ although small. suggests the Deuteron may not be fully described by the spherically symmetric $^3s_1$ state.

- Moreover is nuclear force if due to the exchange of Mesons, the magnetic moments of the Nucleons where in the free state may not be same as when in the nucleus. Correction to magnetic moment due to Mesonic current.
- D-state contribution can be considered by Non-Central Tensor free.

  By Assuming Central potential.

  Schrodinger equation $-\frac{\hbar^2}{2\mu}\nabla^2\psi(r) + v(r)\psi(r) = E\psi(r)$

  where reduced mass $\mu = \frac{m_p m_n}{m_p + m_n} = M/2$

  Now equation can be written as $\frac{\hbar^2}{M}\frac{d^2u(r)}{dr^2} + v(r)u(r) = Eu(r)$

  where $\psi(r) = \frac{u(r)}{r}$ and $E = -W = -2.226$ MeV = binding energy of Deuteron

- **Various types of potential**

  Square well potential $V(r) = -V_0 \quad r \le r_0$

  $= 0 \quad r > r_0$

  Exponential $V(r) = -V_0 e^{-r/r_0}$, Gaussion, $V(r) = -V_0 e^{-r^2/r_0^2}$

Yukawa $V(r) = \frac{-V_0 e^{-r/r_0}}{r/r_0}$

- **Ground state of deutron ($_1D^2$)**

Deutron is a single two nucleon (one P and One n) bound system which is found in nature. The important experimental determinal properties about deutron are given below:

(i) The B.E. of deutron/nucleon is very small compare to other nuclei i.e. it is a weakly system.

(ii) The ground state spin of deutron $I_d = 1$ (iii) The ground state parity of deutron = even (+)

(iv) The quadrupole momento of deutron $(Q_d) \neq 0$

(v) The magnetic moment of deutron is slightly different from the sum of intrinsic mag. moments of neutron + proton i.e. $(\mu_n + \mu_p) - \mu_d \to 0$

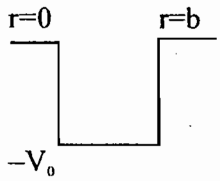
These factor represents that the ground state of Deutron is a mixture of $\underset{(L=0)}{^3S}$ & $\underset{(L=2)}{^3D}$ states in which ^3S contribution is 96% and ^3D contribution is only 4%.

This indicate that nuclear force are non-central as $L \neq \text{constant}$ and they are spin dependent. In deutron both p and n have spin parallel to each other.

- **Schrodinger wave equation for Deutron and its solution for central force (L = 0):**

As deutron is a bound system, let us consider the case of rectangular pot. well represented as

$$\begin{bmatrix} V = -V_0 & 0 < r < b \\ \quad = 0 & \text{elsewhere} \end{bmatrix}$$



The schrodinger equation is $\boxed{\nabla^2\psi + \frac{2\mu}{\hbar^2}(E - V)\psi = 0}$

$$\psi = \psi(r,\theta,\phi) = \sum_{\ell} R_\ell(r)\, Y_{\ell m}(\theta,\phi) = \sum \frac{u_\ell(r)}{r} Y_{\ell m}(\theta,\phi)$$

where reduce mass $\mu = \frac{m_n m_p}{m_n + m_p} \Rightarrow \mu \sim M/2\,(m_n \sim m_p = M)$

For $\ell = 0$. We get,

$$\boxed{k_1 \cot k_1 b = -\alpha}$$

or $$\cot k_1 b = -\frac{\alpha}{k_1} = -\frac{\sqrt{\frac{ME_d}{\hbar^2}}}{\sqrt{\frac{M}{\hbar^2}(V_0 - E_d)}} = -\sqrt{\frac{E_d}{V_0 - E_d}}$$

as E_d is very small, then an approximate solution of equation is obtained as $\cot\sqrt{\frac{M}{\hbar^2}(V_0 - E_d)}.b \to 0$

$$\Rightarrow \quad \cot\sqrt{\frac{M}{\hbar^2}}V_0.b = \cot \pi/2, \cot 3\pi/2......$$

A minimum value of V_0 can be obtained by setting $b\sqrt{\frac{M}{\hbar^2}V_{0m}} = \pi/2$

$$\Rightarrow \quad V_{0m} = \frac{\pi^2\hbar^2}{4Mb^2} \Rightarrow \boxed{V_{0m}b^2 =} \text{ constant}$$

$V_0 \sim 25$ MeV is obtained by putting b is 2×10^{-15} m

The exact solution of equation (9) is obtained by graphical method in which $V_{0m} \sim 38$ MeV

Solved Examples

Example-1: At a centre-of-mass energy of 5 MeV, the phase describing the elastic scattering of a neutron by a certain nucleus has the following values, $\delta_0 = 30°, \delta_1 = 10^0$. Assuming all other phase shifts to be negligible, plot $d\sigma/d\Omega$ as a function of scattering angle. Explicitly calculate $d\sigma/d\Omega$ at 30°, 45° and 90°. What is the total cross section σ?

Soln. The differential cross section is given $\frac{d\sigma}{d\Omega} = \frac{1}{k^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)\right|^2$

Supposing only the first and second terms are important, we have

$$\frac{d\sigma}{d\Omega} \approx \frac{1}{k^2}\left|e^{i\delta_0}\sin\delta_0 + 3e^{i\delta_1}\sin\delta_1\cos\theta\right|^2$$

$$= \frac{1}{k^2}\left[\left(\cos\delta_0\sin\delta_0 + 3\cos\delta_1\sin\delta_1\cos\theta\right) + i\left(\sin^2\delta_0 + 3\sin^2\delta_1\cos\delta\right)\right]^2$$

$$= \frac{1}{k^2}\left[\sin^2\delta_0 + 9\sin^2\delta_1\cos^2\theta + 6\sin\delta_0\sin\delta_1\cos(\delta_1-\delta_0)\cos\theta\right]^2$$

$$= \frac{1}{k^2}\left[0.25 + 0.27\cos^2\theta + 0.49\cos\theta\right]$$

where 'k' is the wave number of the incident neutron in the centre of mass frame. Assume that the mass of the nucleus is far larger than that of the neutron m_n. Then

$$k^2 \approx \frac{2m_nE}{\hbar^2} = \frac{2m_nc^2E}{(hc)^2} = \frac{2\times938\times5}{197^2\times10^{-30}} = 2.4\times10^{29}m^{-2} = 2.4\times10^{25}\ m^{-2}$$

0	0^0	30^0	45^0	90^0	180^0
$k^2\frac{d\sigma}{d\Omega}$	1	0.88	0.73	0.25	0
$\frac{d\sigma}{d\Omega}\times10^{26}(cm^2)$	4.2	3.7	3.0	1.0	0

The total cross section is:

$$\sigma = \int \frac{d\sigma}{d\Omega} = \frac{2\pi}{k^2}\int_0^{\pi}\left(0.25+0.49\cos\theta+0.27\cos^2\theta\right)\sin\theta d\theta$$

$$= \frac{4\pi}{k^2}\left(0.25+\frac{1}{3}\times 0.27\right) = 1.78\times10^{-25}\text{cm}^2 \approx 0.18\text{ barn}$$

Example–2: Neutrons of 1000 eV kinetic energy are incident on a target composed of carbon. If the inelastic cross section is 400×10–24 cm², what upper and lower limits can you place on the elastic scattering cross section?

Soln. At 1 keV kinetic energy, only s-wave scattering is involved. The phase shift δ must have a positive imaginary part for inelastic process to take place. The elastic and inelastic cross sections are respectively given by

$$\sigma_e = \pi\lambda^2\left|e^{2i\delta}-1\right|^2, \sigma_{in} = \pi\lambda^2\left(1-\left|e^{2i\delta}\right|^2\right)$$

The reduced mass of the system is $\mu = \frac{m_n m_e}{m_e+m_n} \approx \frac{12}{13}m_n$

For E = 1000 eV, $\lambda = \frac{\hbar}{\sqrt{2\mu E}} = \frac{\hbar c}{\sqrt{2\mu c^2 E}} = \frac{197}{\sqrt{2\times\frac{12}{13}\times 940\times10^{-3}}}\text{m} = 150\text{ fm}$

$$\pi\lambda = 707\times10^{-24}\text{ cm}^2$$

As, $1-\left|e^{2i\delta}\right|^2 = \frac{\sigma_{in}}{\pi\lambda^2} = \frac{400\times10^{-24}}{707\times10^{-24}} = 0.566$

We have, $\left|e^{2i\delta}\right| = \sqrt{1-0.566} = 0.659 \quad \Rightarrow e^{2i\delta} = \pm 0.659$

Hence, the elastic cross section $\sigma_e = \pi\lambda^2\left|e^{2i\delta}-1\right|^2$

has maximum and minimum values $(\sigma_e)_{max} = 707\times10^{-24}(-0.659-1)^2 = 1946\times10^{-24}\text{ cm}^2$

$$(\sigma_e)_{min} = 707\times10^{-24}(-0.659-1)^2 = 82\times10^{-24}\text{ cm}^2$$

Example-3: Disregarding nucleon spin, set a lower bound on the elastic center of mass proton-neutron forward differential cross-section.

Soln. The forward p–n differential cross section is given by

$$\left.\frac{d\sigma}{d\Omega}\right|_{0^\circ} = \left|f(0)\right|^2 \geq \left|\text{Im} f(0)\right|^2 = \left(\frac{k}{4\pi}\sigma_t\right)^2$$

where the relation between $\text{Im} f(0)$ and σ_t is given by the optical theorem. As $k = p/\hbar$ we have

$$\left.\frac{d\sigma}{d\Omega}\right|_{0^\circ} \geq \left(\frac{pc}{4\pi\hbar c}\sigma_\ell\right)^2 = \left(\frac{10^4\times40\times10^{-27}}{4\pi\times1.97\times10^{-11}}\right)^2 \text{m}^2 = 2.6\times10^{-24}\text{ cm}^2 = 2.6\text{ barn}$$

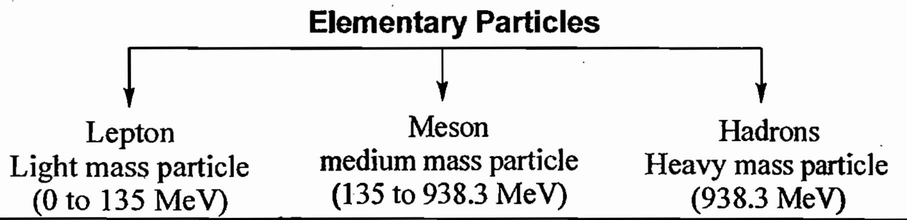
Chapter-6: Particle Physics

- **Classification of fundamental force:**
  The interaction of matter and radiation is governed by four fundamental forces
  (a) Gravitational forces (b) Electromagnetic forces
  (c) Weak forces (contact forces) (d) Strong forces.

- **The four fundamental interaction:**

Interaction	Particle affected	Range	Relative strength	Charac. time	Particle exchange	Role in universe
Strong	Quarks Hadrons	$\sim 10^{-15}$ m	1	10^{-23} sec	Gluons Mesons	Holds quark together to form nucleon. Hold nucleons together to form atomic nuclei.
Electromagnetic	Charged particles	∞	$\sim 10^{-3}$	10^{-20} sec	Photons	Determine structure of atoms, solids and liquid is important factor in astronomical universe.
Weak	Quark & leptons	$\sim 10^{-18}$m	$\sim 10^{-13}$	10^{-10} sec	Intermediate bosons	Mediates transformations of quarks & leptons, helps determine compositions of atomic nuclei
Gravitational	All	∞	10^{-39}	10^{16}	Graviton→ Not experimentally detected	Assemble matter into planets, stars and galaxies.

- **Classification on the basis of mass:**



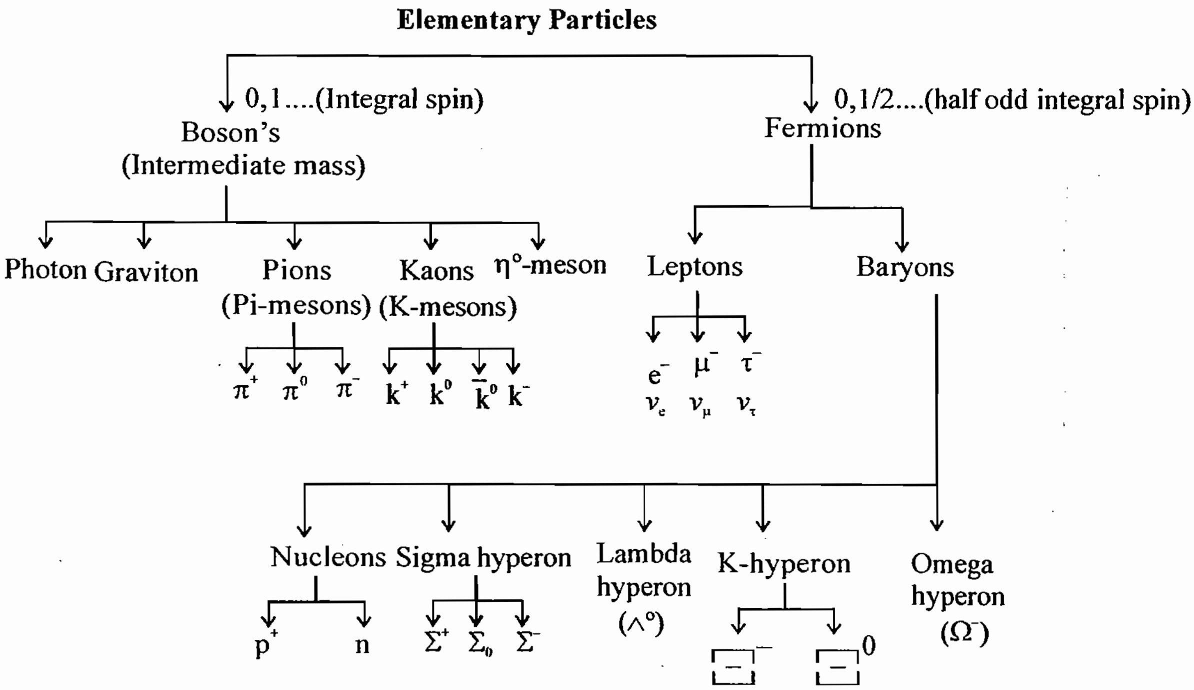
Leptons	Charge	Mass	Mean Life time
$\bar{e}$ Electron $\bar{e}$	–1	0.511 MeV	Stable
υ_e electron nutrino	0	0	Stable
μ^- Muon	–1	105.6 MeV	2.2×10^{-6} Sec
υ_μ Muon Nutrino	0	0	Stable
τ^- tauon	–1	1784 MeV	3.4×10^{-23} Sec
υ_τ tauon nutrino	0	0	Stable

and their six anti particles i.e. e^+ (positron), $\bar{\upsilon}_e, \mu^+, \bar{\upsilon}_\mu, \bar{\tau}, \bar{\upsilon}_\tau$

- **Classification on the Basis of Interactions:**
  Leptons : Those particles which do not participate in strong interaction.
  Hadrons : Those particle which take part in strong interaction.

- **Classification of elementary particles on the basis of spin:**
  (a) Bosons: Spin integral particles (Obey Bose-Einstein statistics)
  (b)Fermions: Spin half odd integral particles (Obey Fermi-Dirac statistics)

- **Classification of elementary particles:**



- **Conservation laws in elementary particle reactions:**
- **Exact conservation laws:**
  (i) Conservation of linear momentum
  (ii) Conservation of angular momentum
  (iii) Conservation of charge
  (iv) Conservation of baryon number
  (v) Conservation of lepton number

- **Approximate conservation laws:**
  (i) Isospin or isotopic spin(I):
  It arose however from the idea that pairs of particles like nucleons and triplets like pions hardly differ in their mass and may be considered as isotopes and that their charges, differing from each other by unity, suggest space qunatization similar to electron spin and orbit in a magnetic field.

- **A multiplet number** (M) is defined as the number of their different charge states. For instance, for nucleons i.e. protons or neutrons - the multiplet number M = 2. Similarly, for the triplet of pions, $M = 3$; $M = 2$ for two kaons and two antikaons; $M = 1$ for one neutral η^0.

  Now, $M = 2I + 1 \Rightarrow I = (M-1)/2$

- Isospin is treated as a vector $\vec{I}$ of magnitude $\sqrt{I(I+1)}$, like angular momentum, but I is dimensionless. Its component along Z-axis, is given by I_3 which have the allowed values.
  $I, (I-1), (I-2), \ldots\ldots\ldots\ldots -I$

- For nucleons $(M=2), I = (M-1)/2 = \frac{1}{2}$ and the values of I_3 are $+\frac{1}{2}$ and $-\frac{1}{2}$; $I_3 = +\frac{1}{2}$ is assigned to proton, $I_3 = -\frac{1}{2}$ to neutron.

- For pions $(M=3), I = (M-1)/2 = 1$. Hence, $I_3 = +1, 0, -1$. $I_3 = +1$ is assigned to π^+, $I_3 = 0$ to π^0 and $I_3 = -1$ to π^-

- Isospin is conserved in strong interactions but is violated in electromagnetic and weak interaction. The Z-component of isospin, I_3- is conserved in strong and electromagnetic interaction and not in weak interaction.

(ii) Hypercharge(Y): It is defined as double of the average charge $\bar{Q}$ of the multiplet.

$$Y = 2\bar{Q} \quad \Rightarrow \bar{Q} = Y/2 \quad \Rightarrow \boxed{Q = I_3 + Y/2}$$

- For any strong and electromagnetic interaction, the hypercharge is conserved, i.e. remains invariant. But it need not be conserved in weak interaction.

Example: $p + p \to \Lambda^0 + K^0 + p + \pi^+$

Hypercharge: $1 + 1 = 0 + 1 + 1 + 0 \quad \Rightarrow \Delta Y = 0$

(iii) Strangeness number (S): It is defined as the difference of the hyper change Y and the baryon number B.

$$S = Y - B \quad \Rightarrow Y = S + B$$

i.e., the hyperchange is the sum of the baryon number and the strangeness number

Therefore, $Q = I_3 + \frac{B+S}{2}$

- Strangeness number S is conserved in strong and electromagnetic reactions. For a weak interactions, $\Delta S = 0$ or ± 1

(iv) Parity:

When particle like neutrions are emitted during radioactive decay, they show a preferred spin direction. If a neutrino spins in the direction at which a right-handed screw advances, it is said to posses a helicity +1; if however the spin is in the direction of a left-handed screw, the helicity is -1. As the parity P is related to the spin J, the two quantum numbers are usually combined and is symbolised by J^P. So, $\left(\frac{1}{2}\right)^+$ means the J-value is $\frac{1}{2}$ and P=+1; $\left(\frac{1}{2}\right)^-$ indicates $J = \frac{1}{2}, P = -1$.

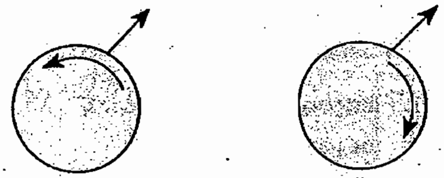


Figure: Helicity of neutrino and antineutrino

- In strong and electromagnetic interactions, parity is conserved but it is not conserved in weak interaction.

(v) Charge conjugation:

Charge conjugation means reversal of the sings of all types of charge i.e. electric, baryonic and laptonic of the particles (Figure below). If a physical law holding for particles also holds for corresponding antiparticles, the principle of charge conjugation is said to be valid.

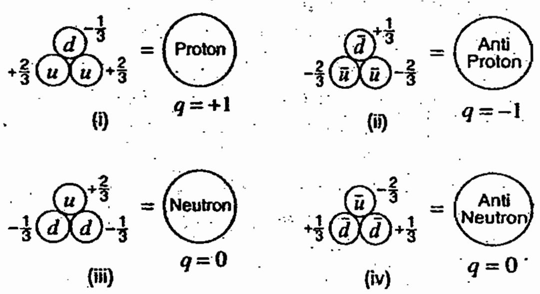


Figure: Formation of proton, antiproton and neutron, antineutron by charge conjugation

- Strong and electromagnetic interactions are charge conjugate invariant. But the weak intervation like β-decay does not obey charge conjugation.

(vi) Time reversal:

The operation T i.e., time-reversal means replacing the time 't' by –t in all equations of motion i.e., reflection of time axis at the origin of time coordinate in relativistic space-time continum. It is thus, like the parity operation, a discrete change.

- T-operation consists in reversing the signs of momenta $(\bar{p} = d\bar{r}/dt)$ and angular momenta $(\bar{L} = \bar{r} \times \bar{p})$.

T also transforms the wave function to its complex conjugate. If 'T' is conserved i.e., time-reversal invariance occurs, then the reversed equation of motion is also a valid equation of motion of the system concerned. All the known fundamental equations of motion are invariant in time-reversal.

- Strong and electromagnetic interactions are invariant under time-reversed transformation.
- **CPT Theorem:**

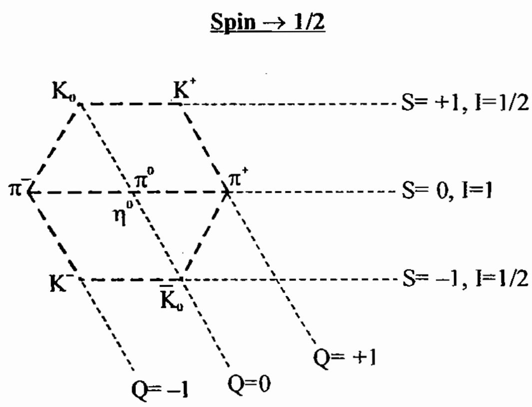
  This is an exact conservation law. It states that all interactions in nature are invariant under joint operations of charge conjugation (C), inversion of space coordinates at origin, i.e. parity (P) and reversal of time (T). The order of operations is immaterial.

  The invariance of CPT transformation implies that if any interaction is not invariant under any one of C, P and T operations, its effect gets compensated by the joint effect of the other two.

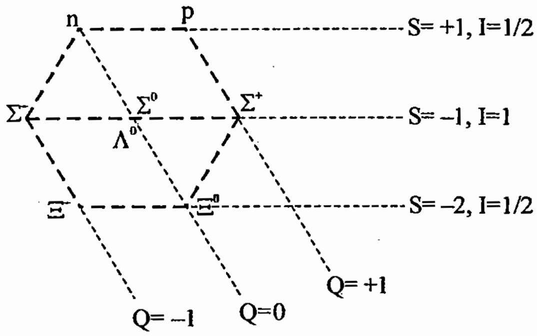
- **Conserved quantities in different type of reactions:**

Conserved Quantities	Strong	Electromagnetic	Weak
1. Charge	✓	✓	✓
2. Linear Mom.	✓	✓	✓
3. Relativistic En	✓	✓	✓
4. Spin	✓	✓	✓
5. Le	✓	✓	✓
6. Lu	✓	✓	✓
7. Li	✓	✓	✓
8. Baryon No. B	✓	✓	✓
9. I_z	✓	✓	×
10. $\vec{I}$	✓	×	×
11. S	✓	✓	×
12. $\gamma = B + S$	✓	✓	×
13. Parity (P)	✓	✓	×
14. Charge conjugation	✓	✓	×
15. Time reversion (T)	✓	✓	✓
16. CP	✓	✓	×
17. CPT	✓	✓	✓

- **Meson Octate:**



- **Baryon Octate:**



- **Relationship between particles and antiparticles:**

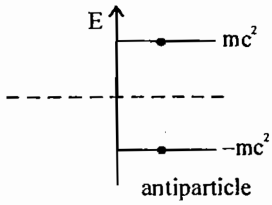
(a) Mass	→	Same
(b) Spin	→	Same
(c) Charge	→	Same but opposite in sign.
(d) Mag. moment	→	Same in magnitude but opposite in sign.
(e) Mean life time of free decay	→	Same
(f) Annihilation	→	In pairs
(g) Creation	→	In pairs
(h) Total isotopic spin	→	same
(i) Third component of isospin(I_3)	→	Same in mag. but opposite in sign.
(j) Intrinsic parity	→	Same for bosons and opposite for Fermions.
(k) Strangeness q.No. (.S)	→	Same in mag. but opposite in sign.
(l) Lepton no. (L)	→	Same in mag. but opposite in sign.
(m) Baryon no. (B)	→	Same in mag. but opposite in sign.
(n) Hypercharge (Y)	→	Same in mag. but opposite in sign.

The particle and antiparticle have same symbol except a bar over particle for antiparticle.

- **Concept of antiparticle by Dirac:**

$$E = \pm\sqrt{p^2c^2 + m_0^2c^4}$$

So, positive solution for particle are negative energy state are filled like a sea.



If electron given enough energy it can come out the filled negative state to positive energy.
The vaccent state in negative energy behaves like holes →antiparticles

- **Quark Model:** The elementary particles can be conceived (as far as isospin and hyper charge are concerned) as being built out of combination of quarks. The original three quarks were called up (Symbol *u*), down (*d*) and strange (*s*)

Each quarks has an anti-quark associated with it $(\bar{u}, \bar{d}, \bar{s})$. Since A quark and its anti-quark have opposite quantum numbers. They can be created from energy. In the reverse process, a quark and its anti-quark annihilate and give energy.

Thus, energy $\rightarrow u+u$
$d+d \rightarrow$ energy
Quark models for proton and antiproton.

Coloured Quark: Quarks and antiquarks have an additional property of some kind that can be manifested in a total of six different ways, rather as electric charge is a property that can be manifested in the two different ways that have come to be called positive and negative. In the case of quarks, this property became known as colour and its three possibilities were called red, green and blue. The antiquark colours are antired, antigreen and antiblue.
According to the colour hypothesis, all three quarks in a baryon have different colours which satisfies the exclusion principle since all are then a different states even if two or three are otherwise identical
The rules for combining colours are the following:
- A colour and its anticolour cancel out. This is called colourless or white.
- All three colours or all three anticolours in combination cancel out and give colourless
- All hadrons are colourless.

Mesons consists of a quark-antiquark pair of a particular colour and its anticolour. Baryons are made up of three quarks, one of each colour. Thus mesons and baryons are white or colourless.
Charm Bottom and OP Quarks: Besides the three quarks (*u*, *d*, *s*) three more quarks are suggested in order to have a significant analysis of the symmetries. These are charm (*c*), top (*t*) and bottom (*b*). Thus over all there are six quarks. The various characteristics of these are given in the following table:

Quark	T	T_3	B	S	Y	Q	E	B	T	*Mass (GeV)*
Up(u)	1/2	+1/2	1/3	0	1/3	2/3	0	0	0	0.39
Down(d)	1/2	−1/2	1/3	0	1/3	−1/3	0	0	0	0.39
Strange(s)	0	0	1/3	−1	−2/3	−1/3	0	0	0	0.51
Charm(c)	0	0	1/3	0	1/3	2/3	1	0	0	1.55
Top(t)	0	0	1/3	0	1/3	−1/3	0	0	0	5.4
Bottom(b)	0	0	1/3	0	1/3	2/3	0	0	1	20

- **Classification of quarks:**

	charge	spin	baryon no.	I_z	I	strangeness no.
u	$+\frac{2}{3}e$	$\frac{1}{2}$	$\frac{1}{3}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0
d	$-\frac{1}{3}e$	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0
s	$-\frac{1}{3}e$	$\frac{1}{2}$	$\frac{1}{3}$	0	0	−1

$K_0 = \bar{s}d; K^+ = \bar{s}u$; $\pi^- = d\bar{u}$, π^0 & η^0 may be $u\bar{u}$ and $d\bar{d}$

$$\pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right) \qquad \text{parity} = -1$$

$$\eta^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \qquad \text{parity} = +1$$

$\pi^+ = u\bar{d};\ K^- = s\bar{u}; \bar{K}_0 = s\bar{d}$

$n = udd; p = uud$

$\Sigma^- = sdd;\ \Sigma^+ = suu; \Xi^- = ssd;\ \Xi^0 = ssu;\ \Omega^- = sss$

λ^0 and Σ^0 may both be sud

$$\Sigma^0 = \frac{1}{\sqrt{2}}(sud + sud) \longrightarrow \text{parity} = +1$$

$$\lambda^0 = \frac{1}{\sqrt{2}}(sud - sud) \longrightarrow \text{parity} = -1$$

Pseudo Scalar Mesons : $J^P = 0^-$

Particle	I	I_3	B	S	Y	Structure
π^+	1	1	0	0	0	$u, \bar{d}$
π^0	1	0	0	0	0	$u\bar{u}, d\bar{d}, s\bar{s}$
π^-	1	−1	0	0	0	u, d
K^+	$1/2$	$+1/2$	0	1	1	$u\bar{s}$
K^0	$1/2$	$-1/2$	0	1	1	$d\bar{s}$
$\bar{K}^0$	$1/2$	$+1/2$	0	−1	−1	$\bar{d}s$
K^-	$1/2$	$-1/2$	0	−1	−1	$\bar{u}s$
n^0	0	0	0	0	0	$s\bar{s}, u\bar{u}, d\bar{d}$

Vector Mesons : $J^P = 1^-$

Particle	I	I_3	B	S	Structure
ρ^+	1	1	0	0	$u\bar{d}$
ρ^0	1	0	0	0	$u\bar{u}, d\bar{d}, s\bar{s}$
ρ^-	1	−1	0	0	$\bar{u}d$
K^+	$1/2$	$+1/2$	0	+1	$u\bar{s}$
K^0	$1/2$	$-1/2$	0	+1	$d\bar{s}$
$\bar{K}^0$	$1/2$	$+1/2$	0	−1	$\bar{d}s$
K^-	$1/2$	$-1/2$	0	−1	$\bar{u}s$
ω^0	0	0	0	0	$u\bar{u}, d\bar{d}, s\bar{s}$

Baryons :

Particle	Q	I	I_3	B	S	Y	Structure
p	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	1	uud
n^0	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	1	udd
Λ^o	0	0	0	1	-1	0	uds
Σ^+	+1	1	+1	1	-1	0	uus
Σ^0	0	1	0	1	-1	0	uds
Σ^-	-1	1	-1	1	-1	0	dds
Ξ^0	0	$\frac{1}{2}$	$\frac{1}{2}$	1	-2	-1	uss
Ξ^-	-1	$\frac{1}{2}$	$-\frac{1}{2}$	1	-2	-1	dss

Baryon Resonances :

Particle	I	I_3	B	S	Y	Structure
Δ^{++}	$\frac{3}{2}$	$\frac{3}{2}$	1	0	1	uuu
Δ^+	$\frac{3}{2}$	$\frac{1}{2}$	1	0	1	uud
Δ^0	$\frac{3}{2}$	$-\frac{1}{2}$	1	0	1	udd
Δ^-	$\frac{3}{2}$	$-\frac{3}{2}$	1	-1	0	uus
Σ^{*+}	1	1	1	-1	0	uds
Σ^{*0}	1	0	1	-1	0	dds
Σ^{*-1}	1	-1	1	-1	0	dds
Ξ^{*0}	$\frac{1}{2}$	$\frac{1}{2}$	1	-2	-1	uss
Ξ^{*-}	$\frac{1}{2}$	$-\frac{1}{2}$	1	-2	-1	dss
Ω^-	0	0	1	-3	-2	sss

Solved Examples

Example-1: If in the following reaction, the incident kaon has a kinetic energy of 1.63 GeV, calculate the total energy to be divided between the four recoiling particles.

$$\bar{K}^- + p^+ \rightarrow \Sigma^- + \pi^+ + \pi^- + \pi^+$$

The mass-energy of $\pi -$mesons are 139.6 MeV, $\Sigma^- = 1197.3\,\text{MeV}$, proton = 938.3 MeV and $\bar{K}^- = 493.8\,\text{MeV}$

Soln. Energy of particles taken together are:

$$\bar{K}^- = 0.4938\ \text{GeV} \quad (\because\ 1\ \text{GeV} = 10^3\ \text{MeV})$$
$$p^+ = 0.9383\ \text{GeV}$$
$$E_{\bar{K}^-} = 1.63\ \text{GeV}$$
$$\Rightarrow \quad \text{Total, } E = 3.0621\ \text{GeV}$$

Energy of the four recoiling particles are:

$$\Sigma^- = 1.1973\ \text{GeV}$$
$$\pi^+ = 0.1396\ \text{GeV}$$
$$\pi^- = 0.1396\ \text{GeV}$$
$$\pi^+ = 0.1396\ \text{GeV}$$
$$\text{Total energy} \quad = 1.6161\ \text{GeV}$$

Therefore, excess energy = 3.0621 – 1.6161 = 1.446 GeV
Therefore, average energy per particle = 1.446/4 = 0.3615 GeV = 361.5 MeV

Example-2: If a pion decays from rest to give a muon of 4.0 MeV energy, what is the kinetic energy of the accompanying neutrino? What is the mass of the neutrino in the process?

Soln. The decay mode of pion is given by, $\pi^- \rightarrow \pi^+ + v_\mu + E$

Therefore, energy, $E = (m_\pi - m_\mu) \times c^2$ (neutrino has zero 'rest' mass)

$$= (273m_e - 207m_e) \times c^2 = 66m_e \times c^2 = 66 \times 0.51\ \text{MeV}$$
$$= 33.7\ \text{MeV}$$

Since meon takes 4.0 MeV of energy, the kinetic energy of the accompanying neutrino is

$$(33.7 - 4.0)\ \text{MeV} = 29.7\ \text{MeV}$$

Therefore, mass of neturino = $(29.7/0.51)m_e = 58.23\ m_e$.

Example-3: Find the value of third component of isotopic spin of Ξ^- in the following strong interaction:

$$\pi^+ + n \rightarrow \Xi^- + K^+ + K^+$$

Soln. The interaction is $\pi^+ + n \rightarrow \Xi^- + K^+ + K^+$

$$I : 1 + \frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$
$$I_3 : +1 - \frac{1}{2} \rightarrow I_3 + \frac{1}{2} + \frac{1}{2}$$

Therefore, I_3 for $\Xi^- = -\frac{1}{2}$

Example-4: Identify the type of the following interaction from conservation laws:

$$\Sigma^0 \rightarrow \Lambda^0 + \gamma \quad (\text{life time} \leq 10^{-14}\ \text{s})$$

Soln. We have $\Sigma^0 \rightarrow \Lambda^0 + \gamma$

Charge, Q : $0 \rightarrow 0 + 0 \quad \Rightarrow \Delta Q = 0$

Baryon number, B : $+1 \to +1+0 \quad \Rightarrow \Delta B=0$

Lepton number, L : $0 \to 0+0 \quad \Rightarrow \Delta L = 0$

Strangeness no. S : $-1 \to -1+0 \quad \Rightarrow \Delta S=0$

Hypercharge, Y : $0 \to 0+0 \quad \Rightarrow Y=0$

Since the strangeness number is conserved, the interaction is either a strong interaction or an electromagnetic one. Its half-life is $\leq 10^{-14}$ s which points to the fact that it cannot be a strong interaction, but is a weak decay. As S is conserved, it cannot be a weak interaction. So, it is an electromagnetic interaction and a $\gamma-$photon is produced.

Example-5: Identify the unknown particle in the reactions given below, using the conservation laws.

(i) $\mu^- + p \to {}_0^1n + ...$ (ii) $\pi^- + p \to K^0 + ...$

Soln. (i) The given reaction is : $\pi^- + p \to {}_0^1n + ...$

The unknown particle must be zero charge and mass, spin $\frac{1}{2}$ and lepton number 1 as they are conserved. Since the interacting particle is π^- meson, the unknown particle is identified as mu-neutrino, ν_μ

(ii) The reaction is $\pi^- + p \to K^0 + ...$

For charge conservation, Q : $-1+1 \to 0+Q \quad \Rightarrow Q=0$

Conservation of baryon no. B : $0+1 \to 0+Q \quad \Rightarrow B=+1$

Strangeness conservation, S : $0+0 \to -1+S \quad \Rightarrow S=+1$

Third component of isospin, I_3: $-1+\frac{1}{2} \to -\frac{1}{2}+I_3 \quad \Rightarrow I_3=0$

Therefore, the unknown particle has charge zero, baryon number +1, strangeness number +1 and third component of isospin 0. So the particle could be Λ^0 or Σ^0.

Example-6: Check if the following reactions are allowed or forbidden.

(i) $\pi^- + p \to \Lambda^0 + \pi^0$ (ii) $p + p^- \to 2\pi^+ + 2\pi^- + 2\pi^0$

Soln. (i) The reaction is : $\pi^- + p \to \Lambda^0 + \pi^0$

Q: $-1+1 \to 0+0 \quad \Rightarrow \Delta Q = 0 \quad$ Q $\to$ conserved

B: $0+1 \to +1+0 \quad \Rightarrow \Delta B = 0 \quad$ B $\to$ conserved

S: $0+1 \to -1+0 \quad \Rightarrow \Delta S \neq 0 \quad$ S $\to$ not conserved

It is a strong interaction where the charge and baryon number are conserved. But since the strangeness number is not conserved, the reaction is forbidden.

(ii) The reaction is : $p + p^- \to 2\pi^+ + 2\pi^- + 2\pi^0$

Q: $+1-1 \to 2-2+0; \quad \Delta Q = 0$

B: $+1-1 \to 0+0+0; \quad \Delta B = 0$

S: $0+0 \to 0+0+0; \quad \Delta S = 0$

Y: $+1-1 \to 0+0+0; \quad \Delta Y = 0$

So, the above reaction is an allowed reaction.

Example-7: An ultra-relativistic proton moves in a magnetic field. Can it radiate π^+, π^- and π^0, electrons and positrons?

Soln. If the energy of the proton is sufficiently large, it can radiate π^0 and π^+ mesons, and also positrons. The reactions are:

$$p \to p + \pi^0; \quad p \to n + \pi^+; \quad p \to n + e^+\nu$$

But π^- mesons and electrons cannot be radiated.

Example-8: Allocate the Isospin to the strange particles from following spins.

(a) $\pi^- + p \to \pi^0 + K^0$ **(b)** $p + p \to \pi^0 + K^+ + p$

(c) $\pi^+ + n \to \pi^0 + K^+$ **(d)** $\pi^- + p \to \Sigma^- + K^+$

(e) $\pi^+ + p \to \Sigma^+ + K^+$ **(f)** $\pi^+ + n \to \Xi^- + K^+ + K^+$

Soln.

(a)

	π^- +	p →	π^0 +	K^0
I_3	−1	½	0	−½

(b)

	p +	p →	π^0 +	K^+ +	p
I_3	½	½	0	½	½

(c)

	π^+ +	n →	π^0 +	K^+
I_3	+1	−½	0	½

(d)

	π^- +	p →	Σ^- +	K^+
I_3	−1	½	−1	½

(e)

	π^+ +	p →	Σ^+ +	K^+
I_3	1	½	+1	½

(f)

	π^+ +	n →	Ξ^- +	K^+ +	K^+
I_3	1	−½	−½	½	½

Example-9: Which of the following reactions are allowed and forbidden under the conservation of strangeness, conservation of baryon number and conservation of charge.

$$\pi^+ + n \to \Lambda^0 + K^+$$
$$\to K^0 + K^+$$
$$\to \bar{K}^0 + \Sigma^+$$
$$\to \pi^- + p$$

$$\pi^- + p \to \Lambda^0 + K^0$$
$$\to \pi^0 + \Lambda^0$$

Soln. **(i)**

	π^+ +	n →	Λ^0 +	K^+	
Q	1	0	0	1	$\Delta Q = 0$
B	0	1	1	0	$\Delta B = 0$
S	0	0	−1	1	$\Delta S = 0$

Allowed

(ii)

	π^+ +	n →	K^0 +	K^+	
Q	1	0	0	1	$\Delta Q = 0$
B	0	1	0	0	$\Delta B \neq 0$
S	0	0	1	1	$\Delta S \neq 0$

Not allowed

(iii)

	π^+ +	n →	$\bar{K}^0$ +	Σ^+	
Q	1	0	0	1	$\Delta Q = 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	−1	−1	$\Delta S = 2$

Not allowed

(iv)

	π^+ +	n →	π^- +	p	
Q	1	0	−1	1	$\Delta Q \neq 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	0	0	$\Delta S = 0$

Not allowed.

(v)

	π^- +	p →	π^0 +	Λ^0	
Q	−1	1	0	0	$\Delta Q = 0$
B	0	1	1	0	$\Delta B = 0$
S	0	0	−1	1	$\Delta S = 0$

Allowed.

(vi)

	π^- +	p →	π^0 +	Λ^0	
Q	−1	1	0	0	$\Delta Q = 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	0	−1	$\Delta S \neq 0$

Not allowed.

Example-10: Calculate the energy of the neutron produced when a slow negative pion is captured by a proton. Should neutron be treated relativistically?

Soln. $\pi^- + p^+ \to n^0 + \gamma + Q$

$139 + 938 \to 939 + h\nu + Q$ or $Q = 138\,MeV = E_\gamma + E_n$

From conservation of momentum $m_n v_n = E_\gamma / c$

and $\frac{E_n}{E_\gamma} = \frac{1}{2}\frac{m_n v_n^2}{c m_n v_n} = \frac{1}{2}\frac{v_n}{c} = \frac{1}{2}\frac{E_\gamma}{m_n c^2} = \frac{E\gamma}{1878}$

or $\frac{E_n}{E_\gamma + E_n} = \frac{E_\gamma}{1878 + E_\gamma}$ $\therefore \frac{E_n}{138} = \frac{E_\gamma}{1878 + E_\gamma}$

and $\frac{E_n}{138} = \frac{138 - E_n}{1878 + 138 - E_n} = \frac{138 - E_n}{2016 - E_n}$

or $E_n = 9MeV$

Using relativistic relations

$m_n v_n = m_{0n} v_n / (1-\beta^2)^{1/2} = E_\gamma / c$ and $E_n = m_{0n}c^2[(1-\beta^2)^{-1/2} - 1]$

We get $E_n = 8.8MeV$

Example-11: Classify the following processes in terms of the type of interaction.

Soln: $\pi^- + p \to \Lambda^0 + K^0$; $\pi^- + p \to \pi^0 + n$; $p + \gamma \to p + \pi^0$; $\Sigma^0 \to \Lambda^0 + \gamma$; $\pi^0 \to \gamma + \gamma$; $K^0 \to \pi^+ + \pi^-$; $\Lambda^0 \to p + \pi^-$; $\Xi^- \to \Lambda^0 + \pi^-$; and $\Lambda^0 \to p + e^- + \bar{\nu}$.

In the reactions $\pi^- + p \to \Lambda^0 + K^0$ and $\pi^- + p \to \pi^0 + n$, the interaction is a short-range force between hadrons (π, p, n, Λ and K) corresponding to one pion-exchange. These reactions obey selection rules $\Delta B = 0, \Delta Q = 0, \Delta Y = 0, \Delta\pi = 0, \Delta T = 0, \Delta T_3 = 0$

hence the interaction is strong.

In the reactions $p + \gamma \to \pi^0 + p$, $\Sigma^0 \to \Lambda^0 + \gamma$ and $\pi^0 \to \gamma + \gamma$, the interaction is electromagnetic, as the electromagnetic force arises due to the mechanical effect of the emission and absorption of virtual photons. These reactions obey all above mentioned selection rules except the rule for conservation of isospin T, hence, the interaction in these processes is electromagnetic.

In the reactions $K^0 \to \pi^+ + \pi^-$, $\Lambda^0 \to p + \pi^-$, $\Xi^- \to \Lambda^0 + \pi^-$ and $\Lambda^0 \to p + e^- + \bar{\nu}$, the interaction is weak, as the weak interaction is due to the leptonic decay of strange and non-strange particles and due to non-leptonic decay of strange particles. These reactions do not conserve parity, strangeness, isospin and the third component of isospin, hence the interaction in these processes is weak.

Example-12: What are the possible values of isotopic spin for the following systems? (a) $A\pi^+$ meson and an antiproton, (b) two neutrons, (c) $a\pi^+$ meson and $a\Lambda^c$, (d) $a\pi^+$ and $a\pi^0$ meson, (e) au and $a\bar{u}$ quark, (f) a c, b and an s quark.

Soln. $I(u)=\frac{1}{2}=I(d)$ while $I(s)=I(c)=I(b)=I(t)=0$

(a) Using the composition law for isospin, we get

$I(\pi^+\bar{p})=\frac{3}{2},\frac{1}{2}, \quad I_3(\pi^+\bar{p})=I_3(\pi^+)+I_3(\bar{p})=1-\frac{1}{2}=\frac{1}{2}$

Since $I_3=\frac{1}{2}$ can be a legitimate isospin projection for both $I=\frac{3}{2}$ and $I=\frac{1}{2}$, we conclude that the composite state $(\pi^+\bar{p})$ can have either $I=\frac{3}{2}$ or $I=\frac{1}{2}$.

(b) A composite state of two neutrons can have $I(nn)=1,0$

$I_3(nn)=I_3(n)+I_3(n)=-\frac{1}{2}-\frac{1}{2}=-1.$

Since $I_3=-1$ cannot be an isospin projection of a state with $I=0$, we conclude that a composite state of two neutrons can only be $I=1$.

(c) The Λ^0 particle is an isosinglet ($I=0$) and, consequently, we have

$I(\pi^+\Lambda^0)=1$

This composite state is therefore $I=1$.

(d) We can have $I(\pi^+\pi^0)=2,1,0$

$I_3(\pi^+\pi^0)=I_3(\pi^+)+I_3(\pi^0)=1+0=1$

A state with two π mesons can have $I=2$, $I=1$ or $I=0$. However, $I=0$ has no projection of $I_3=1$, and the composite state $(\pi^+\pi^0)$ must therefore have $I=2$ or $I=1$.

(e) For a composite system of a u and a $\bar{u}$ quark, we have

$I(u\bar{u})=1,0$

$I_3(u\bar{u})=I_3(u)+I_3(\bar{u})=\frac{1}{2}-\frac{1}{2}=0$

Since the projection $I_3=0$ is possible for both $I=1$, 0 states, we conclude that the composite state $(u\bar{u})$ can have $I=1$ or $I=0$.

(f) All quarks with flavor quantum numbers are isosinglets. Thus, we have

$I(bcs)=0$

This composite state would therefore be a baryon with unique isospin $I=0$.