

# Raster Scan Graphics & Transformations

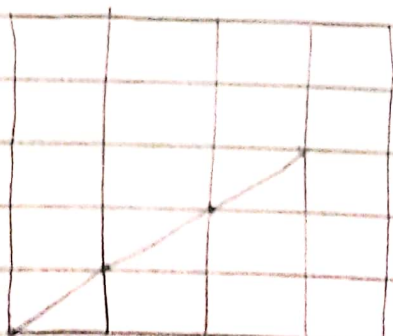
Pixel means discrete set of bright.

\* Line drawing algorithm :

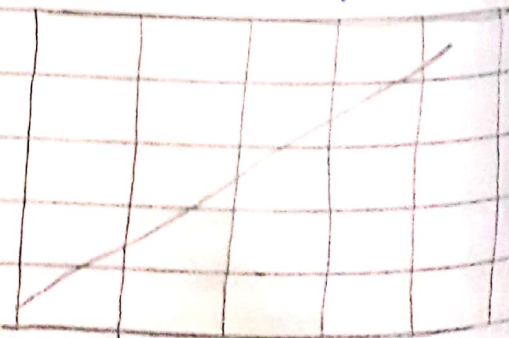
The cathode ray tube is a raster display. It considers a matrix of discrete finite areas (pixel) each of which can be lit or not. It is not possible to directly draw a straight line from one point to another. The process of determining which pixels provide the best approximation to the desired line is known as rasterization.

When combined with this process of generating picture in scan like order it is known as scan conversion.

For horizontal & vertical  $45^\circ$  lines the choice of raster element is obvious for any other the choice is more difficult to show in following fig:



Rasterization Scan Conversion



Rasterization of straight line.

It is useful to consider the general requirement for such i.e., what are the desirable characteristics for these line and that they start.

The primary design criteria are that straight line appear as a straight line & that they start at end accurately.

Display line should have constant brightness along their length, independent of the line length and orient orientation. finally the line should drawn rapidly.

As with most designed criteria not all can be completely satisfied.

The very nature of a raster display precludes the generation of completely straight line except for special case. It is possible for align to precisely being of end at satisfied locations.

Additional design consideration for line drawing algorithm include end points order i.e., rasterizing point  $P_2P_1$  which ends the same result at rasterizing the point  $P_1P_2$ . End point shape i.e., controlling the shape of end of the rasterizing of line.

Eg: Triangle, circle, polygon.



# \* Digital Differential Analyzer. [DDA]

The line end points are  $(x_1, y_1)$  &  $(x_2, y_2)$  assume not equal.

Integer is the functions.

Note that many integer functions are float fun<sup>ct</sup> i.e, integer  $(-8.5 = -9)$  rather than  $(-8)$  the algorithm assume this is the case.

Sign returns  $-1, 0, 1$  for arguments ~~less~~  $< 0, = 0, > 0$  resp. approx. the line length.

Algorithm :

if  $\text{abs}(x_2 - x_1) \geq \text{abs}(y_2 - y_1)$  then

length =  $\text{abs}(x_2 - x_1)$

else

length =  $\text{abs}(y_2 - y_1)$

end if

select the larger of  $\Delta x$  or  $\Delta y$ .

$\Delta x = (x_2 - x_1) / \text{length}$

$\Delta y = (y_2 - y_1) / \text{length}$

$x = x + 0.5$

$y = y + 0.5$

begin main loop

$i = 1$

while ( $i \leq \text{length}$ )

set pixel ( $\text{int } x, \text{int } y$ )

$x = x + \Delta x$

$y = y + \Delta y$

$i = i + 1$

end while

finish

One technique for obtaining a rasterized straight line is to solve the governing differential eq<sup>n</sup> for a straight line i.e.,

$$\frac{dy}{dx} = \text{constant.}$$

$$(or) \quad dy = \frac{y_2 - y_1}{x_2 - x_1} dx$$

The solution of the finite difference approximation

$$y_{i+1} = y_i + \Delta y$$

$$y_{i+1} = y_i + \frac{y_2 - y_1}{x_2 - x_1} \Delta x. \quad \text{--- (2-1)}$$

Where,  $x_1, y_1$  &  $x_2, y_2$  are the end points of the req. straight line &  $y_i$  is the initial value for any given step along the line.

In fact, in the eq<sup>n</sup> (2-1) represent the recursion related for successive values of  $y$  along the req. line.

If its use to rasterize line it is called digital differentiation analyzer.

For a simple DDA either  $\Delta x$  or  $\Delta y$  whichever is larger is chosen as 1 raster unit.

Eg: Consider a line from 0,0 to 5,5.

$$x_1 = 0$$

$$y_1 = 0$$

$$x_2 = 5$$

$$y_2 = 5$$

$$\therefore \text{Length} = 5.$$



$$\Delta x = \frac{5}{5} = 1.$$

$$\Delta y = \frac{5}{5} = 1.$$

i	setpixel	x	y
1	(0,0)	0.5	0.5
2	(1,1)	1.5	1.5
3	(2,2)	2.5	2.5
4	(3,3)	3.5	3.5
5	(4,4)	4.5	4.5

The result are shown in above fig.

Note that the selected pixels are equally spaced along the line. The appearance of the line is quite accepted.

The end point at (0,0) is apparently exact.

The pixel corresponding to the end point at (5,5) is not accurated.

The lines appears to be too short.

Eg: 0,0 to -8,-4.

$$x_1 = 0 \quad x_2 = -8$$

$$y_1 = 0 \quad y_2 = -4.$$

$$\text{length} = -8.$$

$$\Delta x = \frac{-8}{-8} = 1$$

$$\Delta y = \frac{-4}{-8} = 0.5$$

i	set pixel	x	y
1	(0,0)	-0.5 <del>+0.5</del>	+0.5 0
2	(-1,0)	-1.5	-0.5
3	(-2,-1)	-2.5	-1.0
4	(-3,-1)	-3.5	-1.5
5	(-4,-2)	-4.5	-2.0
6	(-5,-2)	-5.5	-2.5
7	(-6,-3)	-6.5	-3.0
8	(-6,-3)	-7.5	-3.5

30-12-19

→ \* Bresenham's Line rasterization Algorithm for 1st octant.

The line end points are  $x_1, y_1$  &  $x_2, y_2$ . assume <sup>not</sup> equal  
of  $x, y, \Delta x, \Delta y$  are assume integer &  $e$  is real.

Initialize variable

$$x = x_1$$

$$y = y_1$$

$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$

$$m = \Delta y / \Delta x$$

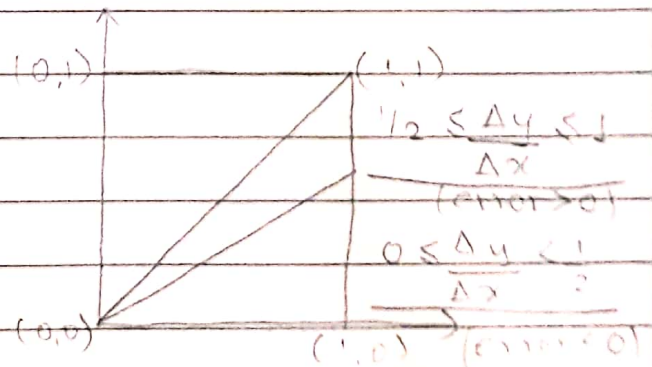
$$e = m - 1/2 \text{ (initialize to compensate for a non zero intercept).}$$

for  $i = 1$  to  $\Delta x$

set pixel  $(x, y)$

while  $(e > 0)$

$$y = y + 1$$





$e = e - 1$   
end while

$x = x + 1$

$e = e + m$

next i

finish

eg: 1)  $e = 1/4 - 1$       $\frac{1}{4} - \frac{4}{4} \Rightarrow -\frac{3}{4}$

2)  $e = -3/4 + 3/8 = \frac{-6}{8} + \frac{3}{8} = \frac{-3}{8}$

3)  $e = -1/2 + 3/8 = \frac{-4}{8} + \frac{3}{8} = \frac{-1}{8}$

4)  $e = -1/8 + 3/8 = \frac{2}{8} = \frac{1}{4}$

5)  $e = \frac{1}{4} - \frac{3}{2} = \frac{1}{4} - \frac{6}{4} = \frac{-5}{4}$

6)  $e = \frac{-1}{6} + \frac{2}{3} = \frac{-1}{6} + \frac{4}{6} = \frac{3}{6} = \frac{1}{2}$

Eg:  $x=0$  ;  $y=0$  ,  $\Delta x=5$  ,  $\Delta y=5$  ,  $m=1$ .

$$\therefore e = m - \frac{1}{2} = 1 - \frac{1}{2} = \frac{1}{2}$$

Eg:  $(0,0)$   $(-8,-4)$ .

$$x=0 \quad ; \quad y=0$$

$$\Delta x = -8 \quad ; \quad \Delta y = -4$$

$$m = \frac{\Delta y}{\Delta x} = \frac{-4}{-8} = \frac{1}{2}$$

$$e = \frac{1}{2} - \frac{1}{2} = 0$$

- It is originally developed by with digital plotters, this algorithm is equally suited for use with CRT raster device.
- The algorithm seek to select the optimum raster locations that represent a straight line.
- To accomplish this the algorithm always increment by 1 unit either  $x$  or  $y$  depending on the slope of the line.
- The increment in the other variable either 0 or 1 it is determined by examining the distance between actual lines & nearest grid locations.
- This distance is called error.
- The algorithm is cleverly constructed so that only the sign of error term need to be examine.



This is illustrated in above fig. for the line in first octant, i.e., for the line with slope between 0 & 1.

In the above fig. note that if the slope of the required line through (0,0) is greater than  $\frac{1}{2}$  then its intercept with the line  $x=1$  is closer to the line  $y=1$  than the line  $y=0$ .

The raster points (1,1) better represent the path of the line than at (1,0).

If the slope is ( $< \frac{1}{2}$ ) the opp. is true.

For a slope of precisely half there is no clear choice.

Here the algorithm chooses the raster point (1,0).

31-12-19

Not all lines pass precisely through a raster point. Where a line of a slope  $\frac{3}{8}$  initially passes through the raster point at (0,0) & subsequently crosses 3 pixels. Also illustrated is the calculation of the error in representing the line by discrete pixel. It is desirable to check only the error term, it is initialize to  $-\frac{1}{2}$ .

If the slope of the line is greater than or equal to half its value & at the next raster point by 1 unit i.e., 1,0.

$$e = e + m$$

↳ slope

$$e = -1/2 + 3/8 = \frac{-4}{8} + \frac{3}{8} = \frac{-1}{8}$$

$e$  is negative, therefore the line passes below the middle of the pixel.

Pixel at the same horizontal level better approx. the location of the line of  $y$  is not increment.

Again incrementing the error term by the slope is

$$e = \frac{-1}{8} + \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$$

At the next raster point is  $(2, 0)$  here.

$e$  is positive, which shows that the line passes above the mid point.

The raster element at the next higher vertical location  $(2, 1)$  better approx. the position of the line, hence  $y$  is increment by 1 unit.

Before considering the next pixel it is necessary to reinitialize the error term.

This is accomplished by subtracting 1 from it.

$$e = \frac{1}{4} - 1 = \frac{1}{4} - \frac{4}{4} = \frac{1-4}{4} = \frac{-3}{4}$$

Note that the intercept of vertical line of  $x=2$  of the desire line is  $-1$  with respect to the line  $y=1$ . Reinitialize to  $-\frac{1}{2}$  related to 0 for the error term ends as yields as  $-\frac{3}{4}$ .

Continue the next raster unit

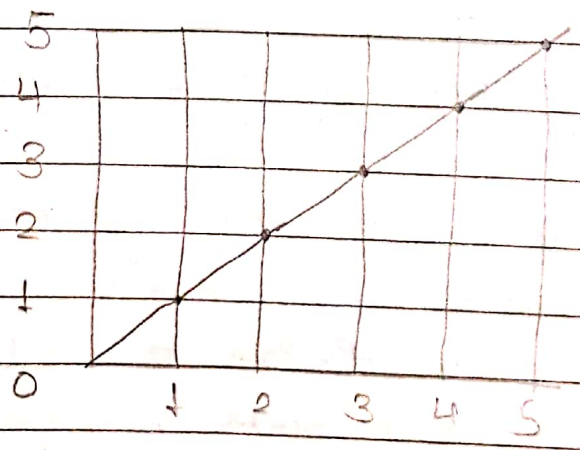
$$e = \frac{-3}{4} + \frac{3}{8} = \frac{-6}{8} + \frac{3}{8} = \frac{-3}{8}$$

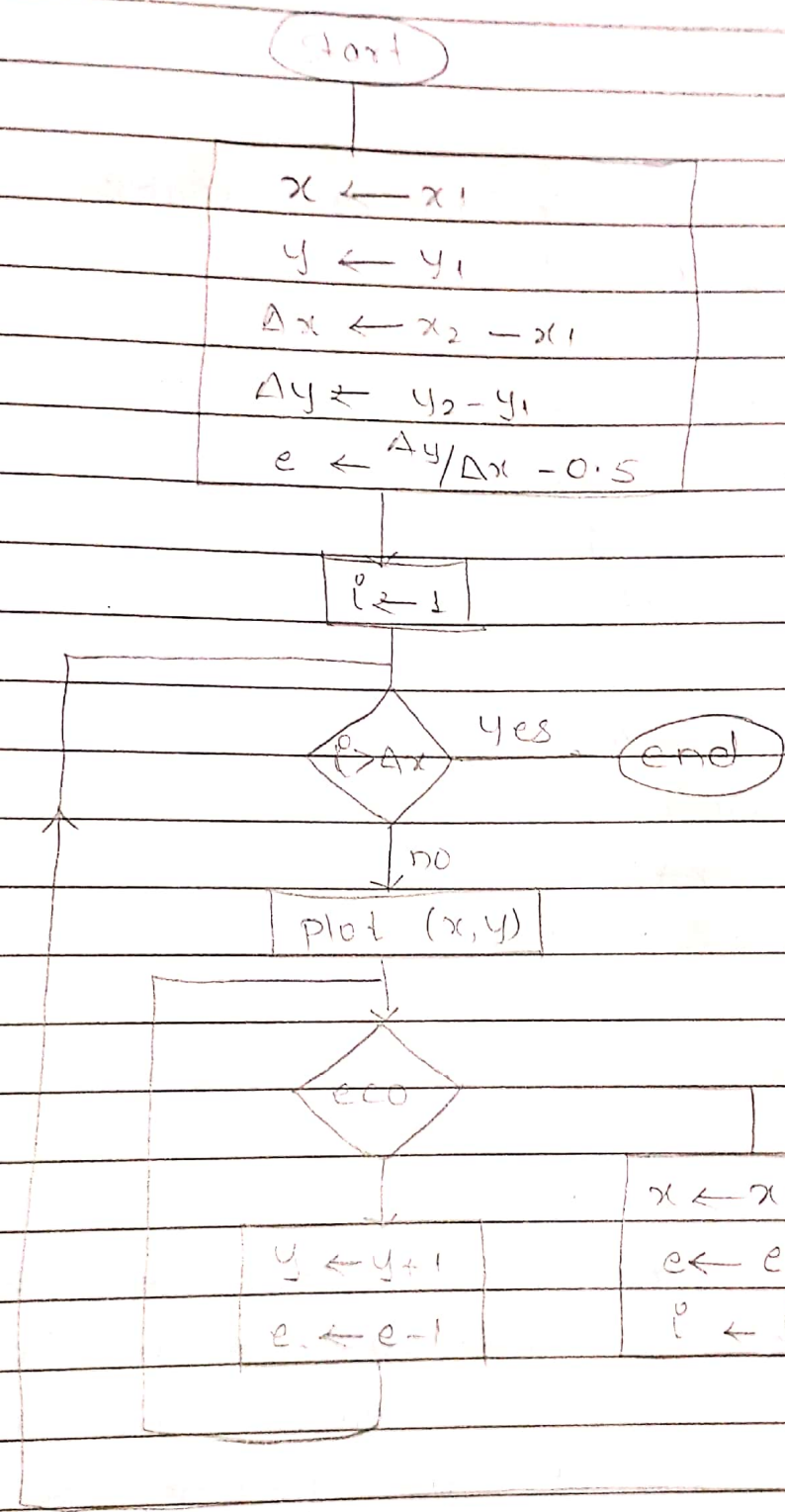


$e$  is negative, the  $y$  value is not incremented.  
It is easy to see that error term is major of the  $y$  intercept of the desired line of each raster element reference to  $-\frac{1}{2}$ .

Eg.  $x=0$   $y=0$   $\Delta x=5$   $\Delta y=5$   $m=1$   $e=1-\frac{1}{2}=\frac{1}{2}$

i	set pixel	e	x	y
1	(0,0)	$\left\{ \begin{array}{l} 1/2 \\ -1/2 \end{array} \right.$	0	0
<del>2</del>	<del>(0,1)</del>		0	1
2	(1,0)	$\left\{ \begin{array}{l} 1/2 \\ -1/2 \end{array} \right.$	1	1
3	(2,2)	$\left\{ \begin{array}{l} 1/2 \\ -1/2 \end{array} \right.$	2	2
4	(3,3)	$\left\{ \begin{array}{l} 1/2 \\ -1/2 \end{array} \right.$	3	3
5	(4,4)	$\left\{ \begin{array}{l} 1/2 \\ -1/2 \end{array} \right.$	4	4





01-01-2020

# Integer Bresenham Integer Algorithm for 1st octant

$(x_1, y_1), (x_2, y_2)$  assume are not equal.



Initialize variables

$$x = x_1$$

$$y = y_1$$

$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$

$$\bar{e} = 2 * \Delta y - \Delta x \quad \} \text{ initialize } \bar{e} \text{ to compensate}$$

for  $i = 1$  to  $\Delta x$  } begin loop

Set pixel  $(x, y)$

while  $(\bar{e} > 0)$

$$y = y + 1$$

$$\bar{e} = \bar{e} - 2 * \Delta x$$

end while

$$x = x + 1$$

$$\bar{e} = \bar{e} + 2 * \Delta y$$

next  $i$

finish

This algorithm is presented above requires use of floating point arithmetic to calculate the slope of the line & to evaluate the error term.

The speed of the algorithm is increased by using integer arithmetic. Because only the sign of error term is important, the simple transformation

$$\bar{e} = 2e\Delta x \quad \text{of the error term, \&}$$

In the previous algorithm, yields of integer algorithm this allows algorithm to be efficiently incremented

In hardware or firmware.

The integer algorithm for 1st octant i.e.,  
 $0 \leq \Delta y \leq \Delta x$ .

	Increment x by -1		Increment y by 1
Increment y by 1	$x = x - 1$ $y = y + 1$	$x = x + 1$ $y = y + 1$	Increment x by 1
Increment x by -1	$x = x - 1$ $y = y - 1$	$x = x + 1$ $y = y - 1$	Increment x by 1
Increment y by -1			Increment y by -1

Condition for general Bresenham Algorithm.

Bresenham

\* Generalize ^ Integer Algorithm for All quadrants

All various variables assume integer.

The sign function  $(-1, 0, 1)$  as its argument  
 $< 0, = 0$  &  $> 0$ .

Initialize variables

$$x = x_1$$

$$y = y_1$$

$$\Delta x = \text{abs}(x_2 - x_1)$$

$$\Delta y = \text{abs}(y_2 - y_1)$$

$$s_1 = \text{sign}(x_2 - x_1)$$

$$s_2 = \text{sign}(y_2 - y_1)$$



Interchange  $\Delta x$  &  $\Delta y$  depending on slope of line

If  $\Delta y > \Delta x$  then,

$$10 = \Delta x \quad |$$

$$\Delta x = \Delta y$$

$$\Delta y = 10.$$

Interchange = 1 else interchange = 0  
end if

$\bar{e} = 2 * \Delta y - \Delta x$  (initialize error term to  
compensate non-zero intercept)

$i = 1$  to  $\Delta x$

set pixel  $(x, y)$

while  $(\bar{e} > 0)$

if interchange = 1 then  $x = x + S_1$

else

$$y = y + S_2$$

end if

$$\bar{e} = \bar{e} - 2 * \Delta x$$

end while

If interchange = 1 then

$$y = y + S_2$$

else

$$x = x + S_1$$

end if

$$\bar{e} = \bar{e} + 2 * \Delta y$$

next  $i$

finish.

eg:  $(x_1, y_1) = (0, 0)$   $(x_2, y_2) = (-8, -4)$ .

$$x = 0$$

$$y = 0$$

$$\Delta x = \text{abs}(x_2 - x_1)$$

$$= \text{abs}(-8 - 0)$$

$$= -8$$

$$\Delta y = \text{abs}(y_2 - y_1)$$

$$= \text{abs}(-4 - 0)$$

$$= -4$$

$$S_1 = \text{sign}(x_2 - x_1)$$

$$= \text{sign}(-8 - 0)$$

$$= -8$$

$$S_2 = \text{sign}(y_2 - y_1)$$

$$= \text{sign}(-4 - 0)$$

$$= -4$$

### \* Assignment :

1. What is computer graphics & explain advantages of CG.
2. Explain applications of CG.
3. Explain CRT in details.
4. Explain colour CRT monitors.
5. Explain direct view storage tube in details.
6. Explain line drawing algorithm.
7. Write an algorithm for DDA
8. Write an algorithm for line Brekenham's line rasterization algorithm.
9. — II — for Bresenham's integer algorithm.
10. — II — Generalize Bresenham Algorithm.